SOLUTION TO PROBLEM 1.1.

$10\log_{10}$(Average P_b) = Y =$10\log_{10}(\varsigma)$ -G_dG_c [dB] – $G_d(\bar{\rho}$ [dB]). Therefore, G_d is the negative of the slope of the curve in the figure. That is, $G_d = -[Y_1 - Y_2]/[10\log10(\bar{\rho}_1) - 10\log10(\bar{\rho}_2)]$. If we choose $10\log10(\bar{\rho}_1)$ = 20 dB and $10\log10(\bar{\rho}_2)$ = 30 dB, then from the curve we find that $Y_1 = -60$ and $Y_2 = -90$. It follows that the diversity gain $G_d = -[-60 - (-90)]/[20 - 30] = -(30)/(-10) = 3$.

The coding gain (in dB) is obtained from the Y-intercept of the curve obtained by plotting $10\log_{10}(P_b)$ versus $10\log10(\bar{\rho})$. Denote the Y-intercept by B = $10\log_{10}(\varsigma)$ -G_dG_c [dB]. It follows from the figure in the problem that B [dB]= 0 [dB] = ς [dB] -G_dG_c [dB]. If $\varsigma = 1$, then ς [dB] = 0 dB, so G_dG_c [dB] = 0 [dB], which implies that G_c [dB] = 0/3 = 0 [dB]. That is, $G_c = 1$.

Since the diversity order is equal to the diversity gain for most MIMO systems without coding (and in particular for those that use QPSK modulation), then $G_d = 3$ implies that $N_d = 3$ for this system, which means that there are 3 independent spatial diversity channels.

The maximum diversity gain for an N_txN_r MIMO system is N_tN_r. Since the diversity order is equal to the diversity order in this problem, it follows that $N_tN_r = 3$. The only way for this to be true is if $N_t = 3$ and $N_r = 1$ or if $N_t = 1$ and $N_r = 3$. In both cases, the total number of antennas is 4.

SOLUTION TO PROBLEM 1.3

$$\det(A) = ad - bc$$

$$\det(B) = eh - fg$$

$$\det(A)\det(B) = adeh - adfg - bceh + bcfg$$

$$AB = \begin{bmatrix} ae + bg & af + bh \\ ce + dg & cf + dh \end{bmatrix}$$

$$\begin{aligned} \det(AB) &= (ae + bg)(cf + dh) - (af + bh)(ce + dg) \\ &= aecf + aedh + bgcf + bgdh - afce - afdg - bhce - bhdg \\ &= aedh + bgcf - afdg - bhce \end{aligned}$$

$$\Rightarrow \det(A)\det(B) = \det(AB)$$

SOLUTION TO PROBLEM 1.4:

We seek to prove that for any square matrix, $\mathbf{A}$, $\mathbf{A}\mathbf{A}^H$ and $\mathbf{A}^H\mathbf{A}$ are both Hermitian matrices. We will first show that $\mathbf{A}\mathbf{A}^H$ is Hermitian.

To simplify the proof, we will use the fact that for any two matrices, $\mathbf{E}$ and $\mathbf{F}$, having dimensions of $m \times n$ and $n \times p$, respectively, we can express the (i,j)-th component of $\mathbf{EF}$ as follows:

$$[\mathbf{EF}]_{i,j} = \sum_{k=1}^{n} \mathbf{E}_{i,k}\mathbf{F}_{k,j}. \tag{1}$$

Let $\mathbf{C} \triangleq \mathbf{A}\mathbf{A}^H$ and assume that $\mathbf{A}$ is dimensioned $m \times n$. It follows that

$$\begin{aligned} [\mathbf{C}]_{i,j} &= \left[\mathbf{A}\mathbf{A}^H\right]_{i,j} \\ &= \sum_{k=1}^{n} [\mathbf{A}]_{i,k}[\mathbf{A}^H]_{k,j} \\ &= \sum_{k=1}^{n} [\mathbf{A}]_{i,k}[\mathbf{A}]^*_{j,k}. \end{aligned} \tag{2}$$

It follows that

$$\begin{aligned} [\mathbf{C}]^*_{j,i} &= \sum_{k=1}^{n} [\mathbf{A}]^*_{j,k}\left([\mathbf{A}]^*_{i,k}\right)^* \\ &= \sum_{k=1}^{n} [\mathbf{A}]^*_{j,k}[\mathbf{A}]_{i,k} \end{aligned} \tag{3}$$

It follows that $[\mathbf{C}]_{i,j} = [\mathbf{C}]^*_{j,i}$. By the definition in 1.9.1(e), this proves that $\mathbf{C}$ is Hermitian. Similar steps can be used to prove that $\mathbf{A}^H\mathbf{A}$ is also Hermitian.