Chapter 1

Introduction

1.1 Exercise

1. Which of the following can be considered as composite materials? a) ice made of salt water; b) cable protected by a carbon fiber sheet on the surface; c) portland cement concrete; d) low carbon steel, and; e) carbon nanotubes dispersed in an asphalt binder.

Answer:

The correct choices are c and e. For others, a) salt does not exhibit any distinct interface or microstructure in ice; b) because the carbon fiber sheet is only a single layer as a reinforcement, it is a structure system instead of a material system; d) the carbon has chemical reaction with iron already, which can be treated as one material phase. Sometimes, people still consider b) as a composite material, but it cannot be treated as a homogeneous material. In micromechanics, it is not a composite material but a bi-layered structure system.

2. Classify the following cases as either an inhomogeneity and an inclusion: a) a homogeneous cement paste with one region shrinking by drying (the moduli are assumed to be the same for different moisture contents); b) a homogeneous polymer containing a air void, and; c) a glass plate containing a glass fiber.

Answer:

a) The drying region is an inclusion because the material has the same stiffness but with an nonmechanical strain (eigenstrain).

b) The air void is an inhomogeneity because it has different stiffness (zero).

c) The glass fiber is an inhomogeneity because the stiffness of the fiber generally is different from glass matrix.

3. Following the case study in Section 1.5, use the classic solution of a thick wall cylinder under a uniform pressure to derive the bulk modulus for a 2D case.

Answer:

The bulk modulus is defined as:

$K = \frac{\sigma_m}{\varepsilon_m}$

In a 2D case, to obtain the bulk modulus of a porous material with an area ratio ϕ, we can use a cylinder with an inner to outer radius ratio at $r_i/r_o = \sqrt{\phi}$ to approximate the material as an RVE. A testing load of $\sigma_r^0 = \sigma_r|_{r=r_o} = \sigma_m$ is applied on the outer surface, from the displacement of the outer surface $u_r^0 = u_r$, we can see the effective area change of the cylinder is $_m = 2u_r^0/r_o$. Here u_r^0 can be obtained by the classic solution of a thick wall cylinder as follows.

Displacement governing equation (which is the so-called Navier Equation for the polar coordinate with axial symmetry) reads:

$u_{r,rr} + \frac{1}{r}u_{r,r} - \frac{1}{r^2}u_r = 0$

The general solution of ur is derived by solving Euler type of ODE as

$u_r = Ar + B/r$

where A and B are to be determined by the boundary conditions (BCs), $\sigma_r|_{r=r_o} = \sigma_m$ and $\sigma_r|_{r=r_i} = 0$. For plane stress condition, the stress in the axial direction is zero. We can write,

$\sigma_r = \frac{E}{1-v^2}\left(u_{r,r} + v\frac{u_r}{r}\right) = \frac{E}{1-v^2}$

Therefore, we can use the BCs to deterimine A and B

and then we can easily calculate K

$.A = \frac{\sigma_m r_0^2(1-v)}{R(r_0^2-r_i^2)}$

$B = \frac{\sigma_m r_0^2 r_i^2(1+v)}{E(r_0^2-r_i^2)}$

Then

$$K = \frac{\sigma_m}{\varepsilon_m} = \frac{E\,(1-\ )}{2\,[(1-v)+\ (1+v)]}$$

Notice that for 2D problems, their are plain stress and plain strain cases. For the plain strain case, the stress related to displacement is different. Using the same procedure, one can obtain K with a higher value than the above one as the displacement or strain in the axial direction is zero.

However, if the RVE is set up in the same way but with a 3D hydrostatic, one can obtain the solution by the superposition of plain stress solution and the unaxial loading along the axial direction with a stress σ_m.

4. Extend the above problem to the 3D case, and derive the bulk modulus of a porous material with air void.

Answer:

The displacement governing equation is

$\frac{d}{dr}\left[\frac{1}{r^2}\frac{d}{dr}\left(r^2 u\right)\right] = 0$

$u_r = Ar + B/r^2$

$\varepsilon_{rr} = u_{r,r}$

$\varepsilon_{\theta\theta} = u_r/r$

$\varepsilon_{\varphi\varphi} = u_r/r$

$\sigma_{rr} = \lambda \left(u_{r,r} + \frac{2u_r}{r}\right) + 2\mu u_{r,r}$

Apply the boundary condition

$\begin{cases} (\lambda + 2\mu) \left(A - \frac{2B}{r_0^3}\right) + 2\lambda \left(A + \frac{B}{r_0^3}\right) = \sigma_m \\ (\lambda + 2\mu) \left(A - \frac{2B}{r_i^3}\right) + 2\lambda \left(A + \frac{B}{r_i^3}\right) = 0 \end{cases}$

Then

$A = \frac{\sigma_m r_0^3}{(r_0^3 - r_i^3)(3\lambda + 2\mu)}$

$B = \frac{\sigma_m r_i^3 r_0^3}{4\mu(r_0^3 - r_i^3)}$

The bulk modulus is

$$K = \frac{(1 -)\,(3\lambda + 2\mu)\,4\mu}{3\,[4\mu + (3\lambda + 2\mu)\Phi]}$$