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# Chapter 1

1. **THINK** In this problem we're given the radius of Earth and asked to compute its circumference, surface area, and volume.

**EXPRESS** Assuming Earth to be a sphere of radius

$$R_E = \left(6.37\times10^6\ \text{m}\right)\left(10^{-3}\ \text{km/m}\right) = 6.37\times10^3\ \text{km},$$

we find that the corresponding circumference, surface area, and volume are

$$C = 2\pi R_E, \quad A = 4\pi R_E^2, \quad V = \frac{4\pi}{3}R_E^3.$$

These geometric formulas are given in Appendix E.

**ANALYZE** Using the formulas, we find (a) the circumference to be

$$C = 2\pi R_E = 2\pi(6.37\times10^3\ \text{km}) = 4.00\times10^4\ \text{km},$$

(b) the surface area to be

$$A = 4\pi R_E^2 = 4\pi\left(6.37\times10^3\ \text{km}\right)^2 = 5.10\times10^8\ \text{km}^2,$$

and (c) the volume to be

$$V = \frac{4\pi}{3}R_E^3 = \frac{4\pi}{3}\left(6.37\times10^3\ \text{km}\right)^3 = 1.08\times10^{12}\ \text{km}^3.$$

**LEARN** From the formulas, we see that $C \sim R_E$, $A \sim R_E^2$, and $V \sim R_E^3$. The ratios of volume to surface area and surface area to circumference are $V/A = R_E/3$ and $A/C = 2R_E$.

5. **THINK** This problem deals with conversion of furlongs to rods and chains, all of which are units for distance.

**EXPRESS** Given that $1\ \text{furlong} = 201.168\ \text{m}$, $1\ \text{rod} = 5.0292\ \text{m}$, and $1\ \text{chain} = 20.117\ \text{m}$, the relevant conversion factors are

$$1.0\ \text{furlong} = 201.168\ \text{m} = (201.168\ \cancel{\text{m}})\frac{1\ \text{rod}}{5.0292\ \cancel{\text{m}}} = 40\ \text{rods}$$

and

$$1.0 \text{ furlong} = 201.168 \text{ m} = (201.168 \cancel{\text{m}})\frac{1 \text{ chain}}{20.117 \cancel{\text{m}}} = 10 \text{ chains}.$$

Note the cancellation of m (meters), the unwanted unit.

**ANALYZE** Using the above conversion factors, we find the distance $d$ (a) in *rods* to be

$$d = 4.0 \text{ furlongs} = \left(4.0 \cancel{\text{furlongs}}\right)\frac{40 \text{ rods}}{1 \cancel{\text{furlong}}} = 160 \text{ rods}$$

and (b) in *chains* to be

$$d = 4.0 \text{ furlongs} = \left(4.0 \cancel{\text{furlongs}}\right)\frac{10 \text{ chains}}{1 \cancel{\text{furlong}}} = 40 \text{ chains}.$$

**LEARN** Since 4 furlongs is about 800 m, this distance is approximately equal to 160 rods ($1 \text{ rod} \approx 5 \text{ m}$) and 40 chains ($1 \text{ chain} \approx 20 \text{ m}$). So our results make sense.

17. **THINK** In this problem we are asked to rank five clocks based on their performance as timekeepers.

**EXPRESS** We first note that none of the clocks advance by exactly 24 h in a 24 h period but this is not the most important criterion for judging their quality for measuring time intervals. What is important here is that a clock advance by the same (or nearly the same) amount in each 24 h period. The clock reading can then easily be adjusted to give the correct interval.

**ANALYZE** The chart below gives the corrections (in seconds) that must be applied to the reading on each clock for each 24 h period. The entries were determined by subtracting the clock reading at the end of the interval from the clock reading at the beginning.

| CLOCK | Sun. –Mon. | Mon. –Tues. | Tues. –Wed. | Wed. –Thurs. | Thurs. –Fri. | Fri. –Sat. |
|---|---|---|---|---|---|---|
| A | –16 | –16 | –15 | –17 | –15 | –15 |
| B | –3 | +5 | –10 | +5 | +6 | –7 |
| C | –58 | –58 | –58 | –58 | –58 | –58 |
| D | +67 | +67 | +67 | +67 | +67 | +67 |
| E | +70 | +55 | +2 | +20 | +10 | +10 |

Clocks C and D are both good timekeepers in the sense that each is consistent in its daily drift; thus, C and D are easily made "perfect" with simple and predictable corrections. The correction for clock C is less than the correction for clock D, so we judge clock C to be the best and clock D to be the next best. The correction that must be applied to clock A is in the range from 15 s to 17 s. For clock B it is the range from –5 s to +10 s, and for clock E it is in the range from –70 s to –2 s. After C and D, A has the smallest range of correction, B has the next smallest range, and E has the greatest range. From best to worst, the ranking of the clocks is C, D, A, B, E.

**LEARN** Of the five clocks, the readings in clocks A, B, and E jump around from one 24 h period to another, making it difficult to correct them.

23. **THINK** This problem consists of two parts. In the first part, we are asked to find the mass of water, given its volume and density; the second part deals with the mass flow rate of water, which is expressed as kg/s in SI units.

**EXPRESS** From the definition of density, $\rho = m/V$, we see that mass can be calculated as $m = \rho V$, the product of the volume of water and its density. With 1 g = $1 \times 10^{-3}$ kg and 1 cm$^3$ = $(1 \times 10^{-2}\ \text{m})^3 = 1 \times 10^{-6}\ \text{m}^3$, the density of water in SI units (kg/m$^3$) is

$$\rho = 1\ \text{g/cm}^3 = \left(\frac{1\ \text{g}}{\text{cm}^3}\right)\left(\frac{10^{-3}\ \text{kg}}{\text{g}}\right)\left(\frac{\text{cm}^3}{10^{-6}\ \text{m}^3}\right) = 1\times 10^3\ \text{kg/m}^3.$$

To obtain the flow rate, we divide the total mass of the water by the time taken to drain it.

