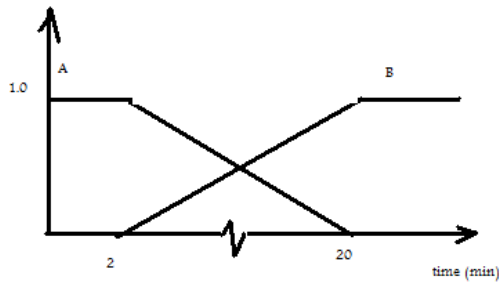


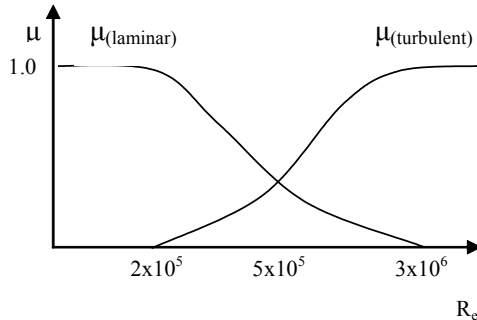
CHAPTER 1

Introduction

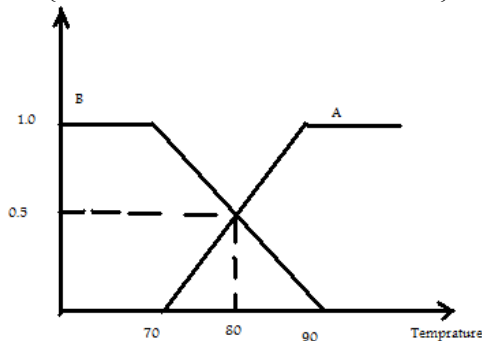
- 1.1 Assume that Extra Fast takes 2 minutes or less (A) Slow Takes 20 minutes or more (B)
 Note that times between 2 min. and 20 min. can be considered as Slow.
 $A = \{1/0 + 1/2 + 0/20 + 0/25\}$
 $B = \{0/1 + 0/2 + 1/20 + 1/25\}$



1.2



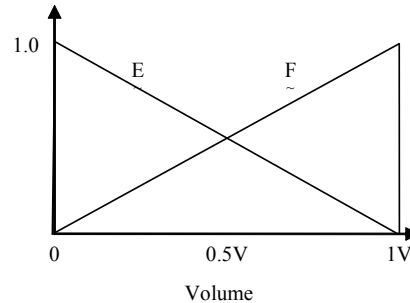
- 1.3 For $T > 90^\circ$ Curling occurs (A)
 For $T < 70^\circ$ Curling does not occur (B)
 $A = \{0/0 + 0/70 + 0.5/80 + 1/90 + 1/100\}$
 $B = \{1/0 + 1/70 + 0.5/80 + 0/90 + 0/100\}$



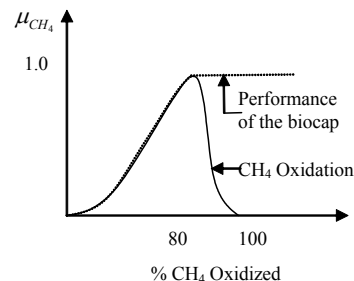
- 1.4 According to the diameters of base(d) and height (h) ratios, solids can be classified into these types. If $0 < d/h < 1$, it is a rod (its membership is $\mu_{A(n)}$); if $1 < d/h < \infty$, it is a disk ($\mu_{C(n)}$); if $d/h = 1$, it is a right cylinder ($\mu_{B(n)}$).

- 1.5 Let V be the volume of the glass, then the condition that glass is full is given by, $F = \frac{0}{0V} + \frac{0.5}{0.5V} + \frac{1}{V}$ & empty,

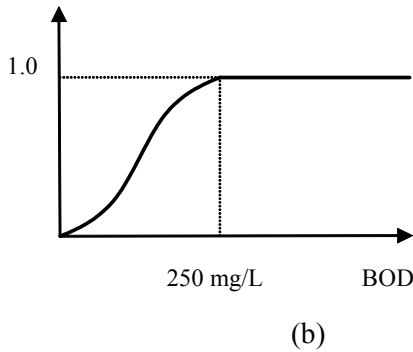
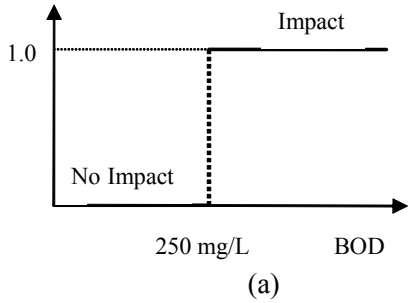
$$E = \frac{1}{0V} + \frac{0.5}{0.5V} + \frac{0}{V}.$$



