Chapter 2: STATICS—A REVIEW

Solution 2.1

The free-body diagram of the disk is shown in Figure S2.1.

Note: For force balance (equilibrium) the reaction R must pass through O since the other two forces (T and mg) intersect at O. Hence, even though we did not neglect friction, the frictional force at B is zero for this system.

Equilibrium Equations:

$$\rightarrow \sum F_x = 0: \quad T\sin\theta - R = 0 \quad \Rightarrow \quad R = T\sin\theta \qquad \text{(i)}$$

$$\uparrow \sum F_y = 0: \quad T\cos\theta - mg = 0 \quad \Rightarrow \quad T = \frac{mg}{\cos\theta} \qquad \text{(ii)}$$

Note: The moment balance is satisfied in the given configuration. Hence, no new equation is generated through this condition.

Substitute (ii) in (1): $R = mg\tan\theta$ (iii)

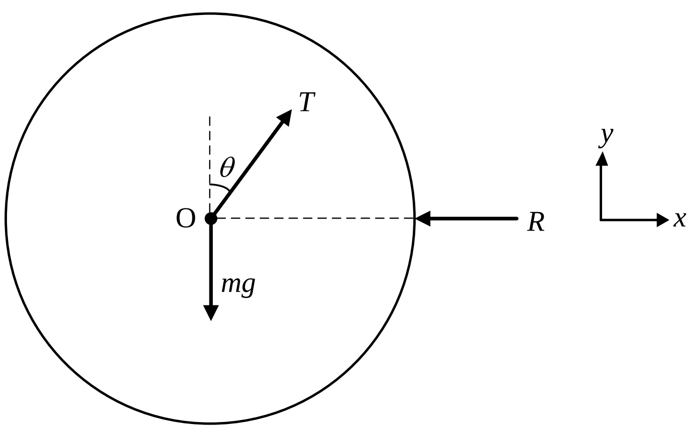


Figure S2.1: Free-body diagram of the disk.

From geometry (right-angled triangle AOB):

$$\sin\theta = \frac{r}{L} = \frac{300}{500} = 0.6; \; \cos\theta = \frac{400}{500} = 0.8; \; \tan\theta = \frac{300}{400} = 0.75;$$

Substitute numerical values.

(ii): $T = \dfrac{20\times 9.81}{0.8} = 245\,\text{N}$

(iii): $R = 20\times 9.81\times 0.75 = 147\,\text{N}$

Solution 2.2

The forces acting at point C of the traffic light are shown in Figure S2.2.

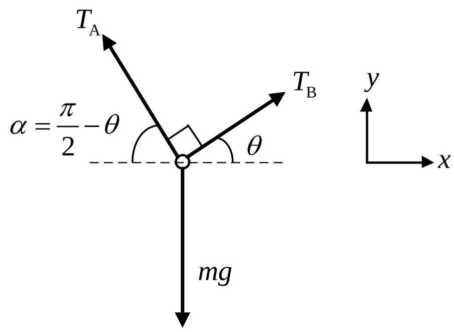


Figure S2.2: Forces at point C of the traffic light.

From geometry, ACB is a right-angled triangle with $\alpha = \frac{\pi}{2} - \theta$.

Hence,

$$\sin\theta = \frac{3}{5} = 0.6;\ \cos\theta = \frac{4}{5} = 0.8;\ \tan\theta = \frac{3}{4} = 0.75$$

Equilibrium Equations:

$$\rightarrow \sum F_x = 0: \quad -T_A \sin\theta + T_B \cos\theta = 0 \quad \Rightarrow \quad -T_A \tan\theta + T_B = 0 \qquad \text{(i)}$$

$$\uparrow \sum F_y = 0: \quad T_A \cos\theta + T_B \sin\theta - mg = 0 \quad \Rightarrow \quad \frac{T_A}{\tan\theta} + T_B = \frac{mg}{\sin\theta} \qquad \text{(ii)}$$

$$\text{(ii)} - \text{(i)}: \quad T_A(\tan\theta + \frac{1}{\tan\theta}) = \frac{mg}{\sin\theta} \qquad \text{(iii)}$$

Substitute numerical values:

$$\text{(iii)}: \ T_A(\frac{1}{0.75} + 0.75) = \frac{50 \times 9.81}{0.6} \quad \Rightarrow \quad T_A = 392\,\text{N}$$

$$\text{(i)}: \ T_B = T_A \tan\theta = 392 \times 0.75 = 294\,\text{N}$$

Solution 2.3

Free-body diagram of the disk with the handle is shown in Figure S2.3.