**ANALYZE** (a) From $m = \rho V$, the mass of a cubic meter of water is

$$m = \rho V = (1\times10^3\ \text{kg/m}^3)(1\ \text{m}^3) = 1\times10^3\ \text{kg}.$$

(b) The total mass of water in the container is

$$M = \rho V = (1\times10^3\ \text{kg/m}^3)(5700\ \text{m}^3) = 5.70\times10^6\ \text{kg},$$

and the time elapsed is $t$ = (10 h)(3600 s/h) = $3.6 \times 10^4$ s. Thus, the mass flow rate $R$ is

$$R = \frac{M}{t} = \frac{5.70\times10^6\ \text{kg}}{3.6\times10^4\ \text{s}} = 158\ \text{kg/s}.$$

**LEARN** In terms of volume, the drain rate can be expressed as

$$R' = \frac{V}{t} = \frac{5700\ \text{m}^3}{3.6\times10^4\ \text{s}} = 0.158\ \text{m}^3/\text{s} \approx 42\ \text{gal/s}.$$

The greater the flow rate, the less time is required to drain a given amount of water.

33. **THINK** In this problem we are asked to differentiate between three types of tons: *displacement* ton, *freight* ton, and *register* ton, all of which are units of volume.

**EXPRESS** The three different tons are defined in terms of *barrel bulk*, with $1 \text{ barrel bulk} = 0.1415 \text{ m}^3 = 4.0155 \text{ U.S. bushels}$ (using $1 \text{ m}^3 = 28.378 \text{ U.S. bushels}$). Thus, in terms of U.S. bushels, we have

$$1 \text{ displacement ton} = (7 \text{ barrels bulk})\left(\frac{4.0155 \text{ U.S. bushels}}{1 \text{ barrel bulk}}\right) = 28.108 \text{ U.S. bushels}$$

$$1 \text{ freight ton} = (8 \text{ barrels bulk})\left(\frac{4.0155 \text{ U.S. bushels}}{1 \text{ barrel bulk}}\right) = 32.124 \text{ U.S. bushels}$$

$$1 \text{ register ton} = (20 \text{ barrels bulk})\left(\frac{4.0155 \text{ U.S. bushels}}{1 \text{ barrel bulk}}\right) = 80.31 \text{ U.S. bushels.}$$

**ANALYZE** (a) The difference between 73 "freight" tons and 73 "displacement" tons is

$$\begin{aligned}\Delta V &= 73(1 \text{ freight ton} - 1 \text{ displacement ton}) = 73(32.124 \text{ U.S. bushels} - 28.108 \text{ U.S. bushels})\\ &= 293.168 \text{ U.S. bushels} \approx 293 \text{ U.S. bushels.}\end{aligned}$$

(b) Similarly, the difference between 73 "register" tons and 73 "displacement" tons is

$$\begin{aligned}\Delta V &= 73(1 \text{ register ton} - 1 \text{ displacement ton}) = 73(80.31 \text{ U.S. bushels} - 28.108 \text{ U.S. bushels})\\ &= 3810.746 \text{ U.S. bushels} \approx 3.81\times 10^3 \text{ U.S. bushels.}\end{aligned}$$

**LEARN** With $1 \text{ register ton} > 1 \text{ freight ton} > 1 \text{ displacement ton}$, we expect the difference found in (b) to be greater than that in (a). This is indeed the case.

39. **THINK** This problem compares the U.K. gallon with the U.S. gallon, two non-SI units for volume. The interpretation of the type of gallons, whether U.K. or U.S., affects the amount of gasoline one calculates for traveling a given distance.

**EXPRESS** If the consumption rate is $R$ (miles/gallon), then the amount of gasoline (gallons) needed for a trip of distance $d$ (miles) would be

$$V(\text{gallons}) = \frac{d \text{ (miles)}}{R \text{ (miles/gallon)}}.$$

Since the car was manufactured in the U.K., the consumption rate is calibrated based on the U.K. gallon, and the correct interpretation should be "40 miles per U.K. gallon." In the U.K., one would think of the gallon as the U.K. gallon; however, in the U.S., the word "gallon" would naturally be interpreted as U.S. gallon. Note also that since $1 \text{ U.K. gallon} = 4.546\,090\,0 \text{ L}$ and $1 \text{ U.S. gallon} = 3.785\,411\,8 \text{ L}$, the relationship between the two is

$$1 \text{ U.K. gallon} = (4.546\,090\,0 \text{ L})\left(\frac{1 \text{ U.S. gallon}}{3.785\,411\,8 \text{ L}}\right) = 1.200\,95 \text{ U.S. gallons.}$$

**ANALYZE** (a) The amount of gasoline actually required is

$$V' = \frac{750 \text{ miles}}{40 \text{ miles/U.K. gallon}} = 18.75 \text{ U.K. gallons} \approx 18.8 \text{ U.K. gallons.}$$

This means that the driver mistakenly believes that the car should need 18.8 U.S. gallons.

(b) Using the conversion factor found above, this is equivalent to

$$V' = (18.75 \text{ U.K. gallons})\left(\frac{1.200\,95 \text{ U.S. gallons}}{1 \text{ U.K. gallon}}\right) \approx 22.5 \text{ U.S. gallons.}$$

**LEARN** A U.K. gallon is greater than a U.S. gallon by roughly a factor of 1.2 in volume. Therefore, 40 mi/U.K. gallon is less fuel efficient than 40 mi/U.S. gallon.

41. **THINK** This problem involves converting *cord*, a non-SI unit for volume, to an SI unit.

**EXPRESS** Using the (exact) conversion 1 in. = 2.54 cm = 0.0254 m for length, we have

$$1 \text{ ft} = 12 \text{ in.} = (12 \text{ in.})\left(\frac{0.0254 \text{ m}}{1 \text{ in.}}\right) = 0.3048 \text{ m.}$$

Thus, $1 \text{ ft}^3 = (0.3048 \text{ m})^3 = 0.0283 \text{ m}^3$ for volume (these results also can be found in Appendix D).

