

## Chapter 2

$$2.1 \quad \mathbf{C} = \mathbf{A} + \mathbf{B} = \begin{bmatrix} 17 & -10 & 1 \\ -4 & 4 & 4 \\ -3 & -4 & 0 \end{bmatrix}; \quad \mathbf{D} = \mathbf{A} - \mathbf{B} = \begin{bmatrix} 1 & 4 & 3 \\ 10 & -14 & 4 \\ -1 & -4 & 12 \end{bmatrix}$$

$$2.2 \quad \mathbf{C} = 2\mathbf{A} + \mathbf{B} = \begin{bmatrix} 17 & -4 & -12 & -7 \\ -12 & -3 & 0 & 19 \\ -11 & -2 & 2 & 9 \end{bmatrix}; \quad \mathbf{D} = \mathbf{A} - 3\mathbf{B} = \begin{bmatrix} -9 & 5 & -20 & -7 \\ -20 & -19 & 21 & -15 \\ -2 & -8 & -27 & 22 \end{bmatrix}$$

$$2.3 \quad \mathbf{C} = \mathbf{AB} = \begin{bmatrix} -5 \\ 2 \\ -8 \\ 4 \end{bmatrix} \begin{bmatrix} 3 & -7 & 1 & -6 \end{bmatrix} = \begin{bmatrix} -15 & 35 & -5 & 30 \\ 6 & -14 & 2 & -12 \\ -24 & 56 & -8 & 48 \\ 12 & -28 & 4 & -24 \end{bmatrix}$$

$$\mathbf{D} = \mathbf{BA} = \begin{bmatrix} 3 & -7 & 1 & -6 \end{bmatrix} \begin{bmatrix} -5 \\ 2 \\ -8 \\ 4 \end{bmatrix} = -15 - 14 - 8 - 24 = -61$$

$$2.4 \quad \mathbf{C} = \mathbf{AB} = \begin{bmatrix} 1 & -10 & -2 & -6 \\ 3 & -8 & -12 & 8 \end{bmatrix} \begin{bmatrix} 2 & -10 \\ 6 & -1 \\ -2 & -10 \\ 5 & 9 \end{bmatrix} = \begin{bmatrix} -84 & -74 \\ 22 & -70 \end{bmatrix}$$

$$\mathbf{D} = \mathbf{BA} = \begin{bmatrix} 2 & -10 \\ 6 & -1 \\ -2 & 10 \\ 5 & 9 \end{bmatrix} \begin{bmatrix} 1 & -10 & -2 & -6 \\ 3 & -8 & -12 & 8 \end{bmatrix} = \begin{bmatrix} -28 & 60 & 116 & 68 \\ 3 & -52 & 0 & -28 \\ 28 & 60 & -116 & -68 \\ 32 & -122 & -118 & -102 \end{bmatrix}$$

$$2.5 \quad \mathbf{C} = \mathbf{AB} = \begin{bmatrix} 5 & 7 & -2 \\ 7 & 9 & 0 \\ -2 & 0 & 8 \end{bmatrix} \begin{bmatrix} 2 & -8 & 1 \\ -8 & 3 & 5 \\ 1 & 5 & 6 \end{bmatrix} = \begin{bmatrix} -48 & -29 & 28 \\ -58 & -29 & 52 \\ 4 & 56 & 46 \end{bmatrix}$$

$$\mathbf{D} = \mathbf{BA} = \begin{bmatrix} 2 & -8 & 1 \\ -8 & 3 & 5 \\ 1 & 5 & 6 \end{bmatrix} \begin{bmatrix} 5 & 7 & -2 \\ 7 & 9 & 0 \\ -2 & 0 & 8 \end{bmatrix} = \begin{bmatrix} -48 & -58 & 4 \\ -29 & -29 & 56 \\ 28 & 52 & 46 \end{bmatrix}$$

$$2.6 \quad \mathbf{C} = \mathbf{AB} = \begin{bmatrix} 3 & 4 & 5 \\ 6 & 3 & 0 \\ -3 & 0 & -7 \end{bmatrix} \begin{bmatrix} 8 & 1 & 6 & -2 \\ 6 & -2 & 5 & 8 \\ 3 & 0 & -1 & 0 \end{bmatrix} = \begin{bmatrix} 54 & -5 & 36 & -26 \\ 10 & -4 & 35 & 20 \\ -6 & -3 & -24 & 6 \end{bmatrix}$$