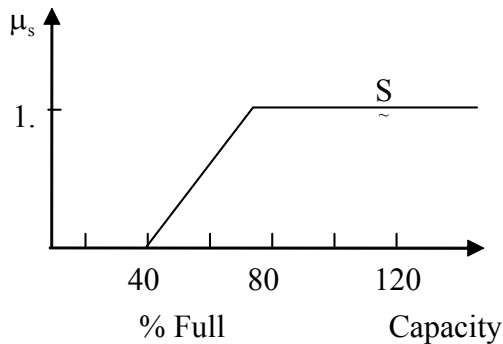
- 1.6 Since landfills are classified as best if they are capable of oxidizing 80% of the methane that originates, the membership function should achieve a full membership here. A reasonable membership function for the % CH_4 oxidation may be as follows:



1.7 The crisp set Impact membership function, figure (a) above has values of 1 for any BOD greater than or equal to 250mg/L while any value of BOD less than 250mg/L has a value of zero. However the membership function for the fuzzy set, figure (b) above, has some Impact values below 250mg/L.



1.8 .

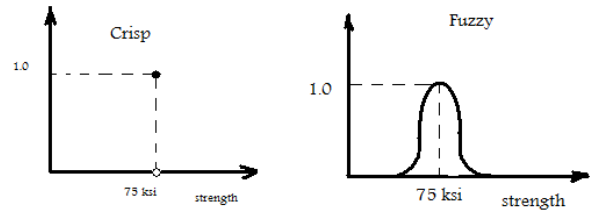


1.9 Crisp:

$$\chi_{LD_{50}} = 1, \text{ for } 0 < LD_{50} \leq 5000 \text{ mg/kg}$$

$$\chi_{LD_{50}} = 0, \text{ for } LD_{50} > 5000 \text{ mg/kg and } LD_{50} \leq 0$$

1.10



1.11 Fuzzy Sets can be represented explicitly by families of parameterized functions, the most common being the following:
 a) Triangular Functions

$$A(x) = \begin{cases} 0, & x \leq 2 \\ \frac{x-a}{m-a}, & x \in [2, m] \\ \frac{b-x}{b-m}, & x \in [m, b] \\ 0, & x \geq b \end{cases}$$

Where m , a , and b denote the modal value, the lower and upper bound values, respectively, for all nonzero values of $A(x)$.

b) Trapezoidal Function

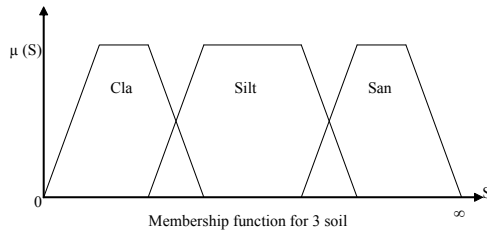
$$A(x) = \begin{cases} 0, & x < a \\ \frac{x-a}{m-a}, & x \in [a, m] \\ 1, & x \in [m, n] \\ \frac{b-x}{b-n}, & x \in [n, b] \\ 0, & x > b \end{cases}$$

c) Gaussian Function

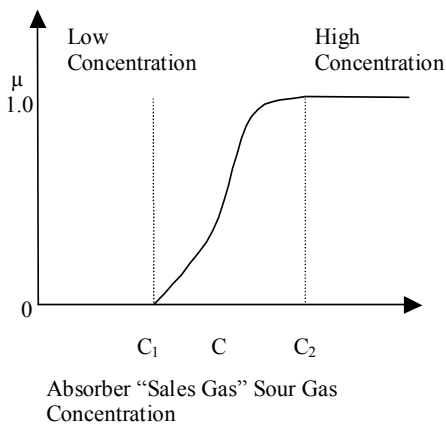
$$A(x) = e^{-k(x-m)^2}$$

Where $k > 0$

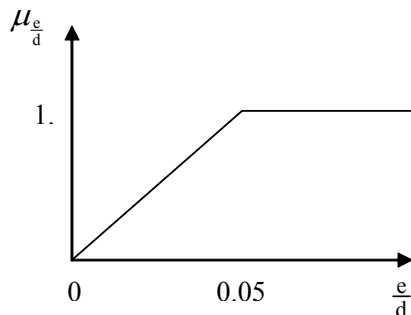
- 1.12 Fuzzy sets are useful in this situation where there is an inherent overlap among soil types. Clay has a smaller particle size than that of Sand however it is often difficult to distinguish Clay from Silt. This is also true for particle size between sand and silt.