**ANALYZE** The volume of a cord of wood is $V = (8 \text{ ft})(4 \text{ ft})(4 \text{ ft}) = 128 \text{ ft}^3$. Using the conversion factor found above, we obtain

$$V = 1 \text{ cord} = 128 \text{ ft}^3 = (128 \text{ ft}^3)\left(\frac{0.0283 \text{ m}^3}{1 \text{ ft}^3}\right) = 3.625 \text{ m}^3,$$

which indicates that

$$1 \text{ m}^3 = \left(\frac{1}{3.625}\right) \text{cord} = 0.276 \text{ cord} \approx 0.3 \text{ cord.}$$

**LEARN** The unwanted units $\text{ft}^3$ all cancel out, as they should. In conversions, units obey the same algebraic rules as variables and numbers.

47. **THINK** This problem involves expressing the speed of light in astronomical units per minute.

**EXPRESS** We first convert meters to astronomical units (AU), and seconds to minutes, using

$$1000 \text{ m} = 1 \text{ km}, \;\; 1 \text{ AU} = 1.50\times10^{8} \text{ km}, \;\; 60 \text{ s} = 1 \text{ min}.$$

**ANALYZE** With these conversion factors, the speed of light can be rewritten as

$$c = 3.0\times10^{8} \text{ m/s} = \left(\frac{3.0\times10^{8} \text{ m}}{1 \text{ s}}\right)\left(\frac{1 \text{ km}}{1000 \text{ m}}\right)\left(\frac{1 \text{ AU}}{1.50\times10^{8} \text{ km}}\right)\left(\frac{60 \text{ s}}{1 \text{ min}}\right) = 0.12 \text{ AU/min}.$$

**LEARN** When we express the speed of light $c$ in AU/min, we readily see that it takes about 8.3 (= 1/0.12) minutes for sunlight to reach Earth (i.e., to travel a distance of 1 AU).

# Chapter 2

3. **THINK** This one-dimensional kinematics problem consists of two parts, and we are asked to solve for the average velocity and average speed of the car.

**EXPRESS** Since the trip consists of two parts, let the displacements during the first and second parts of the motion be $\Delta x_1$ and $\Delta x_2$ and the corresponding time intervals be $\Delta t_1$ and $\Delta t_2$. Now, because the problem is one-dimensional and both displacements are in the same direction, the total displacement is simply $\Delta x = \Delta x_1 + \Delta x_2$, and the total time for the trip is $\Delta t = \Delta t_1 + \Delta t_2$. Using the definition of average velocity given in Eq. 2.1.2, we have

$$v_{\text{avg}} = \frac{\Delta x}{\Delta t} = \frac{\Delta x_1 + \Delta x_2}{\Delta t_1 + \Delta t_2}.$$

To find the average speed, we note that during a time $\Delta t$ if the velocity remains a positive constant, then the speed is equal to the magnitude of velocity, and the distance is equal to the magnitude of displacement, with $d = |\Delta x| = v\,\Delta t$.

**ANALYZE** (a) During the first part of the motion, the displacement is $\Delta x_1 = 40$ km and the time taken is

$$t_1 = \frac{40\text{ km}}{30\text{ km/h}} = 1.33\text{ h}.$$

Similarly, during the second part of the trip, the displacement is $\Delta x_2 = 40$ km and the time interval is

$$t_2 = \frac{40\text{ km}}{60\text{ km/h}} = 0.67\text{ h}.$$

The total displacement is

$$\Delta x = \Delta x_1 + \Delta x_2 = 40\text{ km} + 40\text{ km} = 80\text{ km},$$

and the total time elapsed is

$$\Delta t = \Delta t_1 + \Delta t_2 = 2.00\text{ h}.$$

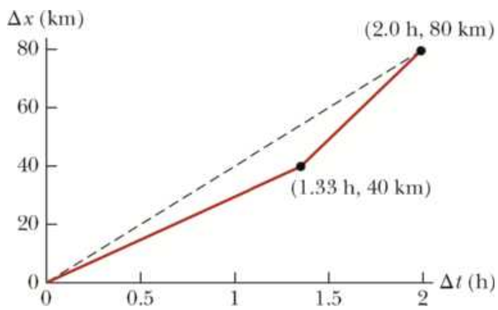
Consequently, the average velocity is

$$v_{\text{avg}} = \frac{\Delta x}{\Delta t} = \frac{80 \text{ km}}{2.0 \text{ h}} = 40 \text{ km/h}.$$

(b) In this case, the average speed is the same as the magnitude of the average velocity:

$$s_{\text{avg}} = 40 \text{ km/h}.$$

(c) The graph of the entire trip, shown below, consists of two connected line segments, the first having a slope of 30 km/h and connecting the origin to ($\Delta t_1$, $\Delta x_1$) = (1.33 h, 40 km) and the second having a slope of 60 km/h and connecting ($\Delta t_1$, $\Delta x_1$) to ($\Delta t$, $\Delta x$) = (2.00 h, 80 km).




The slope of the dashed line drawn from the origin to ($\Delta t$, $\Delta x$) represents the average velocity.

**LEARN** The average velocity is a vector quantity that depends only on the net displacement (also a vector) between the starting and ending points.

5. **THINK** In this one-dimensional kinematics problem, we're given the position function $x(t)$ and asked to calculate the position and velocity of the object at a later time.

