1.7 Plate with width change, Fig. A.11(c).

$P = 3600\ N$, $w_2 = 24$, $w_1 = 16$, $t = 5\ mm$

Polycarbonate, $\sigma_o = 62\ MPa$, $\varepsilon_f = 110$ to 150%

$X_1 = ?$ adequate?

$$S = \frac{P}{w_1 t} = \frac{3600\ N}{16(5)\ mm^2} = 45\ MPa$$

$$X_1 = \frac{\sigma_o}{S} = \frac{62\ MPa}{45\ MPa} = 1.38 \quad ◀$$

The value is a bit low but may be suitable under ideal circumstances. Note that the material is quite ductile. ◀

3.15 Cantilever beam, circular cross sec.

$$v_{max} = \frac{PL^3}{3EI}, \quad I = \frac{\pi r^4}{4} \qquad \text{(Figs. A.4, A.2)}$$

Requirements: L, P, v_{max}

Geometry: r Material: ρ, E

Minimize: (a) $m = \pi r^2 L \rho$

(b) cost, $C_m m$

$$v_{max} = \frac{PL^3}{3E}\frac{4}{\pi r^4}, \quad r^2 = \left(\frac{4PL^3}{3\pi E v_{max}}\right)^{0.5}$$

$$m = \pi L \rho \left(\frac{4PL^3}{3\pi E v_{max}}\right)^{0.5} = f_1(\text{Req.})\, f_2(\text{Mat'l.})$$

$$m = \left[2L^{2.5}\left(\frac{\pi P}{3 v_{max}}\right)^{0.5}\right]\left[\frac{\rho}{\sqrt{E}}\right] = f_1 f_2$$

For the Table 3.13 materials, use the properties given to calculate:

(a) $f_2 = \rho/\sqrt{E}$, (b) $f_2 = C_m \rho/\sqrt{E}$

(a)

Material	Modulus E, GPa	Density ρ, g/cm^3	Mass f_2 $\rho/E^{0.5}$	Mass Rank
1020 steel	203	7.9	0.554	7
4340 steel	207	7.9	0.549	6
7075 Al	71	2.7	0.320	3
Ti-6-4	117	4.5	0.416	4
PC	2.4	1.2	0.775	8
Pine	12.3	0.51	0.145	1
GFRP	21	2.0	0.436	5
CFRP	76	1.6	0.184	2

(3.15, p.2)

Pine has the lowest mass, and CFRP the second lowest.

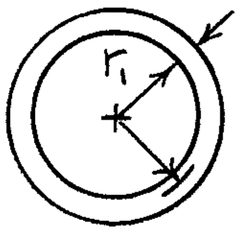
(b)

Material	Rel Cost C_m	Cost f_2 $C_m \rho / E^{0.5}$	Cost Rank
1020 steel	1	0.554	2
4340 steel	3	1.647	3
7075 Al	6	1.923	4
Ti-6-4	45	18.721	7
PC	5	3.873	5
Pine	1.5	0.218	1
GFRP	10	4.364	6
CFRP	200	36.707	8

Pine also has the lowest cost, but now 1020 steel is second.

(c) If pine is suitable, it is the clear choice. If not, then 7075 Al or 4340 steel might be reasonable.

3.17 Column with a tubular section.



$t = 0.2 r_1$

$r_{avg} = r_1 + t/2$

$r_{avg} = 1.1 r_1$

From Fig. A.2(c):

$A = 2\pi r_{avg} t$

$I \approx \pi r_{avg}^3 t$

$$A = 2\pi (1.1 r_1)(0.2 r_1) = 0.44 \pi r_1^2$$

$$I = \pi (1.1 r_1)^3 (0.2 r_1) = 0.2662 \pi r_1^4$$

Requirements: L, P, $X = P_{cr}/P$

Geometry: r_1 Material: ρ, E, (C_m)

Minimize: m, (cost)

$$P_{cr} = \frac{\pi^2 E I}{L^2}, \quad m = AL\rho = 0.44\pi r_1^2 L \rho$$

$$P_{cr} = XP = \frac{\pi^2 E (0.2662 \pi r_1^4)}{L^2}$$

$$r_1^2 = \left(\frac{XPL^2}{0.2662 \pi^3 E}\right)^{0.5}$$

$$m = 0.44 \pi L \rho \left(\frac{XPL^2}{0.2662 \pi^3 E}\right)^{0.5} = f_1 f_2$$

$$m = \left[0.8528 L^2 \left(\frac{XP}{\pi}\right)^{0.5}\right] \left[\frac{\rho}{E^{0.5}}\right]$$

Minimize: $f_2 = \dfrac{\rho}{E^{0.5}}$, $\dfrac{C_m \rho}{E^{0.5}}$

(3.17, p.2)

Material	Modulus E, GPa	Density ρ, g/cm^3	Rel Cost C_m	Mass f_2 $\rho/E^{0.5}$	Mass Rank	Cost f_2 $C_m\rho/E^{0.5}$	Cost Rank
1020 steel	203	7.9	1	0.554	7	0.554	2
4340 steel	207	7.9	3	0.549	6	1.647	3
7075 Al	71	2.7	6	0.320	3	1.923	4
Ti-6-4	117	4.5	45	0.416	4	18.721	7
PC	2.4	1.2	5	0.775	8	3.873	5
Pine	12.3	0.51	1.5	0.145	1	0.218	1
GFRP	21	2	10	0.436	5	4.364	6
CFRP	76	1.6	200	0.184	2	36.707	8

(a) For the space station, light weight is paramount, and the cost of the material unimportant. CFRP is the best choice. Pine may have difficulty with planes of weakness in the material that can be overcome in CFRF by laminating or winding the fibers such that there is no weak plane. ◀

(b) Pine is a good choice, as cost is now important. It is not conveniently made into a tube, but a box section would work. If rot due to moisture or the size of the column is a problem use 1020 steel, as weight does not matter in the garage case. ◀

3.19 Leaf spring as simple beam.

$L = 0.5\,m$, $t = 60\,mm$, $h = 5\,mm$, P at ctr.