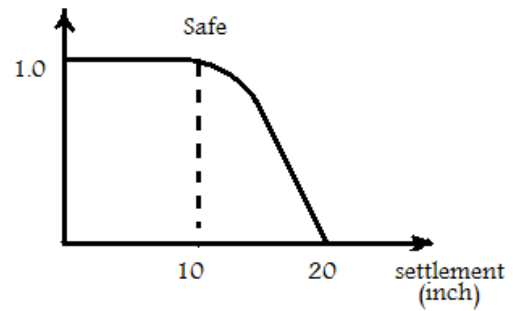
- 1.13 The membership function for the absorber "sales gas" sour gas concentration as a function of concentration C , with C_1 and C_2 is as follows.



- 1.14 The relation shows that the load becomes more eccentric as it approaches $\frac{e}{d} = 0.05$, it remains eccentric thereafter

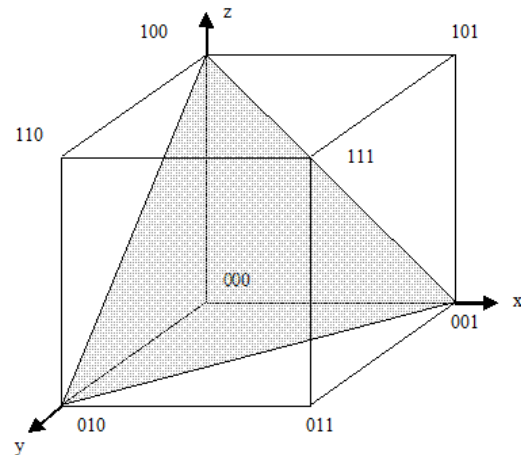


- 1.15



- 1.16 The geometric shape can resemble a disk, a cylinder, or a rod depending on the aspect ratio of d/h . For $d/h \ll 1$ the shape of the object approaches a long rod; in fact, as d/h approaches 0 the shape approaches a line. For $d/h \gg 1$ the object approaches the shape of a flat disk; as d/h approaches infinity the object approaches a circular area. For other values of this aspect ratio, e.g. for $d/h \approx 1$, the shape is typical of what we would call a "right circular cylinder."

- 1.17



CHAPTER 2

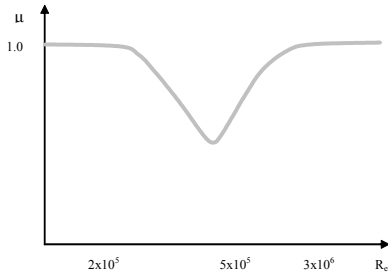
Classical Sets and Fuzzy Sets

2.1

a)

$\tilde{T} \cup \tilde{L}$

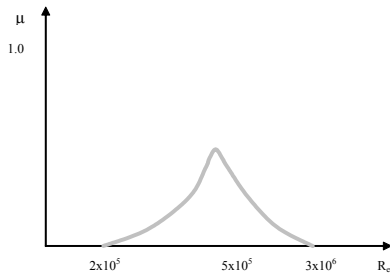
$$\mu_{\tilde{T} \cup \tilde{L}}(R_e) = \mu_{\tilde{T}}(R_e) \vee \mu_{\tilde{L}}(R_e)$$



b)

$\tilde{T} \cap \tilde{L}$

$$\mu_{\tilde{T} \cap \tilde{L}}(R_e) = \mu_{\tilde{T}}(R_e) \wedge \mu_{\tilde{L}}(R_e)$$



c)

$$\tilde{T} = \left\{ \frac{0}{0} + \frac{0.1}{2 \times 10^5} + \frac{0.75}{5 \times 10^5} + \frac{1}{3 \times 10^6} \right\}$$

$$\tilde{L} = \left\{ \frac{1.0}{0} + \frac{0.9}{2 \times 10^5} + \frac{0.25}{5 \times 10^5} + \frac{0}{3 \times 10^6} \right\}$$