**EXPRESS** The position function is

$$x(t) = (3 \text{ m/s})t - (4 \text{ m/s}^2)t^2 + (1 \text{ m/s}^3)t^3.$$

The position of the object at some instant $t_0$ is simply given by $x(t_0)$. For the time interval $t_1 \le t \le t_2$, the displacement is $\Delta x = x(t_2) - x(t_1)$. Similarly, using Eq. 2.1.2, the average velocity for this time interval is

$$v_{\text{avg}} = \frac{\Delta x}{\Delta t} = \frac{x(t_2) - x(t_1)}{t_2 - t_1}.$$

**ANALYZE** (a) Plugging $t$ = 1 s into $x(t)$ yields

$$x(1\text{ s}) = (3\text{ m/s})(1\text{ s}) - (4\text{ m/s}^2)(1\text{ s})^2 + (1\text{ m/s}^3)(1\text{ s})^3 = 0.$$

(b) With $t$ = 2 s we get

$$x(2\text{ s}) = (3\text{ m/s})(2\text{ s}) - (4\text{ m/s}^2)(2\text{ s})^2 + (1\text{ m/s}^3)(2\text{ s})^3 = -2\text{ m}.$$

(c) With $t$ = 3 s we have

$$x(3\text{ s}) = (3\text{ m/s})(3\text{ s}) - (4\text{ m/s}^2)(3\text{ s})^2 + (1\text{ m/s}^3)(3\text{ s})^3 = 0\text{ m}.$$

(d) Similarly, plugging in $t$ = 4 s gives

$$x(4\text{ s}) = (3\text{ m/s})(4\text{ s}) - (4\text{ m/s}^2)(4\text{ s})^2 + (1\text{ m/s}^3)(4\text{ s})^3 = 12\text{ m}.$$

(e) The position at $t$ = 0 is $x$ = 0. Thus, the displacement between $t$ = 0 and $t$ = 4 s is

$$\Delta x = x(4\text{ s}) - x(0) = 12\text{ m} - 0 = 12\text{ m}.$$

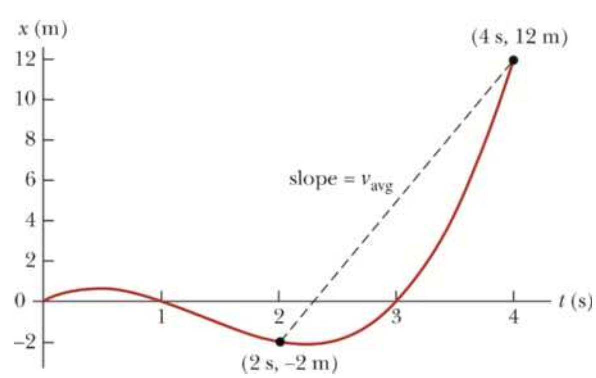
(f) The position at $t$ = 2 s is subtracted from the position at $t$ = 4 s to give the displacement:

$$\Delta x = x(4\text{ s}) - x(2\text{ s}) = 12\text{ m} - (-2\text{ m}) = 14\text{ m}.$$

Thus, the average velocity is

$$v_{\text{avg}} = \frac{\Delta x}{\Delta t} = \frac{14\text{ m}}{2\text{ s}} = 7\text{ m/s}.$$

(g) The position of the object for the interval $0 \le t \le 4$ is plotted below. The slope of the straight line drawn from the point at $(t, x)$ = (2 s, –2 m) to the point at (4 s, 12 m) is the average velocity, the answer for part (f).



**LEARN** Our graphical representation illustrates once again that the average velocity for a time interval depends only on the net displacement between the starting and ending points.

19. **THINK** In this one-dimensional kinematics problem, we're given the speed of a particle at two instants and asked to calculate its average acceleration.

**EXPRESS** We take the initial direction of motion as the $+x$ direction. The average acceleration over a time interval $t_1 \le t \le t_2$ is given by Eq. 2.3.1:

$$a_{\text{avg}} = \frac{\Delta v}{\Delta t} = \frac{v(t_2) - v(t_1)}{t_2 - t_1}.$$

**ANALYZE** Let $v_1 = +18$ m/s at $t_1 = 0$ and $v_2 = -30$ m/s at $t_2 = 2.4$ s. Using Eq. 2.3.1, we find

$$a_{\text{avg}} = \frac{v(t_2) - v(t_1)}{t_2 - t_1} = \frac{(-30\text{ m/s}) - (+18\text{ m/s})}{2.4\text{ s} - 0} = -20\text{ m/s}^2.$$

**LEARN** The average acceleration has magnitude 20 m/s$^2$ and is in the opposite direction to the particle's initial velocity. This makes sense because the velocity of the particle is decreasing over the time interval. With $t_1 = 0$, the velocity of the particle as a function of time can be written as

$$v = v_0 + at = (18\text{ m/s}) - (20\text{ m/s}^2)t.$$

23. **THINK** The electron undergoes a constant acceleration. Given the final speed of the electron and the distance it has traveled, we can calculate its acceleration.

**EXPRESS** Since the problem involves constant acceleration, the motion of the electron can be analyzed using the equations given in Table 2.4.1:

$$v = v_0 + at \qquad (2.4.1)$$

$$x - x_0 = v_0 t + \frac{1}{2}at^2 \qquad (2.4.5)$$

$$v^2 = v_0^2 + 2a(x - x_0). \qquad (2.4.6)$$

The acceleration can be found by solving Eq. 2.4.6.

**ANALYZE** With $v_0 = 1.50\times10^5$ m/s, $v = 5.70\times10^6$ m/s, $x_0 = 0$, and $x = 0.010$ m, we find the average acceleration is

$$a = \frac{v^2 - v_0^2}{2x} = \frac{(5.7\times10^6 \text{ m/s})^2 - (1.5\times10^5 \text{ m/s})^2}{2(0.010 \text{ m})} = 1.62\times10^{15} \text{ m/s}^2.$$

**LEARN** A good idea is to apply other equations in Table 2.4.1 not used for solving the problem as a consistency check. For example, since we now know the value of the acceleration, from Eq. 2.4.1, the time the electron takes to reach its final speed is

$$t = \frac{v - v_0}{a} = \frac{5.70\times10^6 \text{ m/s} - 1.5\times10^5 \text{ m/s}}{1.62\times10^{15} \text{ m/s}^2} = 3.426\times10^{-9} \text{ s}.$$

Substituting the value of $t$ into Eq. 2.4.5, we find that the distance the electron travels is

$$\begin{aligned} x &= x_0 + v_0 t + \frac{1}{2}at^2 \\ &= 0 + (1.5\times10^5 \text{ m/s})(3.426\times10^{-9} \text{ s}) + \frac{1}{2}(1.62\times10^{15} \text{ m/s}^2)(3.426\times10^{-9} \text{ s})^2 \\ &= 0.010 \text{ m}. \end{aligned}$$

This is what was given in the problem statement. So we know the problem has been solved correctly.