$$2.8 \quad \mathbf{AB} = \begin{bmatrix} 14 & -5 & 11 \\ -3 & 18 & -9 \end{bmatrix} \begin{bmatrix} 12 & -7 & 6 & 0 \\ 2 & 9 & 15 & -13 \\ 10 & -4 & -5 & 8 \end{bmatrix} = \begin{bmatrix} 268 & -187 & -46 & 153 \\ -90 & 219 & 297 & -306 \end{bmatrix}$$

$$(\mathbf{AB})^T = \begin{bmatrix} 268 & -90 \\ -187 & 219 \\ -46 & 297 \\ 153 & -306 \end{bmatrix} \quad (1)$$

$$\mathbf{B}^T \mathbf{A}^T = \begin{bmatrix} 12 & 2 & 10 \\ -7 & 9 & -4 \\ 6 & 15 & -5 \\ 0 & -13 & 8 \end{bmatrix} \begin{bmatrix} 14 & -3 \\ -5 & 18 \\ 11 & -9 \end{bmatrix} = \begin{bmatrix} 268 & -90 \\ -187 & 219 \\ -46 & 297 \\ 153 & -306 \end{bmatrix} \quad (2)$$

From Eqs. (1) and (2), we can see that  $(\mathbf{AB})^T = \mathbf{B}^T \mathbf{A}^T$ .

$$\begin{aligned}
 2.9 \quad [A][B][C] &= \begin{bmatrix} -9 & 0 \\ 13 & 20 \\ 8 & -3 \\ -11 & -5 \end{bmatrix} \begin{bmatrix} 15 & -1 & -4 \\ 6 & 16 & 9 \end{bmatrix} \begin{bmatrix} -7 & 10 & 6 & 0 \\ -1 & 2 & -8 & -2 \\ 16 & 12 & 2 & 8 \end{bmatrix} \\
 &= \begin{bmatrix} -135 & 9 & 36 \\ 315 & 307 & 128 \\ 102 & -56 & -59 \\ -195 & -69 & -1 \end{bmatrix} \begin{bmatrix} -7 & 10 & 6 & 0 \\ -1 & 2 & -8 & -2 \\ 16 & 12 & 2 & 8 \end{bmatrix} \\
 &= \begin{bmatrix} 1512 & -900 & -810 & 270 \\ -464 & 5300 & -310 & 410 \\ -1602 & 200 & 942 & -360 \\ 1418 & -2100 & -620 & 130 \end{bmatrix} \\
 ([A][B][C])^T &= \begin{bmatrix} 1512 & -464 & -1602 & 1418 \\ -900 & 5300 & 200 & -2100 \\ -810 & -310 & 942 & -620 \\ 270 & 410 & -360 & 130 \end{bmatrix} \tag{1}
 \end{aligned}$$

$$\begin{aligned}
 [C]^T [B]^T [A]^T &= \begin{bmatrix} -7 & -1 & 16 \\ 10 & 2 & 12 \\ 6 & -8 & 2 \\ 0 & -2 & 8 \end{bmatrix} \begin{bmatrix} 15 & 6 \\ -1 & 16 \\ -4 & 9 \end{bmatrix} \begin{bmatrix} -9 & 13 & 8 & -11 \\ 0 & 20 & -3 & -5 \end{bmatrix} \\
 &= \begin{bmatrix} -168 & 86 \\ 100 & 200 \\ 90 & -74 \\ -30 & 40 \end{bmatrix} \begin{bmatrix} -9 & 13 & 8 & -11 \\ 0 & 20 & -3 & -5 \end{bmatrix} \\
 &= \begin{bmatrix} 1512 & -464 & -1602 & 1418 \\ -900 & 5300 & 200 & -2100 \\ -810 & -310 & 942 & -620 \\ 270 & 410 & -360 & 130 \end{bmatrix} \tag{2}
 \end{aligned}$$