Made from low-alloy (assume 4340) steel.

Required: $h \le 12\,mm$, $k = P/v = 50\,kN/m$

at $v_{max} = 30\,mm$, $X = 1.4$

(a) For $k = 50\,kN/m$, which Table 3.13 materials give lighter weight?

$$v = \frac{PL^3}{48EI}, \quad I = \frac{th^3}{12} \qquad \text{(Figs. A.4, A.2)}$$

Requirements: $k = P/v = 50\,kN/m$

Geometry: h Material: ρ, E, (C_m)

Minimize: m, (cost)

$$k = \frac{P}{v} = \frac{48EI}{L^3} = \frac{4Eth^3}{L^3}, \quad h = L\left(\frac{k}{4Et}\right)^{1/3}$$

$$m = thL\rho = tL^2\left(\frac{k}{4Et}\right)^{1/3}\rho$$

$$m = \left[tL^2\left(\frac{k}{4t}\right)^{1/3}\right]\left[\frac{\rho}{E^{1/3}}\right] = f_1 f_2$$

Minimize $f_2 = \dfrac{\rho}{E^{1/3}}, \ \dfrac{C_m \rho}{E^{1/3}}$

From the table (next page) all but 1020 steel would give a lighter component, but Ti-6-4 and CFRP would be very expensive. ◀

(3.19, p. 2)

Material	Modulus E, GPa	Density ρ, g/cm^3	Rel Cost C_m	Mass f_2 $\rho/E^{1/3}$	Mass Rank	Cost f_2 $C_m\rho/E^{1/3}$	Cost Rank
1020 steel	203	7.9	1	1.344	8	1.34	2
4340 steel	207	7.9	3	1.335	7	4.01	4
7075 Al	71	2.7	6	0.652	3	3.91	3
Ti-6-4	117	4.5	45	0.920	6	41.40	7
PC	2.4	1.2	5	0.896	5	4.48	5
Pine	12.3	0.51	1.5	0.221	1	0.33	1
GFRP	21	2	10	0.725	4	7.25	6
CFRP	76	1.6	200	0.378	2	75.55	8

(b) $h = L\left(\frac{k}{4Et}\right)^{1/3}$ For 1020 steel:

$$h = 500\,\text{mm}\left(\frac{50{,}000\text{ N}}{1000\text{ mm}}\ \frac{1}{4(203{,}000\text{ MPa})(60\text{ mm})}\right)^{1/3}$$

$h = 5.04$ mm (others similarly; see 2nd table)

$$P_{max} = k\,v_{max} = \frac{50{,}000\text{ N}}{1000\text{ mm}}\,30\text{ mm} = 1500\text{ N}$$

$$\sigma = \frac{\sigma_c}{X} = \frac{Mc}{I}, \quad c = \frac{h}{2}, \quad I = \frac{th^3}{12}, \quad M = \frac{PL}{4}$$

(Figs. A.1, A.2, and A.4) $X \geq 1.4$

$$\frac{\sigma_c}{X} = \frac{PL}{4}\,\frac{h}{2}\,\frac{12}{th^3}, \quad X = \frac{2\sigma_c t h^2}{3PL}$$

$$X = \frac{2(260\text{ MPa})(60\text{ mm})(5.04\text{ mm})^2}{3(1500\text{ N})(500\text{ mm})} = 0.352$$

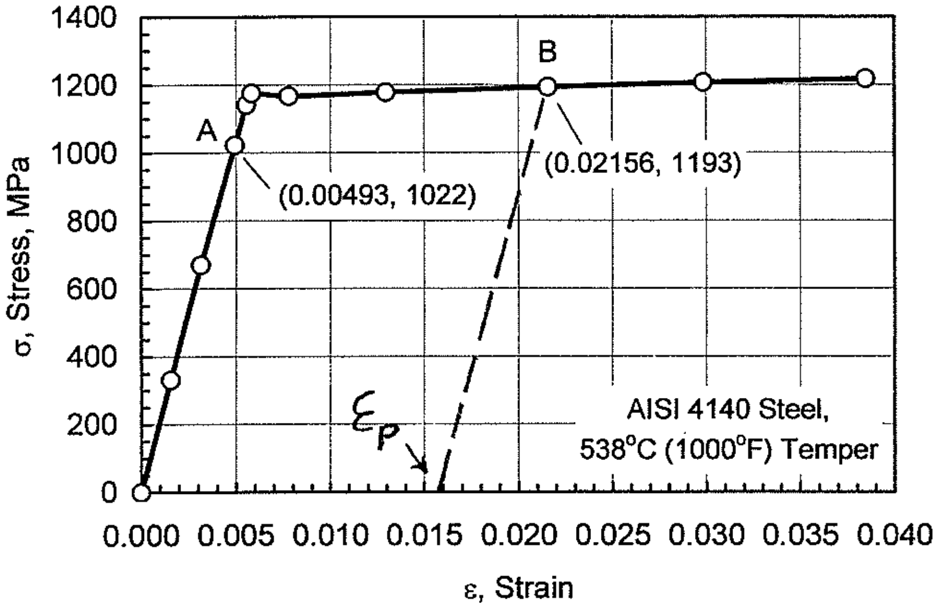
(for 1020 steel; others similarly)

