Chapter 2

2.1 We wish to fabricate a planar waveguide in GaAs for light of wavelength $\lambda_0 = 1.1\,\mu$m that will operate in the single (fundamental) mode. If we assume a planar waveguide like that of Fig. 2.1 with the condition $n_2 - n_1 \gg n_2 - n_3$, what range of values can $n_2 - n_3$ have if $n_2 = 3.4$ and the thickness of the waveguiding layer $t = 3\,\mu$m?

Solution. The cutoff condition is

$$\Delta n = n_2 - n_3 \leq \frac{(2M+1)^2}{32n_2}\left(\frac{\lambda_0}{t}\right)^2 \qquad M = 0, 1, 2, \ldots .$$

Hence for fundamental mode ($M = 0$) propagation only we need

$$\frac{1}{32n_2}\left(\frac{\lambda_0}{t}\right)^2 \leq n_2 - n_3 < \frac{9}{32n_2}\left(\frac{\lambda_0}{t}\right)^2$$
$$\frac{1}{32\times 3.4}\left(\frac{1.15}{3}\right)^2 \leq n_2 - n_3 < \frac{9}{32\times 3.4}\left(\frac{1.15}{3}\right)^2$$
$$0.00135 \leq n_2 - n_3 \ll 0.0122 .$$

2.2 Repeat Problem 2.1 for the case $\lambda_0 = 1.06\,\mu$m, all other parameters remaining unchanged.

Solution.

$$\frac{1}{32\times 3.4}\left(\frac{1.06}{3}\right)^2 \leq n_2 - n_3 < \frac{9}{32\times 3.4}\left(\frac{1.06}{3}\right)^2$$
$$0.00115 \leq n_2 - n_3 < 0.00103$$

2.3 Repeat Problems 2.1 and 2.2 for a waveguide of thickness $t = 6\,\mu$m.

Solution.

$$\frac{1}{32\times 3.4}\left(\frac{1.15}{6}\right)^2 \leq n_2 - n_3 < \frac{9}{32\times 3.4}\left(\frac{1.06}{3}\right)^2$$
$$0.000338 \leq n_2 - n_3 < 0.00304$$
$$\frac{1}{32\times 3.4}\left(\frac{1.06}{6}\right)^2 \leq n_2 - n_3 < \frac{9}{32\times 3.4}\left(\frac{1.06}{6}\right)^2$$
$$0.000287 \leq n_2 - n_3 < 0.00258$$

Note how small the required Δn are in all the cases calculated and also note the strong dependence on wavelength and on waveguide thickness.

2.4 In a planar waveguide like that of Fig. 2.8 with $n_2 = 2.0$, $n_3 = 1.6$, and $n_1 = 1$, what is the angle of propagation of the lowest order mode (θ_0) when cutoff occurs? Is this a maximum or a minimum angle for θ_0?

Solution. At cutoff we know

$$\cos\theta_m = \frac{n_3}{n_2}$$

or

$$\cos\theta_0 = \frac{1.6}{2.0}$$
$$\theta_0 = 36.87^\circ \ .$$

We can tell that this is a maximum angle because

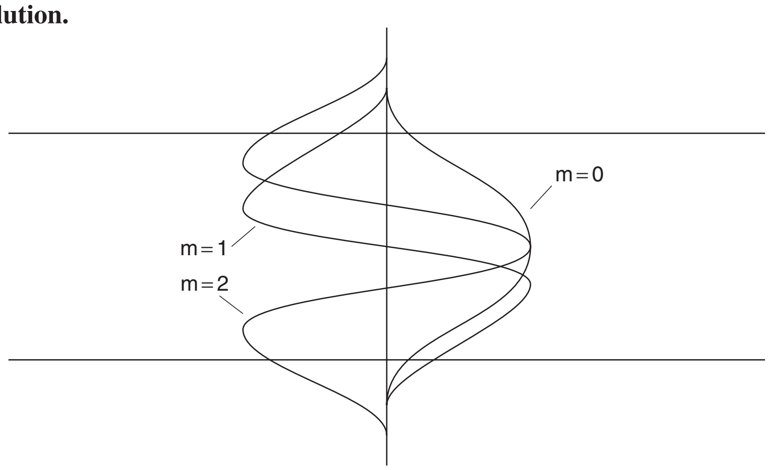
$$\cos\theta_\mathrm{m} = \frac{\beta_\mathrm{m}}{kn_2} \ ,$$

and from physical optics we know the condition required for waveguiding is

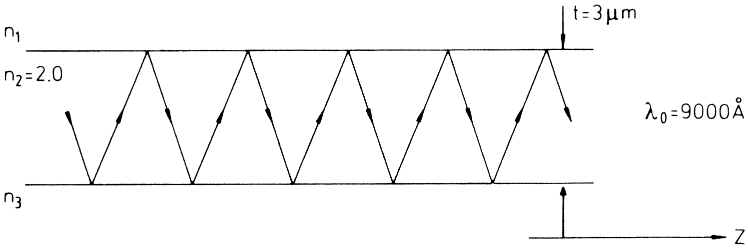
$$\beta_\mathrm{m} \geq kn_3 \ .$$

2.5 Sketch the three lowest order modes in a planar waveguide like that of Fig. 2.8 with $n_1 = n_3 < n_2$.

Solution.

