

Instructor's Manual to Accompany



Chapter 1

1. $S = \{(R, R), (R, G), (R, B), (G, R), (G, G), (G, B), (B, R), (B, G), (B, B)\}$
The probability of each point in S is $1/9$.
2. $S = \{(R, G), (R, B), (G, R), (G, B), (B, R), (B, G)\}$
3. $S = \{(e_1, e_2, \dots, e_n), n \geq 2\}$ where $e_i \in \{\text{heads, tails}\}$. In addition, $e_n = e_{n-1} = \text{heads}$ and for $i = 1, \dots, n-2$ if $e_i = \text{heads}$, then $e_{i+1} = \text{tails}$.

$$\begin{aligned}P\{4 \text{ tosses}\} &= P\{(t, t, h, h)\} + P\{(h, t, h, h)\} \\&= 2 \left[\frac{1}{2} \right]^4 = \frac{1}{8}\end{aligned}$$

4. (a) $F(E \cup G)^c = FE^cG^c$
(b) EFG^c
(c) $E \cup F \cup G$
(d) $EF \cup EG \cup FG$
(e) EFG
(f) $(E \cup F \cup G)^c = E^cF^cG^c$
(g) $(EF)^c(EG)^c(FG)^c$
(h) $(EFG)^c$
5. $\frac{3}{4}$. If he wins, he only wins \$1, while if he loses, he loses \$3.
6. If $E(F \cup G)$ occurs, then E occurs and either F or G occur; therefore, either EF or EG occurs and so

$$E(F \cup G) \subset EF \cup EG$$

Similarly, if $EF \cup EG$ occurs, then either EF or EG occurs. Thus, E occurs and either F or G occurs; and so $E(F \cup G)$ occurs. Hence,

$$EF \cup EG \subset E(F \cup G)$$

which together with the reverse inequality proves the result.

7. If $(E \cup F)^c$ occurs, then $E \cup F$ does not occur, and so E does not occur (and so E^c does); F does not occur (and so F^c does) and thus E^c and F^c both occur. Hence,

$$(E \cup F)^c \subset E^c F^c$$

If $E^c F^c$ occurs, then E^c occurs (and so E does not), and F^c occurs (and so F does not). Hence, neither E or F occurs and thus $(E \cup F)^c$ does. Thus,

$$E^c F^c \subset (E \cup F)^c$$

and the result follows.

8. $1 \geq P(E \cup F) = P(E) + P(F) - P(EF)$
 9. $F = E \cup FE^c$, implying since E and FE^c are disjoint that $P(F) = P(E) + P(FE^c)$.
 10. Either by induction or use

$$\bigcup_1^n E_i = E_1 \cup E_1^c E_2 \cup E_1^c E_2^c E_3 \cup \dots \cup E_1^c \dots E_{n-1}^c E_n$$

and as each of the terms on the right side are mutually exclusive:

$$\begin{aligned} P\left(\bigcup_i E_i\right) &= P(E_1) + P(E_1^c E_2) + P(E_1^c E_2^c E_3) + \dots \\ &\quad + P(E_1^c \dots E_{n-1}^c E_n) \\ &\leq P(E_1) + P(E_2) + \dots + P(E_n) \quad (\text{why?}) \end{aligned}$$

$$11. P\{\text{sum is } i\} = \begin{cases} \frac{i-1}{36}, & i = 2, \dots, 7 \\ \frac{13-i}{36}, & i = 8, \dots, 12 \end{cases}$$

12. Either use hint or condition on initial outcome as:

$$\begin{aligned} P\{E \text{ before } F\} &= P\{E \text{ before } F | \text{initial outcome is } E\}P(E) \\ &\quad + P\{E \text{ before } F | \text{initial outcome is } F\}P(F) \\ &\quad + P\{E \text{ before } F | \text{initial outcome neither } E \text{ or } F\}[1 - P(E) - P(F)] \\ &= 1 \cdot P(E) + 0 \cdot P(F) + P\{E \text{ before } F\} \\ &= [1 - P(E) - P(F)] \end{aligned}$$

Therefore, $P\{E \text{ before } F\} = \frac{P(E)}{P(E) + P(F)}$

13. Condition an initial toss

$$P\{\text{win}\} = \sum_{i=2}^{12} P\{\text{win} | \text{throw } i\}P\{\text{throw } i\}$$