From Eqs. (1) and (2), we can see that

$$([A][B][C])^T = [C]^T [B]^T [A]^T$$



$$\begin{aligned} \mathbf{2.10} \quad \mathbf{C} = \mathbf{B}^T \mathbf{A} \mathbf{B} &= \begin{bmatrix} 5 & -7 & 3 \\ 7 & 8 & -4 \\ -3 & 4 & 9 \end{bmatrix} \begin{bmatrix} 40 & -10 & -25 \\ -10 & 15 & 12 \\ -25 & 12 & 30 \end{bmatrix} \begin{bmatrix} 5 & 7 & -3 \\ -7 & 8 & 4 \\ 3 & -4 & 9 \end{bmatrix} \\ &= \begin{bmatrix} 195 & -119 & -119 \\ 300 & 2 & -199 \\ -385 & 198 & 393 \end{bmatrix} \begin{bmatrix} 5 & 7 & -3 \\ -7 & 8 & 4 \\ 3 & -4 & 9 \end{bmatrix} = \begin{bmatrix} 1,451 & 889 & -2,132 \\ 889 & 2,912 & -2,683 \\ -2,132 & -2,683 & 5,484 \end{bmatrix} \end{aligned}$$

$$\begin{aligned} 2.11 \quad \mathbf{C} = \mathbf{B}^T \mathbf{A} \mathbf{B} &= \begin{bmatrix} 0.6 & -0.8 \\ 0.8 & 0.6 \\ -0.6 & 0.8 \\ -0.8 & -0.6 \end{bmatrix} \begin{bmatrix} 300 & -100 \\ -100 & 200 \end{bmatrix} \begin{bmatrix} 0.6 & 0.8 & -0.6 & -0.8 \\ -0.8 & 0.6 & 0.8 & -0.6 \end{bmatrix} \\ &= \begin{bmatrix} 260 & -220 \\ 180 & 40 \\ -260 & 220 \\ -180 & -40 \end{bmatrix} \begin{bmatrix} 0.6 & 0.8 & -0.6 & -0.8 \\ -0.8 & 0.6 & 0.8 & -0.6 \end{bmatrix} = \begin{bmatrix} 332 & 76 & -332 & -76 \\ 76 & 168 & -76 & -168 \\ -332 & -76 & 332 & 76 \\ -76 & -168 & 76 & 168 \end{bmatrix} \end{aligned}$$

$$2.13 \quad \frac{d\mathbf{A}}{dx} = \begin{bmatrix} 2 \sin x \cos x & -9x^2 & -6 \\ -12x & -2 \cos x \sin x & 0 \\ 0 & 3 \cos x & \sec^2 x \end{bmatrix}$$

$$2.14 \quad \mathbf{C} = \mathbf{A} + \mathbf{B} = \begin{bmatrix} 2x^2 - 3x & -x + 5 \\ 4x^2 - 12x & -x^3 + 8 \\ 2x^3 - 7 & -3x^2 + 5x \\ 2x^3 - 1 & -x^2 + 6x \end{bmatrix}; \quad \frac{d\mathbf{C}}{dx} = \begin{bmatrix} 4x - 3 & -1 \\ 8x - 12 & -3x^2 \\ 6x^2 & -6x + 5 \\ 6x^2 & -2x + 6 \end{bmatrix}$$

$$2.15 \quad [A][B] = \begin{bmatrix} 12 - 30x^4 & -10x^2 - 20x^3 \\ -30x^3 & -2x - 4x^2 + 9x^5 \\ -6x + 14x^2 + 25x^5 & 28x + 8x^3 \end{bmatrix}$$

$$\frac{d[A][B]}{dx} = \begin{bmatrix} -120x^3 & -20x - 60x^2 \\ -90x^2 & -2 - 8x + 45x^4 \\ -6 + 28x + 125x^4 & 28 + 24x^2 \end{bmatrix}$$

$$\mathbf{2.16} \quad \frac{\partial[A]}{\partial x} = \begin{bmatrix} 2x & 0 & 0 \\ 0 & 3y & 0 \\ 0 & 0 & 4z \end{bmatrix}$$
$$\frac{\partial[A]}{\partial y} = \begin{bmatrix} 0 & -2y & 0 \\ -2y & 3x & -z \\ 0 & -z & 0 \end{bmatrix}$$
$$\frac{\partial[A]}{\partial z} = \begin{bmatrix} 0 & 0 & 4z \\ 0 & 0 & -y \\ 4z & -y & 4x \end{bmatrix}$$

$$2.17 \quad \int_0^L \mathbf{A} dx = \begin{bmatrix} 2L^3 & -\frac{L^4}{4} & 9L \\ -\frac{L^4}{4} & \frac{2L^5}{5} & 2L^2 \\ 9L & 2L^2 & \frac{5L^4}{4} \end{bmatrix}$$