2.2 (a) Select ponds number 4 and 5

$$(b) A \cup B = \{0.5/1 + 0.6/2 + 0.8/3 + 1/4 + 1/5\}$$

2.3

$$A \cup B = \{0.2/1 + 0.3/2 + 0.6/3 + 0.9/4\}$$

$$A \cap B = \{0.15/1 + 0.25/2 + 0.5/3 + 0.8/4\}$$

$$A|B = A \cap \bar{B}, \bar{B} = \left\{ \frac{0.8}{1} + \frac{0.7}{2} + \frac{0.5}{3} + \frac{0.2}{4} \right\}$$

$$A|B = \{0.15/1 + 0.85/2 + 0.5/3 + 0.2/4\}$$

$$B|A = B \cap \bar{A},$$

$$\bar{A} = \{0.85/1 + 0.75/2 + 0.4/3 + 0.1/4\}$$

$$B|A = \{0.2/1 + 0.3/2 + 0.4/3 + 0.1/4\}$$

2.4 a)

$$\tilde{D}_1 \cup \tilde{D}_2 = \frac{0}{1.0} + \frac{0.4}{1.5} + \frac{0.8}{2.0} + \frac{0.9}{2.5} + \frac{1.0}{3.0}$$

b)

$$\tilde{D}_1 \cap \tilde{D}_2 = \frac{1}{1.0} + \frac{0.6}{1.5} + \frac{0.2}{2.0} + \frac{0.1}{2.5} + \frac{0}{3.0}$$

$$c) \bar{\tilde{D}}_1 = \frac{0}{1.0} + \frac{0.25}{1.5} + \frac{0.7}{2.0} + \frac{0.85}{2.5} + \frac{1.0}{3.0}$$

$$d) \bar{\tilde{D}}_2 = \frac{0}{1.0} + \frac{0.4}{1.5} + \frac{0.8}{2.0} + \frac{0.9}{2.5} + \frac{1.0}{3.0}$$

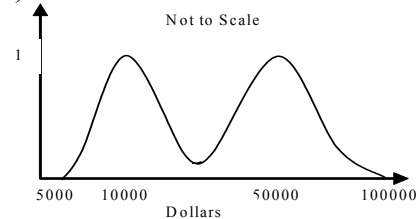
$$e) \tilde{D}_1 / \tilde{D}_2 = \tilde{D}_1 \cap \bar{\tilde{D}}_2 =$$

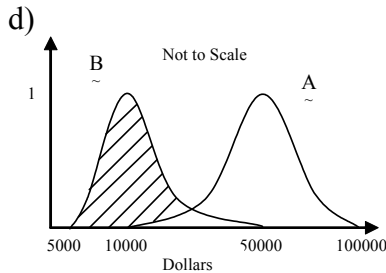
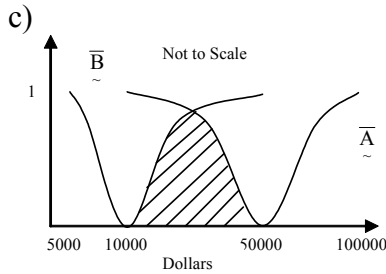
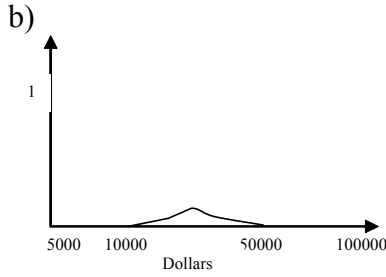
$$\frac{1}{1.0} + \frac{0.6}{1.5} + \frac{0.2}{2.0} + \frac{0.1}{2.5} + \frac{0}{3.0}$$

$$f) \overline{\tilde{D}_1 \cup \tilde{D}_2} = \overline{\tilde{D}_1} \cap \overline{\tilde{D}_2} =$$

$$\frac{0}{1.0} + \frac{0.25}{1.5} + \frac{0.7}{2.0} + \frac{0.85}{2.5} + \frac{1}{3.0}$$

2.5 a)





2.6 a) $\bar{A} \cup \bar{B} = \frac{0.2}{0} + \frac{0.5}{1} + \frac{1}{2} + \frac{0.7}{3} + \frac{0.2}{4}$

b) $\bar{A} \cap \bar{B} = \frac{0.1}{0} + \frac{0.5}{1} + \frac{1}{2} + \frac{0.6}{3} + \frac{0.1}{4}$

c) $\bar{A} = \frac{0.9}{0} + \frac{0.5}{1} + \frac{1}{2} + \frac{0.6}{3} + \frac{0.8}{4}$

d) $\bar{B} = \frac{0.8}{0} + \frac{0.5}{1} + \frac{0}{2} + \frac{0.3}{3} + \frac{0.9}{4}$

