Solutions to Chapter 2 Exercises

2.1. From Eq. (2.4),

$$f(E_f - \Delta E) = \frac{1}{1+e^{-\Delta E/kT}} = \frac{e^{\Delta E/kT}}{e^{\Delta E/kT}+1},$$

and

$$f(E_f + \Delta E) = \frac{1}{1+e^{\Delta E/kT}}.$$

Adding the above two equations yields

$$f(E_f - \Delta E) + f(E_f + \Delta E) = \frac{e^{\Delta E/kT}+1}{e^{\Delta E/kT}+1} = 1.$$

2.2. Neglecting the hole (last) term in Eq. (2.19), one obtains

$$N_c e^{-(E_c - E_f)/kT} + 2N_c e^{-(E_c + E_d - 2E_f)/kT} = N_d.$$

Treating $\exp(E_f/kT)$ as an unknown, the above equation is a quadratic equation with the solution

$$e^{E_f/kT} = \frac{-1+\sqrt{1+8(N_d/N_c)e^{(E_c-E_d)/kT}}}{4e^{-E_d/kT}}.$$

Here only the positive root has been kept. For shallow donors with low to moderate concentration at room temperature, $(N_d/N_c)\exp[(E_c - E_d)/kT] << 1$, and the last equation can be approximated by

$$e^{E_f/kT} = \frac{4(N_d/N_c)e^{(E_c-E_d)/kT}}{4e^{-E_d/kT}} = \frac{N_d}{N_c}e^{E_c/kT},$$

which is the same as Eq. (2.20). If we compare the above relation with Eq. (2.19), it is clear that in this case, $\exp[-(E_d - E_f)/kT] << 1$, and $N_d^+ \approx N_d$ or complete ionization.

If the condition for low to moderate concentration of shallow donors is not met, then $\exp[-(E_d - E_f)/kT]$ is no longer negligible compared with unity. That means $N_d^+ < N_d$ (Eq. (2.19)) or incomplete ionization (freeze-out). [Note that incomplete ionization never occurs for shallow impurities: arsenic, boron, phosphorus, and antimony at room temperature, even for doping concentrations higher than N_c or N_v. This is because in heavily doped silicon, the impurity level broadens and the ionization energy decreases to zero, as discussed in Subsection 9.1.1.2.]

2.3. (a) Substituting Eqs. (2.5) and (2.3) into the expression for average kinetic energy, one obtains

$$\langle \text{K.E.} \rangle = \frac{\int_{E_c}^{\infty} (E-E_c)^{3/2} e^{-(E-E_f)/kT} dE}{\int_{E_c}^{\infty} (E-E_c)^{1/2} e^{-(E-E_f)/kT} dE}.$$

Applying integration by parts to the numerator yields

$$\langle \text{K.E.} \rangle = \frac{(3/2)kT \int_{E_c}^{\infty} (E-E_c)^{1/2} e^{-(E-E_f)/kT} dE}{\int_{E_c}^{\infty} (E-E_c)^{1/2} e^{-(E-E_f)/kT} dE} = \frac{3}{2} kT .$$

(b) For a degenerate semiconductor at 0 K, $f(E) = 1$ if $E < E_f$ and $f(E) = 0$ if $E > E_f$. Here $E_f > E_c$. Therefore,

$$\langle \text{K.E.} \rangle = \frac{\int_{E_c}^{E_f} (E-E_c)^{3/2} dE}{\int_{E_c}^{E_f} (E-E_c)^{1/2} dE} = \frac{3}{5}(E_f - E_c).$$

2.4. With the point charge Q at the center, construct a closed spherical surface S with radius r. By symmetry, the electric field at every point on S has the same magnitude and points outward perpendicular to the surface. Therefore,

$$\oiint_S \mathbf{\mathcal{E}} \cdot \boldsymbol{dS} = 4\pi r^2 \mathcal{E} ,$$

where $\mathcal{E}$ is the magnitude of the electric field on S. 3-D Gauss's law then gives

$$\mathcal{E} = \frac{Q}{4\pi \varepsilon_{si} r^2},$$

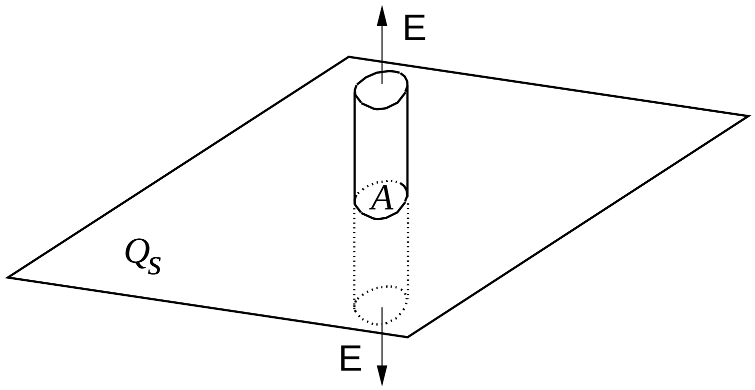
which is Coulomb's law.