(3.19, p.3)

Material	Strength σ_c, MPa	Depth h, mm	Safety Fac, X	Comment
1020 steel	260	5.04	0.35	fails X
4340 steel	1103	5.01	1.48	old design
7075 Al	469	7.16	1.28	fails X
Ti-6-4	1185	6.06	2.32	passes
PC	62	22.14	1.62	fails h
Pine	88	12.84	0.77	fails h, X
GFRP	380	10.74	2.34	passes
CFRP	930	7.00	2.43	passes

(c) All but Ti-6-4, GFRP, and CFRP fail due to h too large or $X < 1.4$. All of these involve a cost increase, by a factor of $7.25/4.01 = 1.8$ for GFRP, and much more for the other two. For GFRP, the weight is reduced by a factor of $7.25/1.335 = 0.54$. Hence, GFRP is a reasonable choice. CFRP is about half the weight of GFRP, but costs 10X more, and so seems an unlikely choice. ◀

4.5 Find: (a) E, (b) L_A and after unload for $L_i = 200$ mm, (c) ε_p for B, (d) L_B and after unload.



(a) $E \approx \dfrac{\sigma_A}{\varepsilon_A} = \dfrac{1022 \text{ MPa}}{0.00493} = 207{,}300 \text{ MPa}$ ◀

(b) $\varepsilon_A = \dfrac{\Delta L_A}{L_i}$, $\Delta L_A = (200 \text{ mm})0.00493 = 0.986 \text{ mm}$

$L_A = L_i + \Delta L_A = 200.99 \text{ mm}$ ◀

after unload, $L_o = L_i = 200.00$ ◀

(c) $\varepsilon_p = \varepsilon - \dfrac{\sigma}{E} = 0.02156 - \dfrac{1193 \text{ MPa}}{207{,}300 \text{ MPa}} = 0.01581$ ◀

(d) $\varepsilon_B = \dfrac{\Delta L_B}{L_i}$, $\Delta L_B = (200)0.02156 = 4.31 \text{ mm}$

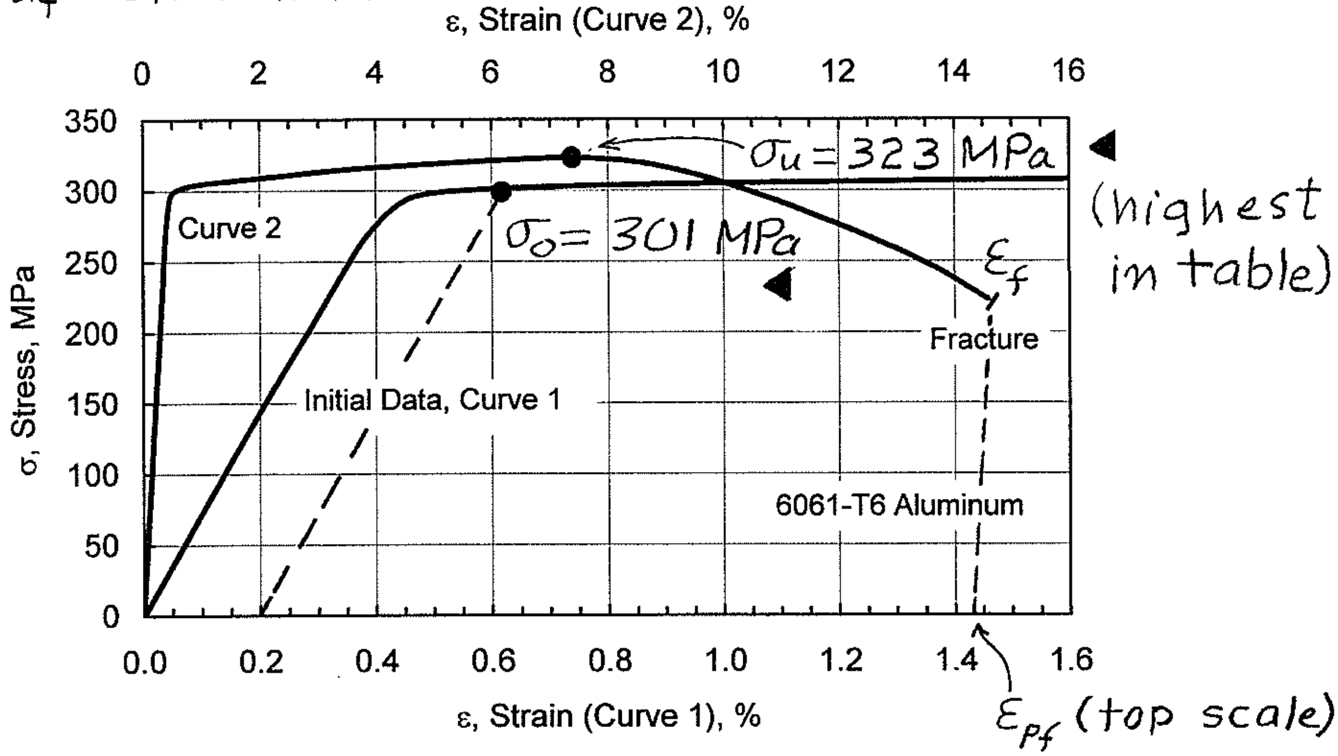
$L_B = L_i + \Delta L_B = 204.31 \text{ mm}$ ◀