31. **THINK** The rocket ship undergoes a constant acceleration from rest, and we want to know the time elapsed and the distance traveled when the rocket reaches a certain speed.

**EXPRESS** Since the problem involves constant acceleration, the motion of the rocket can be readily analyzed using the equations in Table 2.4.1:

$$v = v_0 + at \qquad (2.4.1)$$

$$x - x_0 = v_0 t + \frac{1}{2}at^2 \qquad (2.4.5)$$

$$v^2 = v_0^2 + 2a(x - x_0). \qquad (2.4.6)$$

**ANALYZE** (a) Given $a = 9.8 \text{ m/s}^2$, $v_0 = 0$, and $v = 0.1c = 3.0\times10^7 \text{ m/s}$, we solve $v = v_0 + at$ for the time:

$$t = \frac{v - v_0}{a} = \frac{3.0\times10^7 \text{ m/s} - 0}{9.8 \text{ m/s}^2} = 3.1\times10^6 \text{ s},$$

which is about 1.2 months. So the rocket takes 1.2 months to reach a speed of 0.1$c$ starting from rest with a constant acceleration of 9.8 m/s$^2$.

(b) To calculate the distance traveled during this time, we evaluate $x = x_0 + v_0 t + \frac{1}{2}at^2$, with $x_0 = 0$ and $v_0 = 0$. The result is

$$x = \frac{1}{2}(9.8\ \text{m/s}^2)(3.1\times10^6\ \text{s})^2 = 4.6\times10^{13}\ \text{m}.$$

**LEARN** In solving parts (a) and (b), we did not use Eq. 2.4.6: $v^2 = v_0^2 + 2a(x - x_0)$. This equation can be used to check our answers. The final speed based on this equation is

$$v = \sqrt{v_0^2 + 2a(x - x_0)} = \sqrt{0 + 2(9.8\ \text{m/s}^2)(4.6\times10^{13}\ \text{m} - 0)} = 3.0\times10^7\ \text{m/s},$$

which is what is given in the problem statement. So we know the problem has been solved correctly.

33. **THINK** The car undergoes a constant negative acceleration to avoid impacting a barrier. Given its initial speed, we want to know the distance it has traveled and the time elapsed prior to the impact.

**EXPRESS** Since the problem involves constant acceleration, the motion of the car can be analyzed using the equations in Table 2.4.1:

$$v = v_0 + at \qquad (2.4.1)$$

$$x - x_0 = v_0 t + \frac{1}{2}at^2 \qquad (2.4.5)$$

$$v^2 = v_0^2 + 2a(x - x_0). \qquad (2.4.6)$$

We take $x_0 = 0$ and $v_0$ = 56.0 km/h = 15.55 m/s to be the initial position and speed of the car. Solving Eq. 2.4.5 with $t$ = 2.00 s gives the acceleration $a$. Once $a$ is known, the speed of the car upon impact can be found by using Eq. 2.4.1.

**ANALYZE** (a) Using Eq. 2.4.5, we find the acceleration to be

$$a = \frac{2(x - v_0 t)}{t^2} = \frac{2\left[(24.0\ \text{m}) - (15.55\ \text{m/s})(2.00\ \text{s})\right]}{(2.00\ \text{s})^2} = -3.56\ \text{m/s}^2,$$

or $|a| = 3.56\ \text{m/s}^2$. The negative sign indicates that the acceleration is opposite to the direction of motion of the car; the car is slowing down.

(b) The speed of the car at the instant of impact is

$$v = v_0 + at = 15.55 \text{ m/s} + (-3.56 \text{ m/s}^2)(2.00 \text{ s}) = 8.43 \text{ m/s},$$

which can also be converted to 30.3 km/h.

**LEARN** In solving parts (a) and (b), we did not use Eq. 2.4.6. This equation can be used as a consistency check. The final speed based on this equation is

$$v = \sqrt{v_0^2 + 2a(x - x_0)} = \sqrt{(15.55 \text{ m/s})^2 + 2(-3.56 \text{ m/s}^2)(24 \text{ m} - 0)} = 8.43 \text{ m/s},$$

which is what was calculated in (b). This indicates that the problem has been solved correctly.

45. **THINK** As the ball travels vertically upward, it has gravitational acceleration. The kinematics is one-dimensional.

**EXPRESS** We neglect air resistance for the duration of the motion (between "launching" and "landing"), so $a = -g = -9.8$ m/s² (we take downward to be the $-y$ direction). We use the equations in Table 2.4.1 (with $\Delta y$ replacing $\Delta x$) because this is constant-acceleration motion:

$$v = v_0 - gt \qquad (2.4.1)$$

$$y - y_0 = v_0 t - \frac{1}{2} g t^2 \qquad (2.4.5)$$

$$v^2 = v_0^2 - 2g(y - y_0). \qquad (2.4.6)$$

We set $y_0 = 0$. Upon reaching the maximum height $y$, the ball momentarily has zero speed ($v = 0$). Therefore, we can relate its initial speed $v_0$ to $y$ via the equation $0 = v^2 = v_0^2 - 2gy$. The time the ball takes to reach maximum height is given by $v = v_0 - gt = 0$, or $t = v_0/g$. Therefore, for the entire trip (from the time it leaves the ground until the time it returns to the ground), the total flight time is $T = 2t = 2v_0/g$.