Equilibrium Equations:

$$\rightarrow \sum F_x = 0: \quad -R_1 + F + P\sin\theta = 0 \qquad \text{(i)}$$

$$\uparrow \sum F_y = 0: \quad R - mg - P\cos\theta = 0 \quad \Rightarrow \quad R = mg + P\cos\theta \qquad \text{(ii)}$$

where,

R_1 = reaction at B (normal because frictionless)

R = normal reaction at A

F = frictional resistance force at A

$$\circlearrowleft \sum M_O = 0: \quad F \times r - P \times L = 0 \quad \Rightarrow \quad F = \frac{L}{r}P \qquad \text{(iii)}$$

Substitute (iii) in (i):

$$R_1 = F + P\sin\theta = \frac{L}{r}P + P\sin\theta \qquad \text{(iv)}$$

Substitute numerical values:

(iii): $F = \frac{1.0}{0.5} \times 200 = 400.0\,\text{N}$

(ii): $R = 50 \times 9.81 + 200 \cos 30^\circ = 663.7\,\text{N}$

(iv): $R_1 = \frac{1.0}{0.5} \times 200 + 200 \sin 30^\circ = 500.0\,\text{N}$

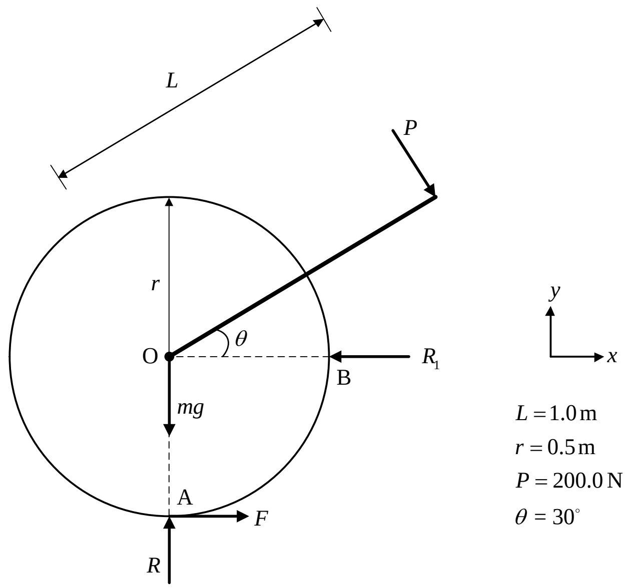


Figure S2.3: Free-body diagram of disk with handle.

Solution 2.4

Free-body diagram of the shaft with the pulley is shown in Figure S2.4.

Note: Ball bearings do not exert moments or axial forces on the shaft.