After unload: $\Delta L_p = L_i \varepsilon_p = (200)0.01581 = 3.16$ mm

$L_o = L_i + \Delta L_p = 203.16 \text{ mm}$ ◀

4.6 Tension test on 6061-T6 Al.

Find E, 0.2% σ_o, σ_u, $100\,\varepsilon_f$, % RA. $d_i = 9.48$, $d_f = 6.25$ mm



$E = 71{,}550$ MPa (fit $y = cx$, $\sigma = 0$ to 199 MPa)

$\varepsilon_{f\%} = 100\,\varepsilon_f = 14.59 = 14.6\%$ (at fracture)

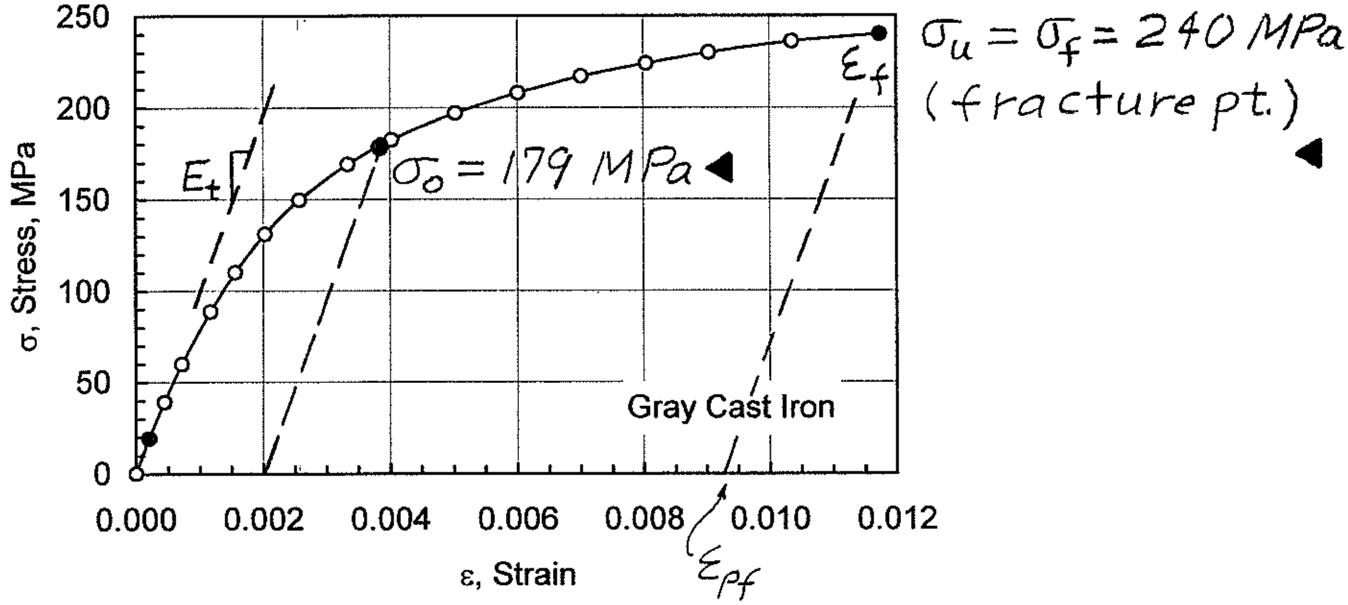
$$\varepsilon_{pf} = \varepsilon_f - \frac{\sigma_f}{E} = 0.1459 - \frac{223\text{ MPa}}{71{,}550\text{ MPa}} = 0.1428$$

$100\,\varepsilon_{pf} = 14.3\%$ (after fracture)

$$\%RA = 100\,\frac{d_i^2 - d_f^2}{d_i^2} = 100\,\frac{9.48^2 - 6.25^2}{9.48^2}\ \frac{\text{mm}^2}{\text{mm}^2}$$

$\%RA = 56.5\%$

4.7 Tension test on gray cast iron. (a) Find E_t, 0.2% σ_o, σ_u, $100\,\varepsilon_f$, %RA. $d_i = 8.57$, $d_f = 8.49$ mm. (b) Compare to ductile iron.



$$E_t = \frac{19.45\text{ MPa}}{0.000199} = 97{,}700\text{ MPa (1st data pt.)}$$

$$\varepsilon_{f\%} = 100\,\varepsilon_f = 1.173 = 1.17\%\ \text{(at fracture)}$$