Now,

$$P\{\text{win}|\text{throw } i\} = P\{i \text{ before } 7\}$$

$$= \begin{cases} 0 & i = 2, 12 \\ \frac{i-1}{5+1} & i = 3, \dots, 6 \\ 1 & i = 7, 11 \\ \frac{13-i}{19-1} & i = 8, \dots, 10 \end{cases}$$

where above is obtained by using Problems 11 and 12.

$$P\{\text{win}\} \approx .49.$$

$$\begin{aligned} 14. \quad P\{A \text{ wins}\} &= \sum_{n=0}^{\infty} P\{A \text{ wins on } (2n+1)\text{st toss}\} \\ &= \sum_{n=0}^{\infty} (1-P)^{2n} P \\ &= P \sum_{n=0}^{\infty} [(1-P)^2]^n \\ &= P \frac{1}{1-(1-P)^2} \\ &= \frac{P}{2P-P^2} \\ &= \frac{1}{2-P} \\ P\{B \text{ wins}\} &= 1 - P\{A \text{ wins}\} \\ &= \frac{1-P}{2-P} \end{aligned}$$

$$\begin{aligned} 16. \quad P(E \cup F) &= P(E \cup FE^c) \\ &= P(E) + P(FE^c) \end{aligned}$$

since E and FE^c are disjoint. Also,

$$\begin{aligned} P(E) &= P(FE \cup FE^c) \\ &= P(FE) + P(FE^c) \text{ by disjointness} \end{aligned}$$

Hence,

$$P(E \cup F) = P(E) + P(F) - P(EF)$$

$$\begin{aligned} 17. \quad \text{Prob}\{\text{end}\} &= 1 - \text{Prob}\{\text{continue}\} \\ &= 1 - P(\{H, H, H\} \cup \{T, T, T\}) \\ &= 1 - [\text{Prob}(H, H, H) + \text{Prob}(T, T, T)]. \end{aligned}$$

$$\text{Fair coin: Prob}\{\text{end}\} = 1 - \left[\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} + \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \right] \\ = \frac{3}{4}$$

$$\text{Biased coin: } P\{\text{end}\} = 1 - \left[\frac{1}{4} \cdot \frac{1}{4} \cdot \frac{1}{4} + \frac{3}{4} \cdot \frac{3}{4} \cdot \frac{3}{4} \right] \\ = \frac{9}{16}$$

18. Let B = event both are girls; E = event oldest is girl; L = event at least one is a girl.

$$(a) P(B|E) = \frac{P(BE)}{P(E)} = \frac{P(B)}{P(E)} = \frac{1/4}{1/2} = \frac{1}{2}$$

$$(b) P(L) = 1 - P(\text{no girls}) = 1 - \frac{1}{4} = \frac{3}{4},$$

$$P(B|L) = \frac{P(BL)}{P(L)} = \frac{P(B)}{P(L)} = \frac{1/4}{3/4} = \frac{1}{3}$$

19. E = event at least 1 six $P(E)$

$$= \frac{\text{number of ways to get } E}{\text{number of samples pts}} = \frac{11}{36}$$

D = event two faces are different $P(D)$

$$= 1 - \text{Prob}(\text{two faces the same})$$

$$= 1 - \frac{6}{36} = \frac{5}{6} P(E|D) = \frac{P(ED)}{P(D)} = \frac{10/36}{5/6} = \frac{1}{3}$$

20. Let E = event same number on exactly two of the dice; S = event all three numbers are the same; D = event all three numbers are different. These three events are mutually exclusive and define the whole sample space. Thus, $1 = P(D) + P(S) + P(E)$, $P(S) = 6/216 = 1/36$; for D have six possible values for first die, five for second, and four for third.

\therefore Number of ways to get $D = 6 \cdot 5 \cdot 4 = 120$.

$$P(D) = 120/216 = 20/36$$

$\therefore P(E) = 1 - P(D) - P(S)$

$$= 1 - \frac{20}{36} - \frac{1}{36} = \frac{5}{12}$$

21. Let C = event person is color blind.

$$P(\text{Male}|C) = \frac{P(C|\text{Male})P(\text{Male})}{P(C|\text{Male})P(\text{Male}) + P(C|\text{Female})P(\text{Female})} \\ = \frac{.05 \times .5}{.05 \times .5 + .0025 \times .5} \\ = \frac{2500}{2625} = \frac{20}{21}$$