2.7 a)

$\bar{M} \cup \bar{F} = \frac{1}{0} + \frac{1}{10} + \frac{1}{20} + \frac{0.7}{40} + \frac{0.9}{80} + \frac{0}{100}$

b)

$\bar{M} \cap \bar{F} = \frac{0.3}{1} + \frac{0.7}{10} + \frac{0.4}{20} + \frac{0}{40} + \frac{0}{80} + \frac{0}{100}$

c)

$\bar{M} = \frac{0}{1} + \frac{0.3}{10} + \frac{0.6}{20} + \frac{1}{40} + \frac{1}{80} + \frac{1}{100}$

d) $\bar{F} = \frac{0.7}{0} + \frac{0}{10} + \frac{0}{20} + \frac{0.5}{40} + \frac{0.8}{80} + \frac{1}{100}$

e) $\bar{C} \cap \bar{F} = \frac{0}{1} + \frac{0}{10} + \frac{0}{20} + \frac{0.5}{40} + \frac{0.8}{80} + \frac{1}{100}$

f) $\bar{M} \cap \bar{C} = \bar{M} \cup \bar{C}$

$= \frac{0.3}{1} + \frac{0.7}{10} + \frac{0.4}{20} + \frac{0}{40} + \frac{0}{80} + \frac{0}{100}$

2.8

$Flow_1 \cap Flow_2$

$= \frac{0}{0} + \frac{0.45}{20} + \frac{0.6}{40} + \frac{0.45}{60} + \frac{0.3}{80} + \frac{0.1}{100}$

$Flow_1 \cup Flow_2$

$= \frac{1}{0} + \frac{0.8}{20} + \frac{0.65}{40} + \frac{0.8}{60} + \frac{0.95}{80} + \frac{1.0}{100}$

$Flow_1 | Flow_2 = Flow_1 \cap \bar{Flow_2}$

$\bar{Flow_2} = \left\{ \frac{1.0}{0} + \frac{0.55}{20} + \frac{0.4}{40} + \frac{0.2}{60} + \frac{0.05}{80} + \frac{0}{100} \right\}$

$Flow_1 | Flow_2$

$= \frac{1.0}{0} + \frac{0.55}{20} + \frac{0.4}{40} + \frac{0.2}{60} + \frac{0.05}{80} + \frac{0}{100}$

$Flow_2 | Flow_1 = Flow_2 \cap \bar{Flow_1}$

$\bar{Flow_1} = \left\{ \frac{0}{0} + \frac{0.2}{20} + \frac{0.35}{40} + \frac{0.55}{60} + \frac{0.7}{80} + \frac{0.9}{100} \right\}$

$Flow_2 | Flow_1$

$= \frac{0}{0} + \frac{0.2}{20} + \frac{0.35}{40} + \frac{0.55}{60} + \frac{0.7}{80} + \frac{0.9}{100}$

2.9 a) $\bar{A} \cup \bar{B} =$

$\frac{0}{0.73} + \frac{0.8}{0.735} + \frac{1}{0.74} + \frac{1}{0.745} + \frac{0.6}{0.750}$

b) $\bar{A} \cap \bar{B} =$

$$\frac{0}{0.73} + \frac{0.4}{0.735} + \frac{0.8}{0.74} + \frac{0.6}{0.745} + \frac{0}{0.750}$$

$$c) \quad \bar{A} = \frac{1}{0.73} + \frac{0.2}{0.735} + \frac{0}{0.74} + \frac{0.4}{0.745} + \frac{1}{0.750}$$

$$d) \quad A|B = A \cap B = \frac{0}{0.73} + \frac{0.6}{0.735} + \frac{0.2}{0.74} + \frac{0}{0.745} + \frac{0}{0.750}$$

$$f) \quad \overline{A \cup B} = \bar{A} \cap \bar{B} = \frac{1}{0.73} + \frac{0.2}{0.735} + \frac{0}{0.74} + \frac{0}{0.745} + \frac{0.4}{0.75}$$

$$e) \quad \overline{A \cap B} = \bar{A} \cup \bar{B} = \frac{1}{0.73} + \frac{0.6}{0.735} + \frac{0.2}{0.74} + \frac{0.4}{0.745} + \frac{1}{0.75}$$