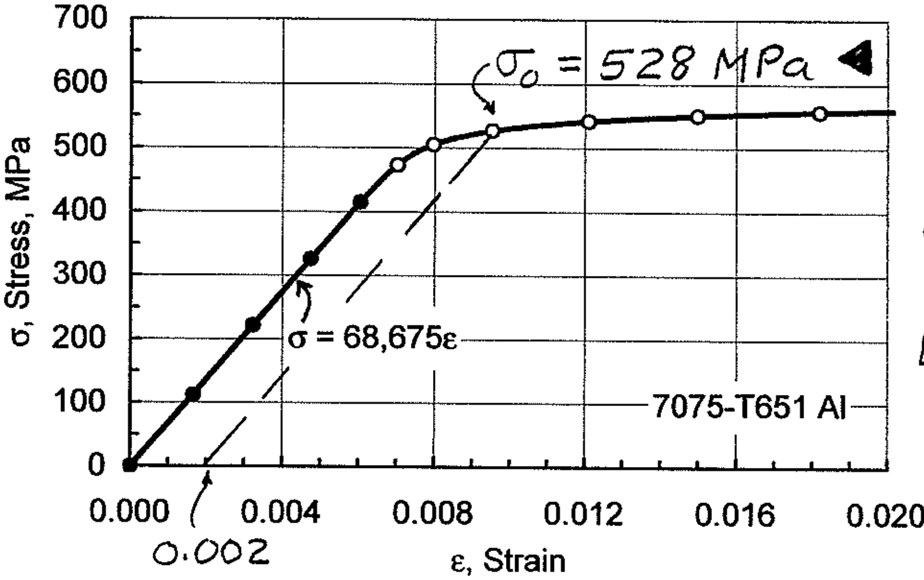
$$\varepsilon_{pf} = \varepsilon_f - \frac{\sigma_f}{E} = 0.01173 - \frac{240\text{ MPa}}{97{,}700\text{ MPa}} = 0.00927$$

$$100\,\varepsilon_{pf} = 0.93\%\ \text{(after fracture)}$$

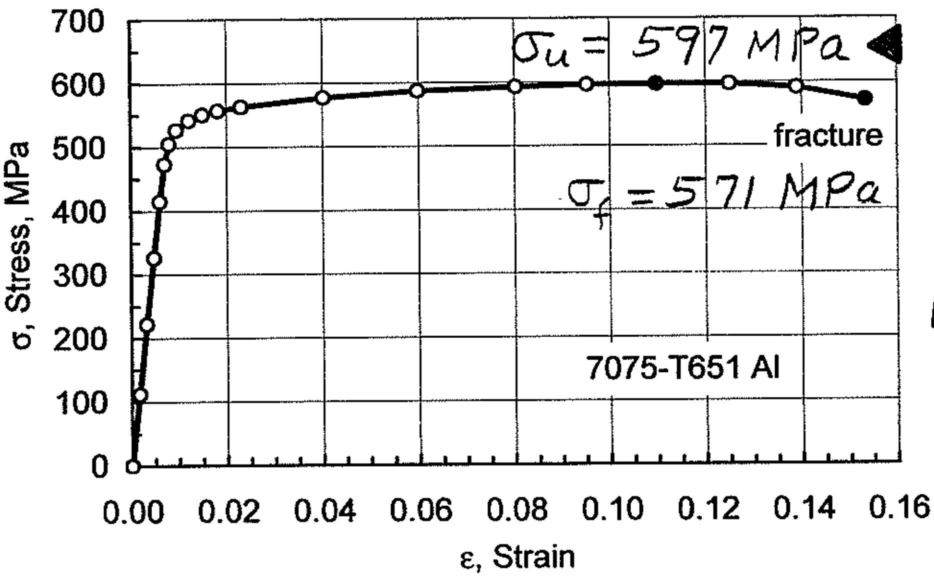
$$\%RA = 100\,\frac{d_i^2 - d_f^2}{d_i^2} = 100\,\frac{8.57^2 - 8.49^2}{8.57^2} = 1.86\%$$

(b) The E, σ_o, and σ_u are considerably lower than for the ductile iron, and ductility, $100\,\varepsilon_f$ and %RA, are drastically lower. This is due to the graphite flakes acting as cracks in gray iron, which does not occur for ductile iron.

4.10 Tension test on 7075-T651 Al, $d_i = 9.07$, $d_f = 7.78$ mm. Find $E, \sigma_o, \sigma_u, 100\,\varepsilon_f$, % RA.



Fit first 5 data pts, by least squares, $E = 68.7$ GPa



At fracture $100\,\varepsilon_f = 15.3\,\%$

After frac. $100\,\varepsilon_{pf} = 14.5\,\%$

After fracture: $\varepsilon_{pf} = \varepsilon_f - \sigma_f / E$

$$\varepsilon_{pf} = 0.1533 - 571/68{,}675 = 0.1450$$

$$\%\,RA = 100\,\frac{d_i^2 - d_f^2}{d_i^2} = 100\,\frac{9.07^2 - 7.78^2}{9.07^2} = 26.4\,\%$$