Since $\mathcal{E} = -dV/dr$, the electric potential at a point on the sphere is

$$V = \frac{Q}{4\pi \varepsilon_{si} r},$$

if one defines the potential to be zero at infinity.

2.5. (a) Construct a cylindrical Gaussian surface perpendicular to the charge sheet as shown:



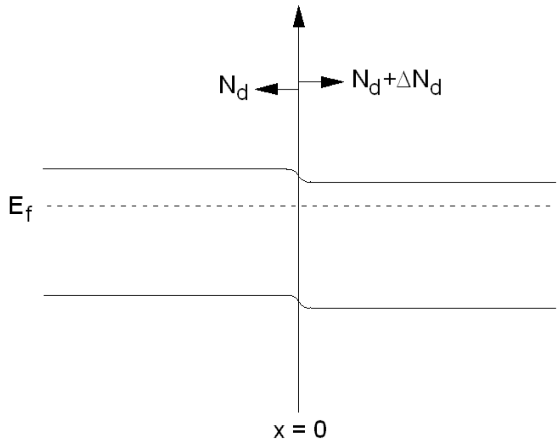
The cross-sectional area is A. At the two ends of the cylinder, the electric field $\mathcal{E}$ is perpendicular to the surface and pointing outward. Along the side surface of the cylinder, the field is parallel to the surface, so $\boldsymbol{\mathcal{E}}\cdot\boldsymbol{dS} = 0$. For an infinitely large sheet of charge, $\mathcal{E}$ is uniform across A from symmetry. Therefore,

$$\oiint_S \mathbf{E}\cdot\boldsymbol{dS} = 2A\mathrm{E}\,.$$

The charge enclosed within the surface is Q_sA. From Gauss's law, one obtains $\mathcal{E} = Q_s/2\varepsilon$.

(b) The field due to the positively charged sheet is $Q_s/2\varepsilon$ pointing away from the sheet. The field due to the negatively charged sheet is also $Q_s/2\varepsilon$, but pointing toward the negatively charged sheet. In the region between the two sheets, the two fields are in the same direction and the total field adds up to Q_s/ε, pointing from the positively charged sheet toward the negatively charged sheet. In the regions outside the two parallel sheets, the fields are equal and opposite to each other, resulting in zero net field.

2.6



For x < 0,

$$\frac{d^2(\Delta\psi_i)}{dx^2} - \frac{q^2 N_d}{\varepsilon_{si} kT}\Delta\psi_i = 0$$

General solution:

$$\Delta\psi_i = Ae^{x/L_D}$$

where

$$L_D \equiv \sqrt{\frac{\varepsilon_{si} kT}{q^2 N_d}}$$

For x > 0,

$$\frac{d^2(\Delta\psi_i)}{dx^2} - \frac{q^2 N_d}{\varepsilon_{si} kT}\Delta\psi_i = -\frac{q}{\varepsilon_{si}}\Delta N_d$$

General solution:

$$\Delta\psi_i = Be^{-x/L_D} + \frac{kT}{qN_d}\Delta N_d$$

Matching $\Delta\psi_i$ and $d\Delta\psi_i/dx$ at x = 0,

$$A = -B = \frac{kT}{2qN_d}\Delta N_d$$

2.7

Electron density per energy $= N(E)f(E) \propto \sqrt{E - E_C}\,e^{-(E-E_f)/kT}$

Differentiate the above and set to zero gives

$$E - E_C = kT/2 = 13\,\text{mV}$$

2.8

(a) The bands are flat in the region to the right → charge neutral.

Electron density: $n = N_C \times \exp[-(E_C - E_f)/kT] = 1.3 \times 10^{16}$ cm^{-3}.

Therefore, n-type with doping density $N_d = N_d^+ = 1.3 \times 10^{16}$ cm^{-3}.

(b) At point A: $n = N_C \times \exp[-0.3/kT] = 2.8 \times 10^{14}$ cm^{-3}.

$p = n_i^2/n = 3.5 \times 10^5$ cm^{-3}.

At point B: $n = N_C \times \exp[-0.4/kT] = 6 \times 10^{12}$ cm^{-3}.