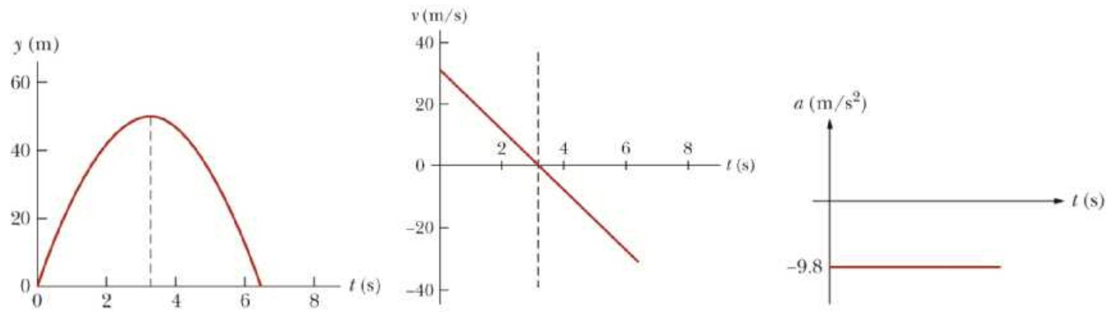
**ANALYZE** (a) At the highest point $v = 0$ and $v_0 = \sqrt{2gy}$. With $y = 50$ m, we find the initial speed of the ball to be

$$v_0 = \sqrt{2gy} = \sqrt{2(9.8 \text{ m/s}^2)(50 \text{ m})} = 31.3 \text{ m/s} \approx 31 \text{ m/s}.$$

(b) Using the result from (a) for $v_0$, the total flight time of the ball is

$$T = \frac{2v_0}{g} = \frac{2(31.3\text{ m/s})}{9.8\text{ m/s}^2} = 6.4\text{ s}.$$

(c) The plots of $y$, $v$, and $a$ as a function of time are shown in the figure. The acceleration graph is a horizontal line at –9.8 m/s². At $t$ = 3.19 s, $y$ = 50 m.




**LEARN** In calculating the total flight time of the ball, we could have used Eq. 2.4.5. At $t = T > 0$, the ball returns to its original position $(y = 0)$. Therefore,

$$y = v_0 T - \frac{1}{2} g T^2 = 0 \;\Rightarrow\; T = \frac{2v_0}{g}.$$

47. **THINK** The wrench is in free fall with an acceleration of $a = -g = -9.8\text{ m/s}^2$.

**EXPRESS** We neglect air resistance, which justifies setting $a = -g = -9.8\text{ m/s}^2$ (taking *down* as the $-y$ direction) for the duration of the fall. This is constant-acceleration motion, which justifies the use of Table 2.4.1 (with $\Delta y$ replacing $\Delta x$):

$$v = v_0 - gt \qquad (2.4.1)$$

$$y - y_0 = v_0 t - \frac{1}{2} g t^2 \qquad (2.4.5)$$

$$v^2 = v_0^2 - 2g(y - y_0). \qquad (2.4.6)$$

Since the wrench had initial speed $v_0 = 0$, knowing its speed of impact allows us to apply Eq. 2.4.6 to calculate the height from which it was dropped.

**ANALYZE** (a) Using $v^2 = v_0^2 + 2a\,\Delta y$, we find the initial height to be

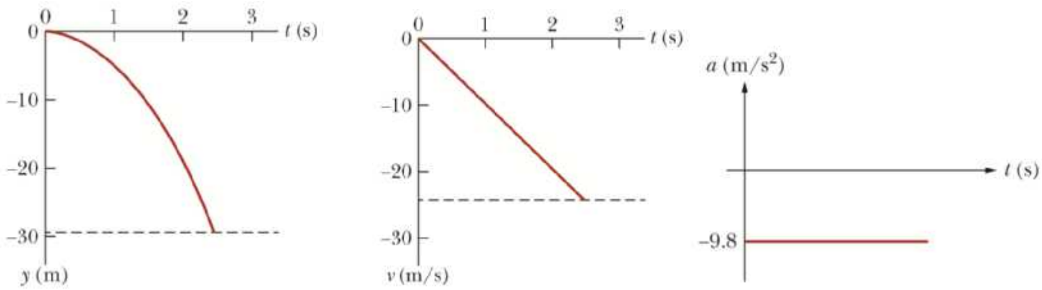
$$\Delta y = \frac{v_0^2 - v^2}{-2a} = \frac{0 - (-24\text{ m/s})^2}{-2(-9.8\text{ m/s}^2)} = -29.4\text{ m}.$$

So it fell through a height of 29.4 m.

(b) Solving $v = v_0 - gt$ for time, we obtain a flight time of

$$t = \frac{v_0 - v}{g} = \frac{0-(-24 \text{ m/s})}{9.8 \text{ m/s}^2} = 2.45 \text{ s}.$$

(c) SI units are used in the graphs shown here, and the initial position is taken as the coordinate origin. The acceleration graph is a horizontal line at –9.8 m/s².




**LEARN** As the wrench falls, with $a = -g < 0$, its speed increases, but its velocity becomes more negative, as indicated by the second graph.

49. **THINK** In this problem a package is dropped from a hot-air balloon that is ascending vertically upward. We analyze the motion of the package under the influence of gravity.

**EXPRESS** We neglect air resistance, which justifies setting $a = -g = -9.8$ m/s² (taking *down* as the $-y$ direction) for the duration of the motion. This allows us to use Table 2.4.1 (with $\Delta y$ replacing $\Delta x$):

$$v = v_0 - gt \qquad (2.4.1)$$

$$y - y_0 = v_0 t - \frac{1}{2} g t^2 \qquad (2.4.5)$$

$$v^2 = v_0^2 - 2g(y - y_0). \qquad (2.4.6)$$

We place the coordinate origin on the ground and note that the initial velocity of the package is the same as the velocity of the balloon, $v_0 = +12$ m/s, and that its initial coordinate is $y_0 = +80$ m. The time the package takes to hit the ground can be found by solving Eq. 2.4.5 with $y = 0$.