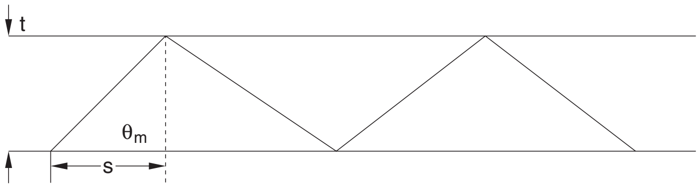


2.6 A mode is propagating in a planar waveguide as shown with $\beta_\mathrm{m} = 0.8\,kn_2$. How many reflections at the $n_1 - n_2$ interface does the ray experience in traveling a distance of 1 cm in the z direction?



Solution.

$$\frac{\beta_m}{kn_2} = 0.8 = \cos\theta_m$$
$$\theta_m = 36.87°$$



from simple geometric considerations the number of bounces from each surface in length L is given by

$$\#\text{ of bounces} = \frac{L}{2s} = \frac{L}{2t\cot\theta_m}$$

for $L = 1$ cm and $t = 3\,\mu$m

$$\#\text{ of bounces} = \frac{1}{2\times 3\times 10^{-4}\times\cot 36.87°} = 1250\ .$$

2.7 Show that the Goos–Hänchen phase shift goes to zero as the cutoff angle is approached for a waveguided optical mode.

Solution. At cutoff for the n_2–n_3 interface,

$$\phi_2 = \theta_c = \sin^{-1}(n_3/n_2)\ .$$

The Goos–Hänchen shift for a TE wave is given by (2.1.21) as

$$\tan\phi_{23} = (n_2^2\sin^2\phi_2 - n_3^2)^{1/2}/(n_2\cos\phi_2)\ ,$$

substituting $\phi_2 = \theta_c = \sin^{-1}(n_3/n_2)$

$$\tan\phi_{23} = (n_2^2\cdot(n_3/n_2)^2 - n_3^2)^{1/2}/n_2\cos(\sin^{-1}[n_3/n_2]) = 0\ .$$

The same result for TM waves can be demonstrated by substituting into equation (2.1.22)

2.8 Calculate the Goos–Hänchen shifts for a TE mode guided with $\beta = 1.85\,k$ in a guide like that of Fig. 2.8, with $n_1 = 1.0$, $n_2 = 2.0$, $n_3 = 1.7$.

Solution.

$$\sin\phi_2 = \frac{\beta}{kn_2} = \frac{1.85k}{k \times 2} = 0.925$$
$$\therefore \phi_2 = 67.7^\circ$$

$$\begin{aligned}\tan\phi_{23} &= \frac{(n_2^2 \sin^2\phi_2 - n_3^2)^{\frac{1}{2}}}{n_2\cos\phi_2} \\ &= \frac{(4(0.925)^2 - (1.7)^2)^{1/2}}{2\cos 67.7^\circ} \\ &= \frac{0.729}{2\cos 67.7^\circ} = 0.961 \\ \phi_{23} &= 43.9^\circ\end{aligned}$$

$$\begin{aligned}\tan\phi_{21} &= \frac{(n_2^2 \sin^2\phi_2 - n_1^2)^{1/2}}{n_2\cos\phi_2} \\ &= \frac{(4(0.925)^2 - 1)^{1/2}}{2\cos 67.7^\circ} \\ &= \frac{1.556}{2\cos 67.7^\circ} = 2.05 \\ \phi_{21} &= 64.0^\circ\end{aligned}$$

The Goos–Hänchen Shifts are

$$\begin{aligned}-2\phi_{23} &= -87.8^\circ \\ -2\phi_{21} &= -128^\circ\ .\end{aligned}$$

2.9 Show by drawing the vectorial relationship between the propagation constants (as in Fig. 2.9) How β, kn_2 and h change in relative magnitude and angle as one goes from the lowest-order mode in a waveguide progressively to higher-order modes.

Solution.

kn_2 = constant
as mode $\uparrow$, θ_m $\uparrow$
$\cos\theta_m$ $\downarrow$
$\beta_m = kn_2 \cos\theta_m$
$\therefore$ as mode $\uparrow$, $\beta_m \downarrow$
$h = \left(n_2^2 k^2 - \beta^2\right)^{1/2}$
$\therefore$ as mode $\uparrow$, $h \uparrow$

See below