$$2.18 \quad \int_0^L [A] dx = \begin{bmatrix} L^2 & \cos L - 1 & \sin L \cos L + L \\ \cos L - 1 & 5L & -L^4 \\ \sin L \cos L + L & -L^4 & L - \frac{L^3}{3} \end{bmatrix}$$



$$2.19 \quad [A][B] = \begin{bmatrix} 10x^2 + 9x^3 + 2x^4 & -9 - 4x^3 - x^5 \\ -9x^2 + 6x^6 & -2x^3 \end{bmatrix}$$

$$\int_0^L ([A][B]) dx = \begin{bmatrix} \frac{10L^3}{3} + \frac{9L^4}{4} + \frac{2L^5}{5} & -9L - L^4 - \frac{L^6}{6} \\ -3L^3 + \frac{6L^7}{7} & -\frac{L^4}{2} \end{bmatrix}$$

$$2.20 \quad \mathbf{AA}^T = \begin{bmatrix} -0.28 & -0.96 & 0 & 0 \\ 0.96 & -0.28 & 0 & 0 \\ 0 & 0 & -0.28 & -0.96 \\ 0 & 0 & 0.96 & -0.28 \end{bmatrix} \begin{bmatrix} -0.28 & 0.96 & 0 & 0 \\ -0.96 & -0.28 & 0 & 0 \\ 0 & 0 & -0.28 & 0.96 \\ 0 & 0 & -0.96 & -0.28 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Thus,  $\mathbf{A}$  is orthogonal.

$$\mathbf{BB}^T = \begin{bmatrix} -0.28 & 0.96 & 0 & 0 \\ 0.96 & -0.28 & 0 & 0 \\ 0 & 0 & -0.28 & 0.96 \\ 0 & 0 & 0.96 & -0.28 \end{bmatrix} \begin{bmatrix} -0.28 & 0.96 & 0 & 0 \\ 0.96 & -0.28 & 0 & 0 \\ 0 & 0 & -0.28 & 0.96 \\ 0 & 0 & 0.96 & -0.28 \end{bmatrix} \\ = \begin{bmatrix} 1 & -0.5376 & 0 & 0 \\ -0.5376 & 1 & 0 & 0 \\ 0 & 0 & 1 & -0.5376 \\ 0 & 0 & -0.5376 & 1 \end{bmatrix}$$

Thus,  $\mathbf{B}$  is not orthogonal.

$$2.21 \quad \left[ \begin{array}{ccc|c} 2 & -3 & 1 & -18 \\ -9 & 5 & 3 & 18 \\ 4 & 7 & -8 & 53 \end{array} \right]$$

$$\left[ \begin{array}{ccc|c} 1 & -1.5 & 0.5 & -9 \\ 0 & -8.5 & 7.5 & -63 \\ 0 & 13 & -10 & 89 \end{array} \right]$$

$$\left[ \begin{array}{ccc|c} 1 & 0 & -0.82353 & 2.1176 \\ 0 & 1 & -0.88235 & 7.4118 \\ 0 & 0 & 1.4706 & -7.3529 \end{array} \right]$$

$$\left[ \begin{array}{ccc|c} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & -5 \end{array} \right]$$

Thus,  $x_1 = -2$   $x_2 = 3$   $x_3 = -5$

$$2.22 \quad \left[ \begin{array}{ccc|c} 20 & -9 & 15 & 354 \\ -9 & 16 & -5 & -275 \\ 15 & -5 & 18 & 307 \end{array} \right]$$

$$\left[ \begin{array}{ccc|c} 1 & -0.45 & 0.75 & 17.7 \\ 0 & 11.95 & 1.75 & -115.7 \\ 0 & 1.75 & 6.75 & 41.5 \end{array} \right]$$

$$\left[ \begin{array}{ccc|c} 1 & 0 & 0.8159 & 13.343 \\ 0 & 1 & 0.14644 & -9.682 \\ 0 & 0 & 6.4937 & 58.444 \end{array} \right]$$

$$\left[ \begin{array}{ccc|c} 1 & 0 & 0 & 6 \\ 0 & 1 & 0 & -11 \\ 0 & 0 & 1 & 9 \end{array} \right]$$

Thus,  $x_1 = 6$   $x_2 = -11$   $x_3 = 9$