**ANALYZE** (a) We solve $0 = y = y_0 + v_0 t - \frac{1}{2} g t^2$ for time using the quadratic formula (choosing the positive root to yield a positive value for $t$):

$$t = \frac{v_0 + \sqrt{v_0^2 + 2gy_0}}{g} = \frac{12 \text{ m/s} + \sqrt{(12 \text{ m/s})^2 + 2\left(9.8 \text{ m/s}^2\right)(80 \text{ m})}}{9.8 \text{ m/s}^2} = 5.447 \approx 5.4 \text{ s}.$$

(b) The velocity of the package when it hits the ground can be calculated using Eq. 2.4.1:

$$v = v_0 - gt = 12\text{ m/s} - (9.8\text{ m/s}^2)(5.447\text{ s}) = -41.38\text{ m/s}.$$

So its final *speed* is about 41 m/s.

**LEARN** Our answers can be verified by using Eq. 2.4.6, which was not used in either (a) or (b). The equation leads to

$$v = -\sqrt{v_0^2 - 2g(y - y_0)} = -\sqrt{(12\text{ m/s})^2 - 2(9.8\text{ m/s}^2)(0 - 80\text{ m})} = -41.38\text{ m/s},$$

which agrees with that calculated in (b).

53. **THINK** This problem involves two objects: a key dropped from a bridge, and a boat moving at a constant speed. We look for conditions such that the key will fall into the boat.

**EXPRESS** The speed of the boat is constant, given by $v_b = d/t$, where $d$ is the distance (12 m) of the boat from the bridge when the key is dropped and $t$ is the time the key takes in falling. To calculate $t$, we take the time to be zero at the instant the key is dropped and compute the time $t$ when $y = 0$ by using $y = y_0 + v_0 t - \frac{1}{2}gt^2$ with $y_0 = 45$ m. Once $t$ is known, the speed of the boat can be calculated.

**ANALYZE** Since the initial velocity of the key is zero, we have $y_0 = \frac{1}{2}gt^2$. Thus, the time the key takes to drop into the boat is

$$t = \sqrt{\frac{2y_0}{g}} = \sqrt{\frac{2(45\text{ m})}{9.8\text{ m/s}^2}} = 3.03\text{ s}.$$

Therefore, the speed of the boat is $v_b = \dfrac{12\text{ m}}{3.03\text{ s}} = 4.0\text{ m/s}.$

**LEARN** From the general expression

$$v_b = \frac{d}{t} = \frac{d}{\sqrt{2y_0/g}} = d\sqrt{\frac{g}{2y_0}},$$

we see that $v_b \sim 1/\sqrt{y_0}$. This agrees with our intuition that the lower the height from which the key is dropped, the greater the speed of the boat in order to catch it.

55. **THINK** The free-falling moist-clay ball strikes the ground with a nonzero speed, and it undergoes deceleration before coming to rest.

**EXPRESS** During contact with the ground, its average acceleration is given by $a_{\text{avg}} = \frac{\Delta v}{\Delta t}$, where $\Delta v$ is the change in its velocity during contact with the ground and $\Delta t = 20.0 \times 10^{-3}$ s is the duration of contact. Thus, we must first find the velocity of the ball just before it hits the ground ($y$ = 0).

**ANALYZE** (a) Now, to find the velocity just *before* contact, we take $t$ = 0 to be when the ball is dropped. Using Eq. 2.4.6 with $y_0 = 15.0$ m, we obtain

$$v = -\sqrt{v_0^2 - 2g(y - y_0)} = -\sqrt{0 - 2(9.8 \text{ m/s}^2)(0 - 15 \text{ m})} = -17.15 \text{ m/s},$$

where the negative option of the square root is chosen since the ball is traveling downward at the moment of contact. Consequently, the average acceleration during contact with the ground is

$$a_{\text{avg}} = \frac{\Delta v}{\Delta t} = \frac{0 - (-17.15 \text{ m/s})}{20.0 \times 10^{-3} \text{ s}} = 857 \text{ m/s}^2.$$

(b) The fact that the result is positive indicates that this acceleration is upward.

**LEARN** Since $\Delta t$ is very small, it is not surprising to have a very large acceleration to stop the motion of the ball. In later chapters, we shall see that the acceleration is directly related to the magnitude and direction of the force exerted by the ground on the ball during the collision.

77. **THINK** The speed of the hot rod changes due to a nonzero acceleration.

**EXPRESS** Since the problem involves constant acceleration, the motion of the hot rod can be analyzed using the equations given in Table 2.4.1. We take the $+x$ direction to be in direction of motion, so

$$v = (60 \text{ km/h})\left(\frac{1000 \text{ m/km}}{3600 \text{ s/h}}\right) = +16.7 \text{ m/s}$$

and $a > 0$. The location where the hot rod starts from rest ($v_0 = 0$) is taken to be $x_0 = 0$.