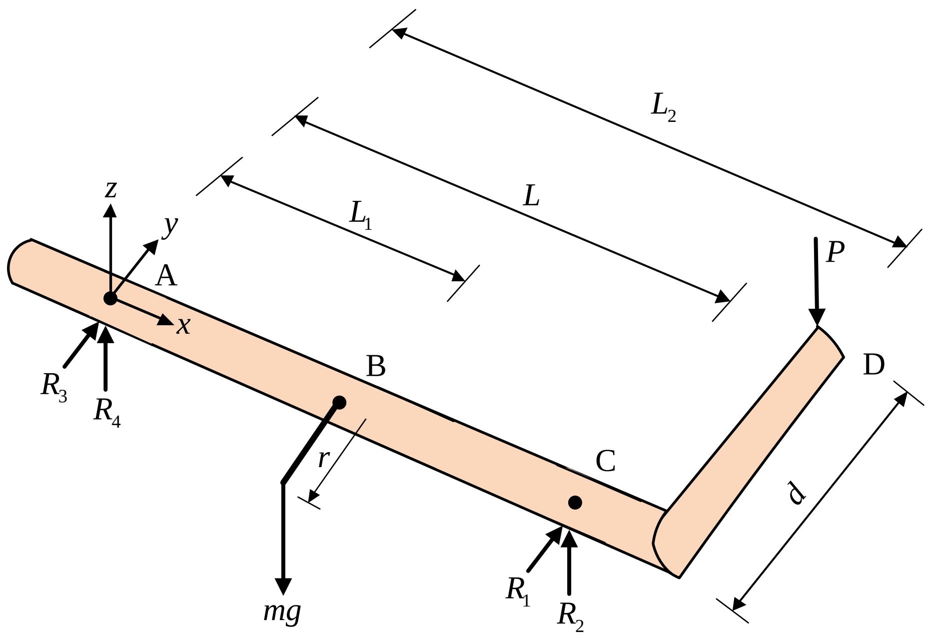


Figure S2.4 Free-body diagram of the shaft with pulley.

Equilibrium Equations:
Note the Cartesian coordinate system (x, y, z) located at A.

$$\sum M_x = 0: \; mg \times r - P \times d = 0 \;\Rightarrow\; P = \frac{r}{d} mg = \frac{0.2}{0.5} \times 20 \times 9.81 = 78.5\,\text{N}$$

$$\sum M_z = 0: \; R_1 \times L = 0 \;\Rightarrow\; R_1 = 0$$

$$\sum F_y = 0: \; R_3 + R_1 = 0 \;\Rightarrow\; R_3 = 0$$

$$\sum M_y = 0: \; mg \times L_1 - R_2 \times L + P \times L_2 = 0$$

$$\Rightarrow\; R_2 = \frac{mgL_1 + PL_2}{L} = \frac{20 \times 9.81 \times 0.6 + 78.5 \times 1.2}{1.0}\,\text{N} = 211.9\,\text{N}$$

$$\sum F_z = 0: \; R_4 - mg + R_2 - P = 0$$

$$\Rightarrow\; R_4 = mg + P - R_2 = 20 \times 9.81 + 78.5 - 211.9 = 62.8\,\text{N}$$

Solution 2.5

The free-body diagram of the structure is shown in Figure S2.5. Note the directions of x and y as given by the Cartesian coordinate frame.

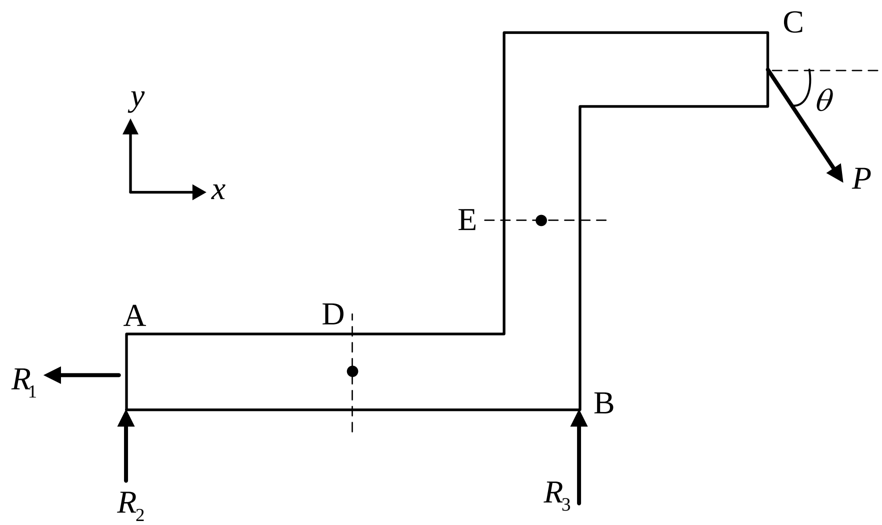


Figure S2.5: Free-body diagram of the structure.