22. Let trial 1 consist of the first two points; trial 2 the next two points, and so on. The probability that each player wins one point in a trial is $2p(1 - p)$. Now a total of $2n$ points are played if the first $(a - 1)$ trials all result in each player winning one of the points in that trial and the n th trial results in one of the players winning both points. By independence, we obtain

$$\begin{aligned} P\{2n \text{ points are needed}\} \\ = (2p(1 - p))^{n-1} (p^2 + (1 - p)^2), \quad n \geq 1 \end{aligned}$$

The probability that A wins on trial n is $(2p(1 - p))^{n-1} p^2$ and so

$$\begin{aligned} P\{A \text{ wins}\} &= p^2 \sum_{n=1}^{\infty} (2p(1 - p))^{n-1} \\ &= \frac{p^2}{1 - 2p(1 - p)} \end{aligned}$$

23. $P(E_1)P(E_2|E_1)P(E_3|E_1E_2) \dots P(E_n|E_1 \dots E_{n-1})$
 $= P(E_1) \frac{P(E_1E_2)}{P(E_1)} \frac{P(E_1E_2E_3)}{P(E_1E_2)} \dots \frac{P(E_1 \dots E_n)}{P(E_1 \dots E_{n-1})}$
 $= P(E_1 \dots E_n)$

24. Let a signify a vote for A and b one for B .

(a) $P_{2,1} = P\{a, a, b\} = 1/3$
(b) $P_{3,1} = P\{a, a\} = (3/4)(2/3) = 1/2$
(c) $P_{3,2} = P\{a, a, a\} + P\{a, a, b, a\}$
 $= (3/5)(2/4)[1/3 + (2/3)(1/2)] = 1/5$
(d) $P_{4,1} = P\{a, a\} = (4/5)(3/4) = 3/5$
(e) $P_{4,2} = P\{a, a, a\} + P\{a, a, b, a\}$
 $= (4/6)(3/5)[2/4 + (2/4)(2/3)] = 1/3$
(f) $P_{4,3} = P\{\text{always ahead}|a, a\}(4/7)(3/6)$
 $= (2/7)[1 - P\{a, a, a, b, b, b|a, a\}$
 $- P\{a, a, b, b|a, a\} - P\{a, a, b, a, b, b|a, a\}]$
 $= (2/7)[1 - (2/5)(3/4)(2/3)(1/2)$
 $- (3/5)(2/4) - (3/5)(2/4)(2/3)(1/2)]$
 $= 1/7$
(g) $P_{5,1} = P\{a, a\} = (5/6)(4/5) = 2/3$
(h) $P_{5,2} = P\{a, a, a\} + P\{a, a, b, a\}$
 $= (5/7)(4/6)[(3/5) + (2/5)(3/4)] = 3/7$

By the same reasoning we have

- (i) $P_{5,3} = 1/4$
- (j) $P_{5,4} = 1/9$
- (k) In all the cases above, $P_{n,m} = \frac{n-n}{n+n}$

25. (a) $P\{\text{pair}\} = P\{\text{second card is same denomination as first}\}$
 $= 3/51$

(b) $P\{\text{pair|different suits}\}$
 $= \frac{P\{\text{pair, different suits}\}}{P\{\text{different suits}\}}$
 $= P\{\text{pair}\}/P\{\text{different suits}\}$
 $= \frac{3/51}{39/51} = 1/13$

26. $P(E_1) = \binom{4}{1} \binom{48}{12} / \binom{52}{13} = \frac{39.38.37}{51.50.49}$

$$P(E_2|E_1) = \binom{3}{1} \binom{36}{12} / \binom{39}{13} = \frac{26.25}{38.37}$$

$$P(E_3|E_1E_2) = \binom{2}{1} \binom{24}{12} / \binom{26}{13} = 13/25$$

$$P(E_4|E_1E_2E_3) = 1$$

$$P(E_1E_2E_3E_4) = \frac{39.26.13}{51.50.49}$$

27. $P(E_1) = 1$

$P(E_2|E_1) = 39/51$, since 12 cards are in the ace of spades pile and 39 are not.