**ANALYZE** (a) Using Eq. 2.3.1, we find the average acceleration to be

$$a_{\text{avg}} = \frac{\Delta v}{\Delta t} = \frac{v - v_0}{t - t_0} = \frac{16.7 \text{ m/s} - 0}{5.4 \text{ s} - 0} = 3.09 \text{ m/s}^2.$$

(b) Assuming constant acceleration $a = a_{avg} = 3.09\ \text{m/s}^2$, the total distance traveled during the 5.4 s time interval is

$$x = x_0 + v_0 t + \frac{1}{2}at^2 = 0 + 0 + \frac{1}{2}(3.09\ \text{m/s}^2)(5.4\ \text{s})^2 = 45\ \text{m}.$$

(c) Using Eq. 2.4.5, we find the time required to travel a distance of $x$ = 250 m:

$$x = \frac{1}{2}at^2 \;\Rightarrow\; t = \sqrt{\frac{2x}{a}} = \sqrt{\frac{2(250\ \text{m})}{3.09\ \text{m/s}^2}} = 12.73\ \text{s}.$$

**LEARN** The displacement of the hot rod as a function of time can be written as $x(t) = \frac{1}{2}(3.09\ \text{m/s}^2)t^2$. Note that we could have chosen Eq. 2.4.7 to solve for (b):

$$x = \frac{1}{2}(v_0 + v)\,t = \frac{1}{2}(16.7\ \text{m/s})(5.4\ \text{s}) = 45\ \text{m}.$$

81. **THINK** Because the particle undergoes a *variable* acceleration, an integration is required to calculate velocity.

**EXPRESS** With a variable acceleration $a(t) = dv/dt$, the velocity of the particle at time $t_1$ is given by Eq. 2.6.1:

$$v_1 = v_0 + \int_{t_0}^{t_1} a(t)\,dt,$$

where $v_0$ is the velocity at time $t_0$. In our situation, we have $a = 5.0t$. We also know that $v_0 = 17\ \text{m/s}$ at $t_0 = 2.0\ \text{s}$.

**ANALYZE** Integrating (from $t$ = 2 s to $t$ = 4 s) the acceleration to get the velocity and using the values given in the problem lead to

$$v = v_0 + \int_{t_0}^{t} a\,dt = v_0 + \int_{t_0}^{t}(5.0t)\,dt = v_0 + \frac{1}{2}(5.0)(t^2 - t_0^2) = 17 + \frac{1}{2}(5.0)(4^2 - 2^2) = 47\ \text{m/s}.$$

**LEARN** The velocity of the particle as a function of $t$ is

$$v(t) = v_0 + \frac{1}{2}(5.0)(t^2 - t_0^2) = 17 + \frac{1}{2}(5.0)(t^2 - 4) = 7 + 2.5t^2$$

in SI units (m/s). Since the acceleration is linear in $t$, we expect the velocity to be quadratic in $t$ and the displacement to be cubic in $t$.

# Chapter 3

1. **THINK** In this problem we're given the magnitude and direction of a vector in two dimensions and asked to calculate its *x* and *y* components.

**EXPRESS** The *x* and the *y* components of a vector $\vec{a}$ lying in the *xy* plane are given by

$$a_x = a\cos\theta, \qquad a_y = a\sin\theta,$$

where $a = |\vec{a}| = \sqrt{a_x^2 + a_y^2}$ is the magnitude and the angle $\theta = \tan^{-1}(a_y/a_x)$ is between $\vec{a}$ and the $+x$ direction. Given that $\theta = 250°$, we see that the vector is in the third quadrant, and we expect both the *x* and the *y* components of $\vec{a}$ to be negative.

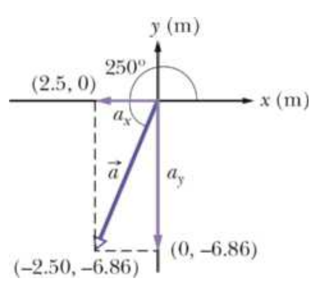
**ANALYZE** (a) The *x* component of $\vec{a}$ is

$$a_x = a\cos\theta = (7.3\text{ m})\cos 250° = -2.5\text{ m}.$$

(b) The *y* component is

$$a_y = a\sin\theta = (7.3\text{ m})\sin 250° = -6.86\text{ m} \approx -6.9\text{ m}.$$

The results are depicted in the figure below:




**LEARN** In considering the variety of ways to compute these, we note that the vector is 70° below the –*x* direction, so the components could also have been found from

$$a_x = -(7.3\text{ m})\cos 70° = -2.50\text{ m}, \quad a_y = -(7.3\text{ m})\sin 70° = -6.86\text{ m}.$$

Similarly, we note that the vector is 20° to the left from the –*y* direction, so one could also achieve the same results by using

$$a_x = -(7.3 \text{ m})\sin 20° = -2.50 \text{ m}, \quad a_y = -(7.3 \text{ m})\cos 20° = -6.86 \text{ m}.$$

As a consistency check, we note that

$$\sqrt{a_x^2 + a_y^2} = \sqrt{(-2.50 \text{ m})^2 + (-6.86 \text{ m})^2} = 7.3 \text{ m}$$

and

$$\tan^{-1}(a_y/a_x) = \tan^{-1}[(-6.86 \text{ m})/(-2.50 \text{ m})] = 250°,$$

which are indeed the values given in the problem statement.

3. **THINK** In this problem we're given the *x* and *y* components of a vector $\vec{A}$ in two dimensions and asked to calculate its magnitude and direction.

**EXPRESS** Vector $\vec{A}$ can be represented in *magnitude-angle* notation ($A$, $\theta$), where

$$A = \sqrt{A_x^2 + A_y^2}$$

is the magnitude and

$$\theta = \tan^{-1}\left(\frac{A_y}{A_x}\right)$$

is the angle $\vec{A}$ makes with the positive direction of the *x* axis. Given that $A_x = -25.0$ m and $A_y = 40.0$ m, the above formulas can be used to calculate $A$ and $\theta$.

**ANALYZE** (a) The magnitude of $\vec{A}$ is

$$A = \sqrt{A_x^2 + A_y^2} = \sqrt{(-25.0 \text{ m})^2 + (40.0 \text{ m})^2} = 47.2 \text{ m}.$$

(b) Because $\tan\theta = \tan(\theta + 180°)$,

$$\tan^{-1}[(40.0 \text{ m})/(-25.0 \text{ m})] = -58° \text{ or } 122°.$$

Noting that the vector is in the second quadrant (by the signs of its *x* and *y* components), we see that 122° is the correct answer. The results are depicted in the figure here