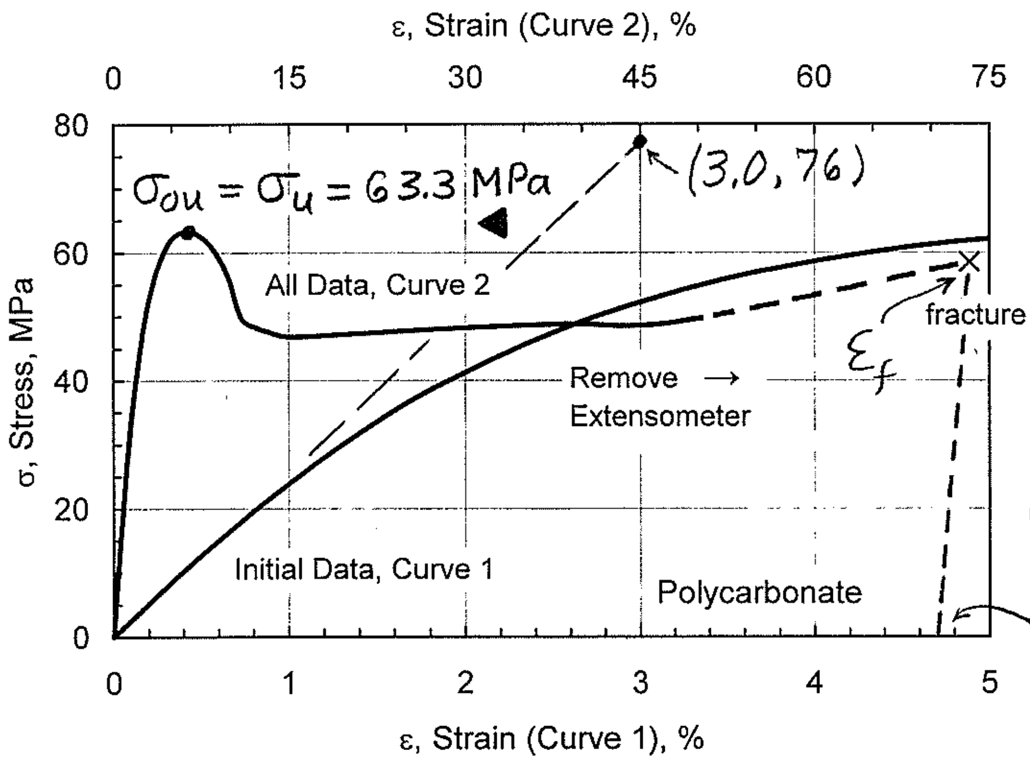
4.13 Tension test on PC, $\sigma-\varepsilon$ data points to removal of extensometer

Initial: $w_i = 12.54$, $t_i = 2.00$, $L_i = 50$ mm

After fracture: $w_f = 10.09$, $t_f = 1.37$, $L_f = 85.5$ mm

Fracture at 1.466 kN

Find: E, σ_{ou}, σ_u, $100\,\varepsilon_f$, % RA



$$E_t = \frac{76 \text{ MPa}}{0.030}$$

$$E_t = 2530 \text{ MPa} \blacktriangleleft$$

$$\sigma_f = \frac{1466 \text{ N}}{12.54 \times 2.00 \text{ mm}^2}$$

$$\sigma_f = 58.5 \text{ MPa}$$

$$\varepsilon_{pf} = \frac{\Delta L_f}{L_i} = \frac{(85.5 - 50)\text{ mm}}{50 \text{ mm}} = 0.71, \quad 100\,\varepsilon_{pf} = 71\,\% \blacktriangleleft$$

Estimate at fracture value; neglect recovery:

$$\varepsilon_f \approx \varepsilon_{pf} + \frac{\sigma_f}{E_t} = 0.71 + \frac{58.5 \text{ MPa}}{2530 \text{ MPa}} = 0.733$$

$100\,\varepsilon_f \approx 73.3\,\% \triangleleft$ (extend plot above to ε_f, σ_f)

$$\% RA = \frac{w_i t_i - w_f t_f}{w_i t_i} 100 = \frac{12.54 \times 2.00 - 10.09 \times 1.37}{12.54 \times 2.00} 100$$

$$= 44.9\,\% \blacktriangleleft$$

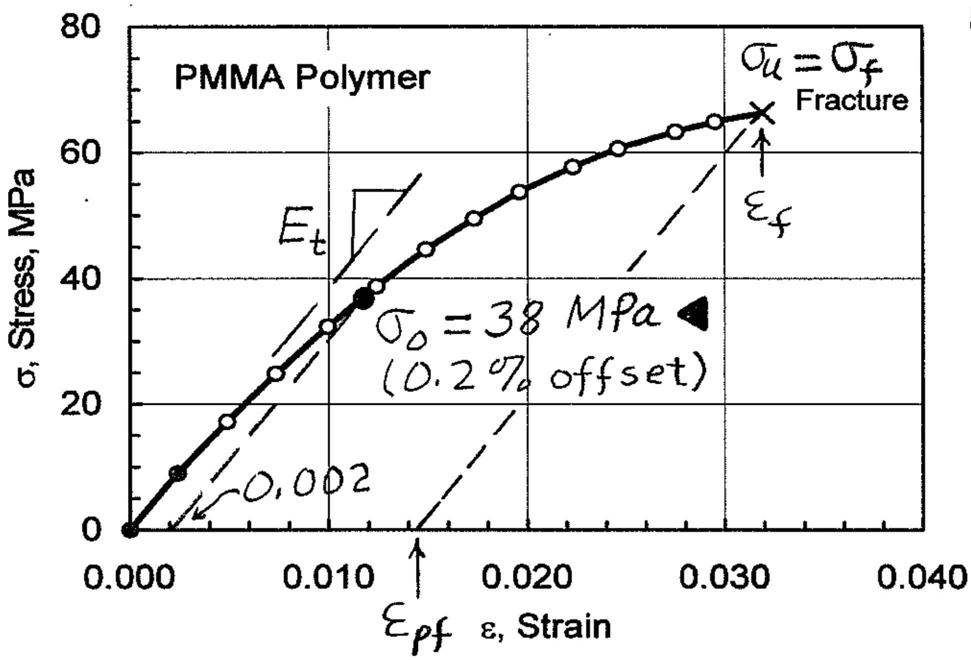
4.14

For given stress-strain data from a tension test on PMMA polymer, determine E, σ_o, σ_u, $100\varepsilon_f$, and $\%RA$.

$$E_t = \frac{9.00 \text{ MPa}}{0.00241}$$

(1st data pt.)