$P(E_3|E_1E_2) = 26/50$, since 24 cards are in the piles of the two aces and 26 are in the other two piles.

$$P(E_4|E_1E_2E_3) = 13/49$$

So

$$P\{\text{each pile has an ace}\} = (39/51)(26/50)(13/49)$$

28. Yes. $P(A|B) > P(A)$ is equivalent to $P(AB) > P(A)P(B)$, which is equivalent to $P(B|A) > P(B)$.

29. (a) $P(E|F) = 0$

(b) $P(E|F) = P(EF)/P(F) = P(E)/P(F) \geq P(E) = .6$

(c) $P(E|F) = P(EF)/P(F) = P(F)/P(F) = 1$

30. (a) $P\{\text{George|exactly 1 hit}\} = \frac{P\{\text{George, not Bill}\}}{P\{\text{exactly 1}\}}$
 $= \frac{P\{G, \text{ not } B\}}{P\{G, \text{ not } B\} + P\{B, \text{ not } G\}}$
 $= \frac{(.4)(.3)}{(.4)(.3) + (.7)(.6)}$
 $= 2/9$

$$\begin{aligned}
 \text{(b)} \quad P\{G|\text{hit}\} &= P\{G, \text{ hit}\}/P\{\text{hit}\} \\
 &= P\{G\}/P\{\text{hit}\} = .4/[1 - (.3)(.6)] \\
 &= 20/41
 \end{aligned}$$

31. Let S = event sum of dice is 7; F = event first die is 6.

$$\begin{aligned}
 P(S) &= \frac{1}{6}P(FS) = \frac{1}{36}P(F|S) = \frac{P(F|S)}{P(S)} \\
 &= \frac{1/36}{1/6} = \frac{1}{6}
 \end{aligned}$$

32. Let E_i = event person i selects own hat. P (no one selects own hat)

$$\begin{aligned}
 &= 1 - P(E_1 \cup E_2 \cup \dots \cup E_n) \\
 &= 1 - \left[\sum_{i_1} P(E_{i_1}) - \sum_{i_1 < i_2} P(E_{i_1}E_{i_2}) + \dots \right. \\
 &\quad \left. + (-1)^{n+1} P(E_1E_2E_n) \right] \\
 &= 1 - \sum_{i_1} P(E_{i_1}) - \sum_{i_1 < i_2} P(E_{i_1}E_{i_2}) \\
 &\quad - \sum_{i_1 < i_2 < i_3} P(E_{i_1}E_{i_2}E_{i_3}) + \dots \\
 &\quad + (-1)^n P(E_1E_2E_n)
 \end{aligned}$$

Let $k \in \{1, 2, \dots, n\}$. $P(E_{i_1}E_{i_2}E_{i_k})$ = number of ways k specific men can select own hats \div total number of ways hats can be arranged $= (n - k)!/n!$. Number of terms in summation $\sum_{i_1 < i_2 < \dots < i_k}$ = number of ways to choose k variables out of n variables $= \binom{n}{k} = n!/k!(n - k)!$.

Thus,

$$\begin{aligned}
 &\sum_{i_1 < \dots < i_k} P(E_{i_1}E_{i_2} \dots E_{i_k}) \\
 &= \sum_{i_1 < \dots < i_k} \frac{(n - k)!}{n!} \\
 &= \binom{n}{k} \frac{(n - k)!}{n!} = \frac{1}{k!}
 \end{aligned}$$

$\therefore P$ (no one selects own hat)

$$\begin{aligned}
 &= 1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \dots + (-1)^n \frac{1}{n!} \\
 &= \frac{1}{2!} - \frac{1}{3!} + \dots + (-1)^n \frac{1}{n!}
 \end{aligned}$$

33. Let S = event student is sophomore; F = event student is freshman; B = event student is boy; G = event student is girl. Let x = number of sophomore girls; total number of students = $16 + x$.

$$\begin{aligned} P(F) &= \frac{10}{16+x} P(B) = \frac{10}{16+x} P(FB) = \frac{4}{16+x} \\ \frac{4}{16+x} &= P(FB) = P(F)P(B) = \frac{10}{16+x} \\ \frac{10}{16+x} &\Rightarrow x = 9 \end{aligned}$$

34. Not a good system. The successive spins are independent and so

$$\begin{aligned} P\{11\text{th is red}|1\text{st 10 black}\} &= P\{11\text{th is red}\} \\ &= P\left[\frac{18}{38}\right] \end{aligned}$$

35. (a) $1/16$
 (b) $1/16$
 (c) $15/16$, since the only way in which the pattern H, H, H, H can appear before the pattern T, H, H, H is if the first four flips all land heads.