Note: A smooth roller support cannot exert a lateral force on the structure
R_1 = horizontal component of the reaction at A
R_2 = vertical component of the reaction at A
R_3 = reaction at B (vertical, because of smooth roller)

(a)
Equilibrium Equations:

$$\rightarrow \sum F_x = 0: \; -R_1 + P\cos\theta = 0 \;\;\Rightarrow\;\; R_1 = P\cos\theta = 200\times\cos 60^\circ = 100\,\text{N}$$

$$\uparrow \sum F_y = 0: \; R_2 + R_3 - P\sin\theta = 0 \;\;\Rightarrow\;\; R_2 + R_3 = P\sin\theta \qquad \text{(i)}$$

$$\circlearrowleft \sum M_A = 0: \quad R_3\times L - P\cos\theta\times h - P\sin\theta\times(L+\frac{L}{2}) = 0$$

$$\Rightarrow R_3 = \frac{h}{L}P\cos\theta + 1.5P\sin\theta = \frac{0.8}{1.0}\times200\times\frac{1}{2} + 1.5\times200\times\frac{\sqrt{3}}{2}\,\text{N} = 340\,\text{N}$$

Substitute in (i):

$$R_2 = P\sin\theta - R_3 = P\sin\theta - (\frac{h}{L}P\cos\theta + 1.5P\sin\theta) = -0.5P\sin\theta - \frac{h}{L}P\cos\theta \qquad \text{(ii)}$$

$$\Rightarrow \quad R_2 = -0.5\times200\times\frac{\sqrt{3}}{2} - \frac{0.8}{1.0}\times200\times\frac{1}{2} = -167\,\text{N}$$

(b)
Make a virtual X-section at D and consider the resulting free body AD. Its free-body diagram is shown in Figure S2.5 (a).

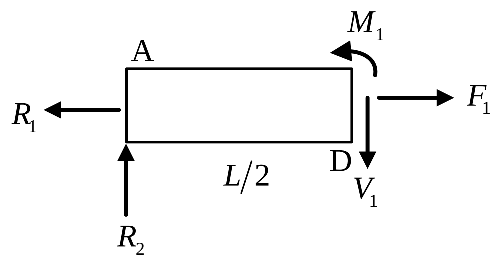


Figure S2.5(a): Free-body diagram of AD.

Note: The shear force V_1 and the bending moment M_1 at the X-section D are marked in their +ve directions by convention. F_1 is marked as +ve when in tension.

Equilibrium Equations:

$$\rightarrow \sum F_x = 0:\ -R_1 + F_1 = 0 \quad \Rightarrow \quad F_1 = R_1 = P\cos\theta = 100\,\text{N}$$

$$\uparrow \sum F_y = 0:\ R_2 - V_1 = 0 \quad \Rightarrow \quad V_1 = R_2$$

(ii):

$$V_1 = -0.5P\sin\theta - \frac{h}{L}P\cos\theta = -167\,\text{N}$$

$$\curvearrowleft \sum M_\text{D} = 0: \quad M_1 - R_2 \times \frac{L}{2} = 0 \quad \Rightarrow \quad M_1 = \frac{R_2 L}{2}$$

$$\text{(ii)} \Rightarrow \quad M_1 = (-0.25P\sin\theta - \frac{h}{2L}P\cos\theta)\times L = -167\times\frac{1.0}{2}\,\text{N}\cdot\text{m} = -83.5\,\text{N}\cdot\text{m}$$

Next, make a virtual X-section at E and consider the resulting free body EC. Its free-body diagram is shown in Figure S2.5(b).

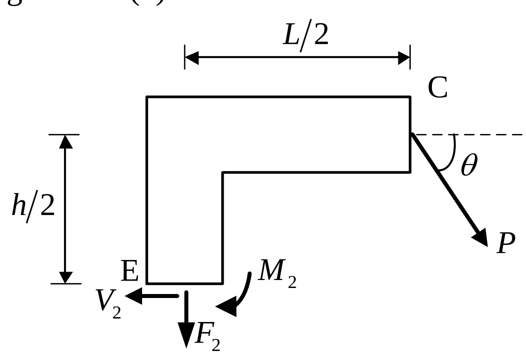


Figure S2.5(b): Free-body diagram of EC.

Note: The shear force V_2 and the bending moment M_2 at E are marked in their +ve directions, by convention. F_2 is marked as +ve when in tension.