2.10

$$A \cup B = \left\{ \frac{0.15}{winter} + \frac{0.55}{spring} + \frac{0.9}{summer} + \frac{0.25}{fall} \right\}$$

$$A \cap B = \left\{ \frac{0.1}{winter} + \frac{0.3}{spring} + \frac{0.52}{summer} + \frac{0.2}{fall} \right\}$$

$$A|B = A \cap \bar{B},$$

$$\bar{B} = \left\{ \frac{0.9}{winter} + \frac{0.45}{spring} + \frac{0.1}{summer} + \frac{0.8}{fall} \right\}$$

$$A|B = \left\{ \frac{0.15}{winter} + \frac{0.33}{spring} + \frac{0.1}{summer} + \frac{0.25}{fall} \right\}$$

$$B|A = B \cap \bar{A},$$

$$\bar{A} = \left\{ \frac{0.85}{winter} + \frac{0.67}{spring} + \frac{0.48}{summer} + \frac{0.75}{fall} \right\}$$

$$B|A = \left\{ \frac{0.1}{winter} + \frac{0.55}{spring} + \frac{0.48}{summer} + \frac{0.2}{fall} \right\}$$

2.11

$$MP = \frac{0}{0} + \frac{0.2}{1} + \frac{0.4}{2} + \frac{0.6}{3} + \frac{0.8}{4} + \frac{1}{5}$$

$$+ \frac{0.8}{6} + \frac{0.6}{7} + \frac{0.4}{8} + \frac{0.2}{9} + \frac{0}{10}$$

$$\overline{MP} = \frac{1}{0} + \frac{0.8}{1} + \frac{0.6}{2} + \frac{0.4}{3} + \frac{0.2}{4} + \frac{0}{5}$$

$$+ \frac{0.2}{6} + \frac{0.4}{7} + \frac{0.6}{8} + \frac{0.8}{9} + \frac{1}{10}$$

$$HP = \frac{0}{0} + \frac{0.1}{1} + \frac{0.2}{2} + \frac{0.3}{3} + \frac{0.4}{4} + \frac{0.5}{5}$$

$$+ \frac{0.6}{6} + \frac{0.7}{7} + \frac{0.8}{8} + \frac{0.9}{9} + \frac{1}{10}$$

$$\overline{HP} = \frac{1}{0} + \frac{0.9}{1} + \frac{0.8}{2} + \frac{0.7}{3} + \frac{0.6}{4} + \frac{0.5}{5}$$

$$+ \frac{0.4}{6} + \frac{0.3}{7} + \frac{0.2}{8} + \frac{0.1}{9} + \frac{0}{10}$$

$$MP/H P = \frac{0}{0} + \frac{0.2}{1} + \frac{0.4}{2} + \frac{0.6}{3} + \frac{0.6}{4} + \frac{0.5}{5}$$

$$+ \frac{0.4}{6} + \frac{0.3}{7} + \frac{0.2}{8} + \frac{0.1}{9} + \frac{0}{10}$$

$$MP \cup HP = \frac{0}{0} + \frac{0.2}{1} + \frac{0.4}{2} + \frac{0.6}{3} + \frac{0.8}{4} + \frac{1}{5}$$

$$+ \frac{0.8}{6} + \frac{0.7}{7} + \frac{0.8}{8} + \frac{0.9}{9} + \frac{1}{10}$$

$$\overline{MP} \cap \overline{HP} = \frac{1}{0} + \frac{0.8}{1} + \frac{0.6}{2} + \frac{0.4}{3} + \frac{0.2}{4} + \frac{0}{5}$$

$$+ \frac{0.2}{6} + \frac{0.3}{7} + \frac{0.2}{8} + \frac{0.1}{9} + \frac{0}{10}$$

2.12

$$Q \cup C = \frac{1}{0} + \frac{1}{1} + \frac{0.8}{2} + \frac{0.4}{5} + \frac{0.6}{7} + \frac{0.8}{9} + \frac{1}{10}$$

$$Q \cap C = \frac{0}{0} + \frac{0}{1} + \frac{0}{2} + \frac{0.3}{5} + \frac{0.1}{7} + \frac{0}{9} + \frac{0}{10}$$

$$\bar{Q} = \frac{0}{0} + \frac{0}{1} + \frac{0.2}{2} + \frac{0.7}{5} + \frac{0.9}{7} + \frac{1}{9} + \frac{1}{10}$$