36. Let B = event marble is black; B_i = event that box i is chosen. Now

$$\begin{aligned} B &= BB_1 \cup BB_2 P(B) = P(BB_1) + P(BB_2) \\ &= P(B|B_1)P(B_1) + P(B|B_2)P(B_2) \\ &= \frac{1}{2} \cdot \frac{1}{2} + \frac{2}{3} \cdot \frac{1}{2} = \frac{7}{12} \end{aligned}$$

37. Let W = event marble is white.

$$\begin{aligned} P(B_1|W) &= \frac{P(W|B_1)P(B_1)}{P(W|B_1)P(B_1) + P(W|B_2)P(B_2)} \\ &= \frac{\frac{1}{2} \cdot \frac{1}{2}}{\frac{1}{2} \cdot \frac{1}{2} + \frac{1}{3} \cdot \frac{1}{2}} = \frac{\frac{1}{4}}{\frac{5}{12}} = \frac{3}{5} \end{aligned}$$

38. Let T_W = event transfer is white; T_B = event transfer is black; W = event white ball is drawn from urn 2.

$$\begin{aligned} P(T_W|W) &= \frac{P(W|T_W)P(T_W)}{P(W|T_W)P(T_W) + P(W|T_B)P(T_B)} \\ &= \frac{\frac{2}{7} \cdot \frac{2}{3}}{\frac{2}{7} \cdot \frac{2}{3} + \frac{1}{7} \cdot \frac{1}{3}} = \frac{\frac{4}{21}}{\frac{5}{21}} = \frac{4}{5} \end{aligned}$$

39. Let W = event woman resigns; A, B, C are events the person resigning works in store A, B, C , respectively.

$$\begin{aligned} P(C|W) &= \frac{P(W|C)P(C)}{P(W|C)P(C) + P(W|B)P(B) + P(W|A)P(A)} \\ &= \frac{.70 \times \frac{100}{225}}{.70 \times \frac{100}{225} + .60 \times \frac{75}{225} + .50 \times \frac{50}{225}} \\ &= \frac{70}{225} / \frac{140}{225} = \frac{1}{2} \end{aligned}$$

40. (a) F = event fair coin flipped; U = event two-headed coin flipped.

$$\begin{aligned} P(F|H) &= \frac{P(H|F)P(F)}{P(H|F)P(F) + P(H|U)P(U)} \\ &= \frac{\frac{1}{2} \cdot \frac{1}{2}}{\frac{1}{2} \cdot \frac{1}{2} + 1 \cdot \frac{1}{2}} = \frac{\frac{1}{4}}{\frac{3}{4}} = \frac{1}{3} \end{aligned}$$

$$\begin{aligned} (b) \quad P(F|HH) &= \frac{P(HH|F)P(F)}{P(HH|F)P(F) + P(HH|U)P(U)} \\ &= \frac{\frac{1}{4} \cdot \frac{1}{2}}{\frac{1}{4} \cdot \frac{1}{2} + 1 \cdot \frac{1}{2}} = \frac{\frac{1}{8}}{\frac{5}{8}} = \frac{1}{5} \end{aligned}$$

$$\begin{aligned} (c) \quad P(F|HHT) &= \frac{P(HHT|F)P(F)}{P(HHT|F)P(F) + P(HHT|U)P(U)} \\ &= \frac{P(HHT|F)P(F)}{P(HHT|F)P(F) + 0} = 1 \end{aligned}$$

since the fair coin is the only one that can show tails.

41. Note first that since the rat has black parents and a brown sibling, we know that both its parents are hybrids with one black and one brown gene (for if either were a pure black then all their offspring would be black). Hence, both of their offspring's genes are equally likely to be either black or brown.

$$\begin{aligned} (a) \quad P(2 \text{ black genes} | \text{at least one black gene}) &= \frac{P(2 \text{ black genes})}{P(\text{at least one black gene})} \\ &= \frac{1/4}{3/4} = 1/3 \end{aligned}$$

(b) Using the result from part (a) yields the following:

$$\begin{aligned} P(2 \text{ black genes} | 5 \text{ black offspring}) &= \frac{P(2 \text{ black genes})}{P(5 \text{ black offspring})} \\ &= \frac{1/3}{1(1/3) + (1/2)^5(2/3)} \\ &= 16/17 \end{aligned}$$

where $P(5 \text{ black offspring})$ was computed by conditioning on whether the rat had 2 black genes.