Equilibrium Equations:

$$\rightarrow \sum F_x = 0:\ -V_2 + P\cos\theta = 0 \quad \Rightarrow \quad V_2 = P\cos\theta = 200\times\frac{1}{2} = 100\,\text{N}$$

$$\uparrow \sum F_y = 0:\ -F_2 - P\sin\theta = 0 \quad \Rightarrow \quad F_2 = -P\sin\theta = -200\times\frac{\sqrt{3}}{2} = -173\,\text{N}$$

(i.e., it is in compression)

$$\circlearrowleft \sum M_E = 0: \quad -M_2 - P\cos\theta\times\frac{h}{2} - P\sin\theta\times\frac{L}{2} = 0$$

$$\Rightarrow M_2 = -\frac{P}{2}(h\cos\theta + L\sin\theta) = -\frac{200}{2}\times(0.8\times\frac{1}{2} + 1.0\times\frac{\sqrt{3}}{2})\text{N}\cdot\text{m} = -127\,\text{N}\cdot\text{m}$$

Solution 2.6

It is seen that both members are two-force members. Assume that the members are in tension (by convention).

From geometry: $cos\theta = \frac{1}{2} \rightarrow \theta = 60°\ ;\ sin\theta = \sqrt{3}/2$

Case (a):

Consider Joint C (Figure S2.6(a)):

$\rightarrow \sum F_x = 0 \quad \rightarrow \quad -F_{BC} - F_{AC}cos\theta = 0$

$\rightarrow \quad F_{BC} = -\frac{1}{2}F_{AC}$

$\uparrow \sum F_y = 0 \quad \rightarrow \quad F_{AC}sin\theta - P = 0$

$\rightarrow \quad F_{AC} = \frac{P}{sin\theta} = \frac{20}{\sqrt{3}/2} = \frac{40}{\sqrt{3}}\text{kN}$ (tension)

$\rightarrow \quad F_{BC} = -\frac{20}{\sqrt{3}}\text{kN}$ (compression)

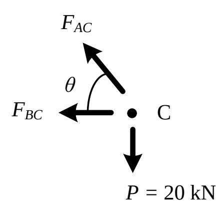


Figure S2.6(a): Forces at joint C.

Consider Joint B (Figure S2.6(b)):

$\rightarrow \sum F_x = 0 \quad \rightarrow \quad -R_B + F_{BC} = 0 \quad \rightarrow \quad R_B = F_{BC} = -\frac{20}{\sqrt{3}}\text{ kN}$

(i.e., acting to the right)

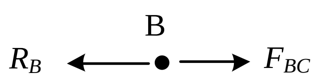


Figure S2.6(b): Forces at joint B.

Consider Joint A (Figure S2.6(c)):

$$R_A = F_{AC} = \frac{40}{\sqrt{3}} \text{ kN}$$

(acting upward at 60° from left horizontal)

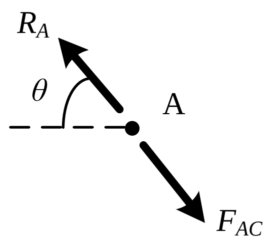


Figure S2.6(c): Forces at joint A.

Case (b):

Joint C (Figure S2.6(d)):
Clearly (by inspection; No need to write equations)

$$F_{AC} = 0$$
$$F_{BC} = P = 20 \text{ kN} \qquad \text{(i.e., in tension)}$$

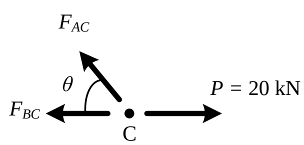


Figure S2.6(d): Forces at joint C.

Joint B(Figure S2.6(e)):

$$R_B = F_{BC} = 20 \text{ kN} \quad \text{(i.e., acting to the left)}$$

Also, $\quad R_A = 0$

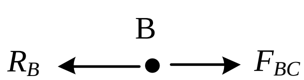


Figure S2.6(d): Forces at joint B.

Solution 2.7
Load P = 200g = 200×9.81 N = 1962.0 N
From geometry: $\theta = 22.5^{\circ}$; $AC = CD = \sqrt{2}$ m

Free-body diagram of the structure of interest is shown in Figure S2.7(a).
There are no moments at A and B because they are pin joints (frictionless).