$E_t = 3730$ MPa ◀



σ, MPa	ε, %
0	0
9.00	0.241
17.20	0.490
24.8	0.733
32.3	0.995
38.7	1.239
44.6	1.487
49.5	1.729
53.7	1.960
57.7	2.23
60.6	2.46
63.3	2.75
64.9	2.95
66.3 ←	3.19

(Final point is fracture)

$\varepsilon_f = 0.0319$, $100\varepsilon_f = 3.19\%$ ◀

$$\varepsilon_{pf} = \varepsilon_f - \frac{\sigma_f}{E} = 0.0319 - \frac{66.3 \text{ MPa}}{3730 \text{ MPa}}$$

$\varepsilon_{pf} = 0.0141$, $100\varepsilon_{pf} = 1.41\%$ ◀

$\%RA \approx 0$ (small) ◀

(No width, thickness change was found.)

$\sigma_u = 66.3$ MPa ◀

Yield strength value is very sensitive to the offset value arbitrarily chosen.

4.21 Tension test on 7075-T651 AL.

(a) Calc. $\tilde{\sigma}$, $\tilde{\varepsilon}$ up to σ_u, plot with σ vs. ε.

(b) Calc. $\tilde{\varepsilon}_p$, plot $\tilde{\sigma}$ vs $\tilde{\varepsilon}_p$ on log-log, fit $\tilde{\sigma} = H\tilde{\varepsilon}_p^{\tilde{n}}$.

Engr Stress σ, MPa	Engr Strain ε, %	Engr Strain ε	True Stress	True Strain	True Plastic Strain
0	0	0	0	0	0
112	0.165	0.00165	112	0.00165	0
222	0.322	0.00322	222	0.00321	0
326	0.474	0.00474	326	0.00473	0
415	0.605	0.00605	415	0.00603	0
473	0.703	0.00703	473	0.00701	0.00012
505	0.797	0.00797	505	0.00794	0.00058
527	0.953	0.00953	527	0.00948	0.00181
542	1.209	0.01209	542	0.01202	0.00413
551	1.498	0.01498	551	0.01487	0.00685
557	1.819	0.01819	557	0.01803	0.00992
563	2.30	0.0230	576	0.0227	0.01435
577	4.02	0.0402	600	0.0394	0.0307
587	5.98	0.0598	622	0.0581	0.0490
593	8.02	0.0802	641	0.0771	0.0678
596	9.52	0.0952	653	0.0909	0.0814
597	10.97	0.1097	662	0.1041	0.0944
597	12.50	0.1250			
591	13.90	0.1390			
571	15.33	0.1533			

(Final point is fracture)