42. Let B = event biased coin was flipped; F and U (same as above).

$$\begin{aligned} P(U|H) &= \frac{P(H|U)P(U)}{P(H|U)P(U) + P(H|B)P(B) + P(H|F)P(F)} \\ &= \frac{1 \cdot \frac{1}{3}}{1 \cdot \frac{1}{3} + \frac{3}{4} \cdot \frac{1}{3} + \frac{1}{2} \cdot \frac{1}{3}} = \frac{\frac{1}{3}}{\frac{9}{12}} = \frac{4}{9} \end{aligned}$$

43. Let B be the event that Flo has a blue eyed gene. Using that Jo and Joe both have one blue-eyed gene yields, upon letting X be the number of blue-eyed genes possessed by a daughter of theirs, that

$$P(B) = P(X = 1|X < 2) = \frac{1/2}{3/4} = 2/3$$

Hence, with C being the event that Flo's daughter is blue eyed, we obtain

$$P(C) = P(CB) = P(B)P(C|B) = 1/3$$

44. Let W = event white ball selected.

$$\begin{aligned} P(T|W) &= \frac{P(W|T)P(T)}{P(W|T)P(T) + P(W|H)P(H)} \\ &= \frac{\frac{1}{5} \cdot \frac{1}{2}}{\frac{1}{5} \cdot \frac{1}{2} + \frac{5}{12} \cdot \frac{1}{2}} = \frac{12}{37} \end{aligned}$$

45. Let B_i = event i th ball is black; R_i = event i th ball is red.

$$\begin{aligned} P(B_1|R_2) &= \frac{P(R_2|B_1)P(B_1)}{P(R_2|B_1)P(B_1) + P(R_2|R_1)P(R_1)} \\ &= \frac{\frac{r}{b+r+c} \cdot \frac{b}{b+r}}{\frac{r}{b+r+c} \cdot \frac{b}{b+r} + \frac{r+c}{b+r+c} \cdot \frac{r}{b+r}} \\ &= \frac{rb}{rb + (r+c)r} \\ &= \frac{b}{b + r + c} \end{aligned}$$

46. Let $X(=B \text{ or } =C)$ denote the jailer's answer to prisoner A . Now for instance,

$$\begin{aligned} &P\{A \text{ to be executed}|X = B\} \\ &= \frac{P\{A \text{ to be executed, } X = B\}}{P\{X = B\}} \\ &= \frac{P\{A \text{ to be executed}\}P\{X = B|A \text{ to be executed}\}}{P\{X = B\}} \\ &= \frac{(1/3)P\{X = B|A \text{ to be executed}\}}{1/2}. \end{aligned}$$

Now it is reasonable to suppose that if A is to be executed, then the jailer is equally likely to answer either B or C . That is,

$$P\{X = B | A \text{ to be executed}\} = \frac{1}{2}$$

and so,

$$P\{A \text{ to be executed} | X = B\} = \frac{1}{3}$$

Similarly,

$$P\{A \text{ to be executed} | X = C\} = \frac{1}{3}$$

and thus the jailer's reasoning is invalid. (It is true that if the jailer were to answer B , then A knows that the condemned is either himself or C , but it is twice as likely to be C .)

47. 1. $0 \leq P(A|B) \leq 1$
2. $P(S|B) = \frac{P(SB)}{P(B)} = \frac{P(B)}{P(B)} = 1$
3. For disjoint events A and D

$$\begin{aligned} P(A \cup D|B) &= \frac{P((A \cup D)B)}{P(B)} \\ &= \frac{P(AB \cup DB)}{P(B)} \\ &= \frac{P(AB) + P(DB)}{P(B)} \\ &= P(A|B) + P(D|B) \end{aligned}$$

Direct verification is as follows:

$$\begin{aligned} &P(A|BC)P(C|B) + P(A|BC^c)P(C^c|B) \\ &= \frac{P(ABC)}{P(BC)} \frac{P(BC)}{P(B)} + \frac{P(ABC^c)}{P(BC^c)} \frac{P(BC^c)}{P(B)} \\ &= \frac{P(ABC)}{P(B)} + \frac{P(ABC^c)}{P(B)} \\ &= \frac{P(AB)}{P(B)} \\ &= P(A|B) \end{aligned}$$

Chapter 2

$$1. P\{X = 0\} = \binom{7}{2} / \binom{10}{2} = \frac{14}{30}$$