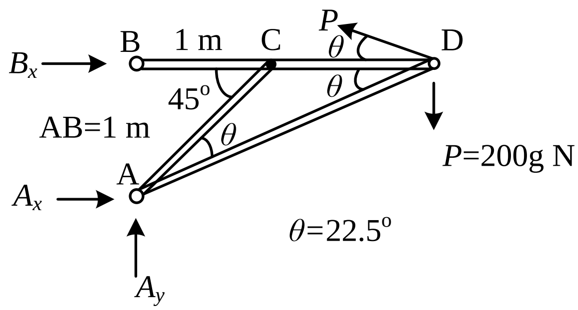


Figure S2.7(a): Free-body diagram of the boom.

Note: BC is a two-force member ==> Reaction at B should be along BC, given by B_x. This will be further confirmed below, by analyzing joint B.

Equilibrium Equations for Boom:

$$\rightarrow \sum F_x = 0 \;\Rightarrow\; A_x + B_x - P\cos\theta = 0 \qquad \text{(i)}$$

$$\uparrow \sum F_y = 0 \;\Rightarrow\; A_y - P + P\sin\theta = 0 \qquad \text{(ii)}$$

$$\circlearrowleft \sum M_A = 0 \;\Rightarrow\; -B_x \times 1 - P\times(1+\sqrt{2}) + P\sin\theta\times(1+\sqrt{2}) + P\cos\theta\times 1 = 0 \qquad \text{(iii)}$$

We get:

(ii): $A_y = P(1-\sin\theta) = 1962.0(1-\sin 22.5^\circ) = 1211.2\text{ N}$

(iii): $B_x = -P(1-\sin\theta)(1+\sqrt{2}) = -1962.0(1-\sin 22.5^\circ)(1+\sqrt{2}) + 1962\cos 22.5^\circ$
$= -1111.1\text{ N}$

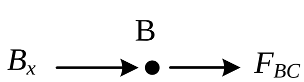
(i): $A_x = P\cos\theta - B_x = 1962.0\cos 22.5^\circ + 1111.1\text{ N} = 2923.7\text{ N}$

Note: Instead of writing (iii) above, we could have taken moments about B, and obtained an equation for A_x. Then (i) would give B_x.

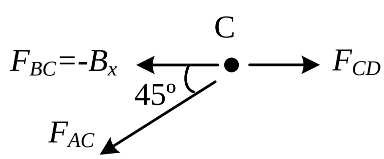
Method of Joints:
To determine the loads in the remaining rods of the truss, we use the method of joints.

Sign Convention: Rods are in tension.

(b)



(c)



(d)

(e)

Figure S2.7: Forces at: (b) joint B; (c) joint C; (d) joint C; (e) joint A.

Joint B (Figure S2.7(b)):

$\rightarrow \sum F_x = 0 \quad \Rightarrow \quad B_x + F_{BC} = 0 \quad \Rightarrow$

Load in BC = F_{BC} = -B_x = 2924.0 N (tension)

Joint C (Figure S2.7(c)):

$\uparrow \sum F_y = 0 \quad \Rightarrow \quad -F_{AC} \sin 45^\circ = 0 \quad \Rightarrow \quad F_{AC} = 0$

$\rightarrow \sum F_x = 0 \quad \Rightarrow \quad -F_{BC} + F_{CD} = 0 \quad \Rightarrow \quad F_{CD} = F_{BC} = 2924.0 \text{ N}$

Joint D (Figure S2.7(d)):

$\uparrow \sum F_y = 0 \quad \Rightarrow \quad -F_{AD} \sin\theta + P \sin\theta - P = 0$

$$\Rightarrow F_{AD} = -\frac{P(1-\sin\theta)}{\sin\theta} = -\frac{1962.0 \times (1-\sin 22.5^\circ)}{\sin 22.5^\circ} = -3165.0 \text{ N}$$

Note: This means, the member AD is in compression (which should be intuitive).

The same result may be obtained by considering Joint A.

Joint A (Figure S2.7(e)):

$\uparrow \sum F_y = 0 \quad \Rightarrow \quad A_y + F_{AD} \sin\theta = 0$

$$\Rightarrow F_{AD} = -\frac{(1-\sin\theta)P}{\sin\theta} = -3165.0 \text{ N}$$

Solution 2.8

Free-body diagrams of the components of the pipe gripper are shown in Figure S2.8.

(a)