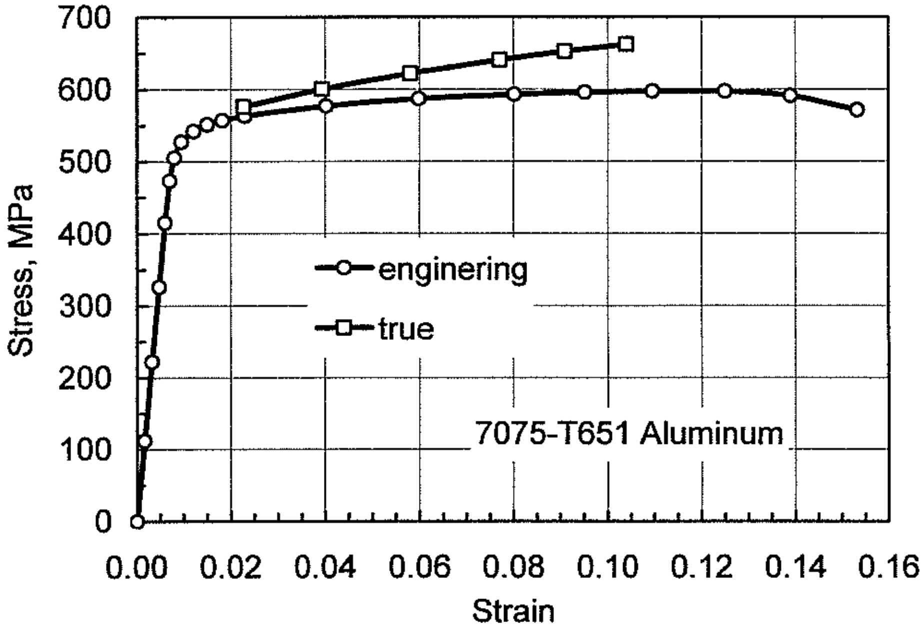
fit

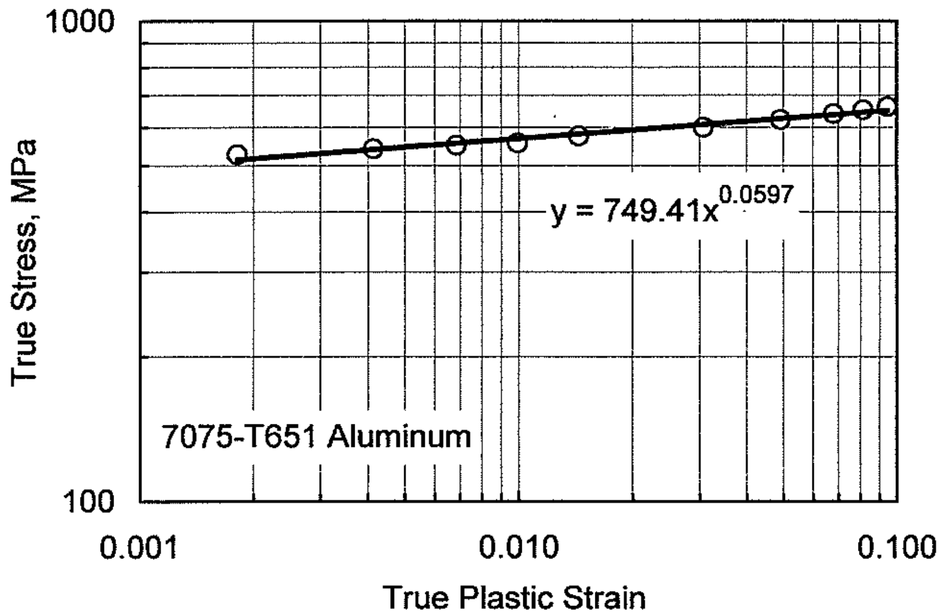
From Prob. 4.10: $\sigma_o = 528$ MPa, $\varepsilon_o = 0.0097$, $2\varepsilon_o = 0.0194$. Below this, use $\tilde{\sigma} = \sigma$.

Table gives calculations, plots on next page, $\tilde{\sigma} = \sigma(1+\varepsilon)$ for $\varepsilon > 2\varepsilon_o$, $\tilde{\varepsilon} = \ln(1+\varepsilon)$

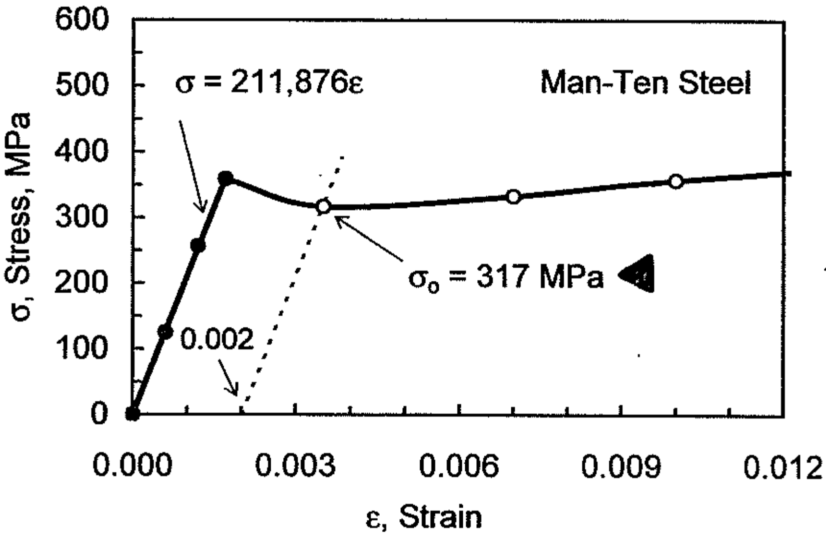
Least squares power eqn. fit on log-log plot gives $H = 749$ MPa, $n = 0.0597$.

(4.21, p.2)


700
600
500
400
300
200
100
0
Stress, MPa
engineering
true
7075-T651 Aluminum
0.00 0.02 0.04 0.06 0.08 0.10 0.12 0.14 0.16
Strain


1000
100
True Stress, MPa
y = 749.41x^0.0597
7075-T651 Aluminum
0.001 0.010 0.100
True Plastic Strain

4.22 For tension test on Man-Ten steel, with $d_i = 6.32$ mm; (a) Find E, σ_o, σ_u, %RA. (b) Determine and plot $\tilde{\sigma}, \tilde{\varepsilon}$, also $\tilde{\sigma}_B$; find $\tilde{\sigma}_{fB}, \tilde{\varepsilon}_f$. (c) Calculate $\tilde{\varepsilon}_p$ and fit $\tilde{\sigma}_B = H\tilde{\varepsilon}_p^{\,n}$.



Plot beginning of data.

Least squares fit of first 4 pts. gives.

$E = 211{,}900$ MPa

$E = 211.9$ GPa

Take highest σ in table to be $\sigma_u = 576$ MPa

$$\%RA = 100\frac{d_i^2 - d_f^2}{d_i^2} = 100\frac{6.32^2 - 3.50^2}{6.32^2} = 69.3\%$$

(b) Calculate $\tilde{\sigma}, \tilde{\varepsilon}$ as below; values in table.

$\tilde{\sigma} = \sigma$ for $\varepsilon < 2\varepsilon_o = 0.007$

$\tilde{\sigma} = \sigma(1+\varepsilon)$ for $\varepsilon > 2\varepsilon_o$ where d unknown

$\tilde{\sigma} = \sigma\frac{A_i}{A} = \sigma\left(\frac{d_i}{d}\right)^2$ where d known

$\tilde{\varepsilon} = \ln(1+\varepsilon)$ where d unknown

$\tilde{\varepsilon} = \ln\frac{A_i}{A} = \ln\left(\frac{d_i}{d}\right)^2$ where d known

(Note that $\tilde{\sigma} = \sigma(1+\varepsilon)$ and $\tilde{\varepsilon} = \ln(1+\varepsilon)$ used $< \sigma_u$.)