$$\bar{C} = \frac{1}{0} + \frac{1}{1} + \frac{1}{2} + \frac{0.6}{5} + \frac{0.4}{7} + \frac{0.2}{9} + \frac{0}{10}$$

$$Q/C = \frac{1}{0} + \frac{1}{1} + \frac{0.8}{2} + \frac{0.3}{5} + \frac{0.1}{7} + \frac{0}{9} + \frac{0}{10}$$

$$\overline{Q \cup C} = \frac{0}{0} + \frac{0}{1} + \frac{0.2}{2} + \frac{0.6}{5} + \frac{0.4}{7} + \frac{0.2}{9} + \frac{0}{10}$$

$$\overline{Q \cap C} = \frac{1}{0} + \frac{1}{1} + \frac{1}{2} + \frac{0.7}{5} + \frac{0.9}{7} + \frac{1}{9} + \frac{1}{10}$$

$$Q \cup \bar{Q} = \frac{1}{0} + \frac{1}{1} + \frac{0.8}{2} + \frac{0.7}{5} + \frac{0.9}{7} + \frac{1}{9} + \frac{1}{10}$$

$$\begin{aligned} Q \cap \bar{Q} &= \frac{0}{0} + \frac{0}{1} + \frac{0.2}{2} + \frac{0.3}{5} + \frac{0.1}{7} + \frac{0}{9} + \frac{0}{10} \\ C \cup \bar{C} &= \frac{1}{0} + \frac{1}{1} + \frac{1}{2} + \frac{0.6}{5} + \frac{0.6}{7} + \frac{0.8}{9} + \frac{1}{10} \\ C \cap \bar{C} &= \frac{0}{0} + \frac{0}{1} + \frac{0}{2} + \frac{0.4}{5} + \frac{0.4}{7} + \frac{0.2}{9} + \frac{0}{10} \end{aligned}$$

2.13

$$\begin{aligned} A &= \frac{0.14}{0} + \frac{0.32}{1} + \frac{0.62}{2} + \frac{0.88}{3} + \frac{1}{4} + \frac{0.88}{5} \\ &\quad + \frac{0.61}{6} + \frac{0.32}{7} + \frac{0.14}{8} + \frac{0.04}{9} + \frac{0}{10} \\ B &= \frac{0.003}{0} + \frac{0.0022}{1} + \frac{0.01}{2} + \frac{0.04}{3} + \frac{0.14}{4} \\ &\quad + \frac{0.32}{5} + \frac{0.61}{6} + \frac{0.88}{7} + \frac{1}{8} + \frac{0.88}{9} + \frac{0.61}{10} \\ \bar{A} &= \frac{0.86}{0} + \frac{0.68}{1} + \frac{0.38}{2} + \frac{0.12}{3} + \frac{0}{4} + \frac{0.12}{5} \\ &\quad + \frac{0.39}{6} + \frac{0.68}{7} + \frac{0.86}{8} + \frac{0.96}{9} + \frac{1}{10} \\ \bar{B} &= \frac{0.997}{0} + \frac{0.998}{1} + \frac{0.99}{2} + \frac{0.96}{3} + \frac{0.86}{4} \\ &\quad + \frac{0.68}{5} + \frac{0.39}{6} + \frac{0.12}{7} + \frac{0}{8} + \frac{0.12}{9} + \frac{0.39}{10} \\ A \cup B &= \frac{0.14}{0} + \frac{0.32}{1} + \frac{0.61}{2} + \frac{0.88}{3} + \frac{1}{4} \\ &\quad + \frac{0.88}{5} + \frac{0.61}{6} + \frac{0.88}{7} + \frac{1}{8} + \frac{0.88}{9} + \frac{0.61}{10} \\ A \cap B &= \frac{0.00}{0} + \frac{0.002}{1} + \frac{0.01}{2} + \frac{0.04}{3} + \frac{0.14}{4} \\ &\quad + \frac{0.32}{5} + \frac{0.61}{6} + \frac{0.32}{7} + \frac{0.14}{8} + \frac{0.04}{9} + \frac{0.01}{10} \\ \bar{A} \cup \bar{B} &= \frac{0.99}{0} + \frac{0.998}{1} + \frac{0.99}{2} + \frac{0.96}{3} + \frac{0.86}{4} \\ &\quad + \frac{0.68}{5} + \frac{0.39}{6} + \frac{0.68}{7} + \frac{0.86}{8} + \frac{0.96}{9} + \frac{0.99}{10} \\ \bar{A} \cap \bar{B} &= \frac{0.86}{0} + \frac{0.68}{1} + \frac{0.39}{2} + \frac{0.12}{3} + \frac{0}{4} + \frac{0.12}{5} \\ &\quad + \frac{0.39}{6} + \frac{0.12}{7} + \frac{0}{8} + \frac{0.12}{9} + \frac{0.39}{10} \\ A/B &= \frac{0.14}{0} + \frac{0.32}{1} + \frac{0.61}{2} + \frac{0.88}{3} + \frac{0.86}{4} \end{aligned}$$