$p = n_i^2/n = 1.7 \times 10^7$ cm^{-3}.

(c) At point A: Net charge = $N_d^+ - n \approx 1.3 \times 10^{16}$ cm^{-3}.

At point B: Net charge = $N_d^+ - n \approx 1.3 \times 10^{16}$ cm^{-3}.

2.9

(a)

Eq. 2.20, $$E_c - E_f = kT \ln\left(\frac{N_c}{N_d}\right).$$

Table 2.1, $N_c = 2.9 \times 10^{19}$ cm^{-3}

Therefore, $E_c - E_f = 0.266$ eV

(b)

Eq. 2.67 $$L_D \equiv \sqrt{\frac{\varepsilon_{si} kT}{q^2 N_d}} = 0.13\ \mu\text{m}$$

(c)

$\rho_n \varepsilon_{si}$ is the *dielectric relaxation time*.

From Fig. 2.10, $\rho_n = 4.3$ Ω-cm for n-type, 10^{15} cm^{-3} density. So $\rho_n \varepsilon_{si} = 4.5$ ps.

(d)

Eq. (2.10) $$N_c = 2g\sqrt{m_t^2 m_l}\left(\frac{2\pi kT}{h^2}\right)^{3/2} \propto T^{3/2}$$

$N_c(100°C) = (373/300)^{3/2} \times N_c(300\ K) = 4.0 \times 10^{19}\ cm^{-3}$

Therefore, $$E_c - E_f = kT\ln\left(\frac{N_c}{N_d}\right) = 0.274\text{ eV} \quad \text{at } 100°C.$$

2.10

(a)

$$E_f - E_i = 0.2\text{ eV}$$

Eq. (2.14), $$n = n_i e^{(E_f - E_i)/kT} = 2.26\times10^{13}\text{ cm}^{-3}$$

Eq. (2.15), $$p = n_i e^{(E_i - E_f)/kT} = 4.43\times10^{6}\text{ cm}^{-3}$$

Ionized donor density $N_d^+ = 10^{16}\ cm^{-3}$.

Therefore, not charge neutral, net positive charge.

(b)

See above.

(c)

The one in the middle so electrons fall toward positive charge in the center.

Solutions to Chapter 3 Exercises

3.1 Equation (3.138) gives

$$\frac{1}{M_p} = \exp\left[-\int_0^W(\alpha_p - \alpha_n)dx\right] - \int_0^W \alpha_n \exp\left[-\int_0^x(\alpha_p - \alpha_n)dx'\right]dx \,. \qquad (1)$$

As $M_p \to \infty$, $1/M_p \to 0$, the above equation gives

$$0 = \exp\left[-\int_0^W(\alpha_p - \alpha_n)dx\right] - \int_0^W \alpha_n \exp\left[-\int_0^x(\alpha_p - \alpha_n)dx'\right]dx \qquad (2)$$

$$= \exp\left(-(\alpha_p - \alpha_n)W\right) + \frac{\alpha_n}{(\alpha_p - \alpha_n)}\left[\exp\left(-(\alpha_p - \alpha_n)W\right) - 1\right].$$

Therefore, $$\alpha_n = \alpha_p \exp\left(-(\alpha_p - \alpha_n)W\right), \qquad (4)$$

or $$W = \ln\left(\alpha_n / \alpha_p\right) / \left(\alpha_n - \alpha_p\right).$$

Similarly, Eq. (3.139) gives the same expression for W as $M_n \to \infty$.

3.2 Let $$u(x) = -\int_0^x f(x')dx' \,. \qquad (1)$$

Therefore $$\frac{du(x)}{dx} = -f(x) \qquad (2)$$

and $$\frac{de^{u(x)}}{dx} = -f(x)e^{u(x)} \,. \qquad (3)$$

Using (3), we have

$$\int_0^W f(x)\exp\left[-\int_0^x f(x')dx'\right]dx = \int_0^W f(x)e^{u(x)}dx \qquad (4)$$

$$= -\int_0^W \frac{de^{u(x)}}{dx}dx = 1 - e^{u(W)} \,.$$

Therefore, $$\int_0^W f(x)\exp\left[-\int_0^x f(x')dx'\right]dx = 1 - \exp\left[-\int_0^W f(x')dx'\right]. \qquad (5)$$

A similar procedure can be used to show that

$$\int_0^W f(x)\exp\left[-\int_x^W f(x')dx'\right]dx = 1 - \exp\left[-\int_0^W f(x')dx'\right] \qquad (6)$$

by letting $u(x) = -\int_x^W f(x')dx'$.