$$\begin{aligned} &+ \frac{0.68}{5} + \frac{0.39}{6} + \frac{0.12}{7} + \frac{0}{8} + \frac{0.04}{9} + \frac{0.01}{10} \\ \bar{A} \cap B &= \frac{0.00}{0} + \frac{0.002}{1} + \frac{0.01}{2} + \frac{0.04}{3} + \frac{0}{4} \\ &\quad + \frac{0.12}{5} + \frac{0.39}{6} + \frac{0.68}{7} + \frac{0.86}{8} + \frac{0.88}{9} + \frac{0.61}{10} \\ A \cup \bar{A} &= \frac{0.86}{0} + \frac{0.68}{1} + \frac{0.61}{2} + \frac{0.88}{3} + \frac{1}{4} \\ &\quad + \frac{0.88}{5} + \frac{0.61}{6} + \frac{0.68}{7} + \frac{0.86}{8} + \frac{0.96}{9} + \frac{0.99}{10} \\ A \cap \bar{A} &= \frac{0.14}{0} + \frac{0.32}{1} + \frac{0.39}{2} + \frac{0.12}{3} + \frac{0}{4} \\ &\quad + \frac{0.12}{5} + \frac{0.39}{6} + \frac{0.32}{7} + \frac{0.14}{8} + \frac{0.04}{9} + \frac{0.01}{10} \\ B \cup \bar{B} &= \frac{0.9997}{0} + \frac{0.998}{1} + \frac{0.99}{2} + \frac{0.96}{3} + \frac{0.86}{4} \\ &\quad + \frac{0.68}{5} + \frac{0.61}{6} + \frac{0.88}{7} + \frac{1}{8} + \frac{0.88}{9} + \frac{0.61}{10} \\ B \cap \bar{B} &= \frac{0.0003}{0} + \frac{0.002}{1} + \frac{0.01}{2} + \frac{0.04}{3} + \frac{0.14}{4} \\ &\quad + \frac{0.32}{5} + \frac{0.39}{6} + \frac{0.12}{7} + \frac{0}{8} + \frac{0.12}{9} + \frac{0.39}{10} \end{aligned}$$

2.14 $A = \text{"fast" chips}$

$$\begin{aligned} A &= \frac{0}{1} + \frac{0}{2} + \frac{0}{3} + \frac{0}{4} + \frac{0.2}{5} + \frac{0.6}{6} + \frac{1}{7} + \frac{1}{8} \\ D &= \text{"hot" chips} \\ D &= \frac{0}{1} + \frac{0}{2} + \frac{0}{3} + \frac{0.5}{4} + \frac{0.1}{5} + \frac{1}{6} + \frac{0.5}{7} + \frac{1}{8} \\ A \cup D &= \frac{0}{1} + \frac{0}{2} + \frac{0}{3} + \frac{0.5}{4} + \frac{0.2}{5} + \frac{1}{6} + \frac{1}{7} + \frac{1}{8} \\ A \cap D &= \frac{0}{1} + \frac{0}{2} + \frac{0}{3} + \frac{0}{4} + \frac{0.1}{5} + \frac{0.6}{6} + \frac{0.5}{7} + \frac{1}{8} \\ \bar{A} &= \frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{0.8}{5} + \frac{0.4}{6} + \frac{0}{7} + \frac{0}{8} \\ \bar{A} \cap \bar{D} &= \frac{0}{1} + \frac{0}{2} + \frac{0}{3} + \frac{0}{4} + \frac{0.2}{5} + \frac{0}{6} + \frac{0.5}{7} + \frac{0}{8} \\ \bar{A} \cap D &= \frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \frac{0.5}{4} + \frac{0.8}{5} + \frac{0}{6} + \frac{0}{7} + \frac{0}{8} \\ \bar{A} \cup \bar{D} &= \frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{0.9}{5} + \frac{0.4}{6} + \frac{0.5}{7} + \frac{0}{8} \end{aligned}$$