Problem Solutions

Flack

Chapter 1

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10.1

$$F = \dot{m}(u_e - u_a) + A_e(p_e - p_a)$$

$$A_e = \pi D_e^2/4 = \pi\, 29^2/4 = 660.5\ ft^2$$

$$F = \frac{175}{32.17}\ \frac{slug}{sec}\left(2437 - 728\right)\frac{ft}{s} + 660.5\left(6.75 - 14.69\right) in^2\ \frac{lbf}{in^2}$$

$$= \underset{\text{mom}}{9297} - \underset{\text{pressure}}{5244}$$

$$\boxed{F = 4052\ lbf}$$

1.2

$$F = \dot{m}(u_e - u_a) + A_e(p_e - p_a)$$

$$A_e = \pi D_e^2/4 = \pi\, 29^2/4 = 660.5\ ft^2$$

$$F = \frac{175}{32.17}\ \frac{slug}{sec}\left(2437 - 728\right)\frac{ft}{s} + 660.5\left(6.75 - 676\right) ft^2\ \frac{lb_f}{ft^2}$$

$$= \underbrace{9297}_{mom} - \underbrace{7}_{pressure}$$

$$\boxed{F = 9290\ lb_f}$$

1.3 Turbofan

$\dot{m}_e = 150\ lb_m/s$ $\qquad V_a = 893\ ft/s$

$\dot{m}_s = 2.2 \times 150\ lb_m/s = 330\ lb_m/s$

$A_f = 336\ in^2$ $\qquad p_f = 14.7\ psia$ $\qquad V_f = 2288\ ft/s$

$A_c = 542\ in^2$ $\qquad p_c = 15.8\ psia$ $\qquad V_c = 1164\ ft/s$

$$F = A_c(p_c - p_a) + A_f(p_f - p_a) + \dot{m}_e(V_c - V_a) + \dot{m}_s(V_s - V_a)$$

$$= 542(15.8 - 14.7) + 336(14.7 - 14.7)$$

$$+ 150(1164 - 893)\frac{lb_m}{s}\frac{sl}{32.2\ lb_m}\frac{ft}{s}$$

$$+ 330(2288 - 893)/32.2$$

$$= 596\ lb_f + 0 + 1262\ lb_f + 14297\ lb_f$$

(core: $596 + 0 + 1262\ lb_f$; fan: $14297\ lb_f$)

$$\boxed{F = 16155\ lb_f}$$

$F_c = 1858 \Rightarrow 11.5\%$

$F_f = 14297 \Rightarrow 88.5\%$

Fully Mixed Turbofan

1.4

$p_a = 14.69 \text{ psia}$ $\quad A_e = 851 \text{ in}^2$

$u_a = 893 \text{ ft/s}$ $\quad p_e = 18.4 \text{ psia}$

$\dot{m} = 150 \text{ lbm/s core}$ $\quad u_e = 1435 \text{ ft/s}$

$\alpha = 2.2$ BP Ratio $\quad \dot{m}_f$ negligible

$$F = A_e (p_e - p_a) + \dot{m}_e (u_e - u_a)$$

$$\dot{m}_e = \dot{m} + 2.2\dot{m} = 3.2\dot{m}$$

$$F = 851 (18.4 - 14.69) \frac{\text{lb}}{\text{in}^2} \text{ in}^2 + \frac{3.2 \times 150 \frac{\text{lbm}}{\text{s}} (1435 - 893) \frac{\text{ft}}{\text{s}}}{32.17 \frac{\text{lbm} \cdot \text{ft}}{\text{s}^2 \text{lbf}}}$$

$$\boxed{F = 11237 \text{ lbf}}$$

1.5

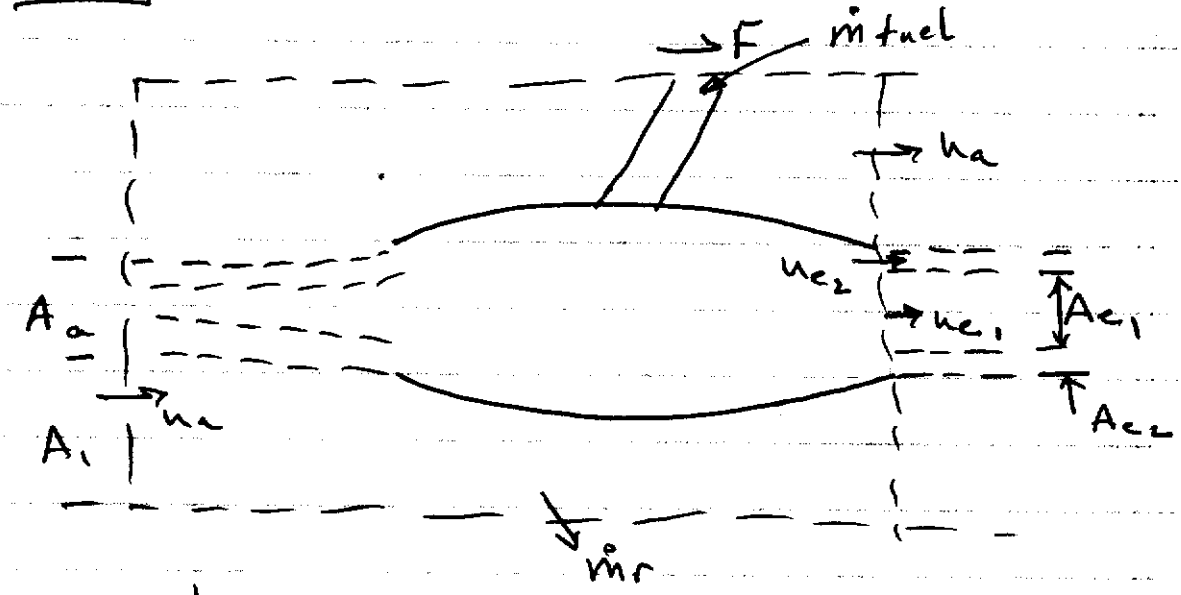
$$F = \dot{m}(u_B - u_a) + A(p_B - p_a) + F_p$$

$$= \frac{30(625-783)}{32.17} + 290(14.7-14.7) + 2517$$

$$= -147 + 2517$$

$$\boxed{F = 2370 \text{ lb}_f}$$

1.6 Mixed Turbofan



x Momentum:

$$F + p_a A_a + p_a A_1 - p_{e_1} A_{e_1} - p_{e_2} A_{e_2} - p_a A_2 + F_{bx} =$$

$$- u_a \rho_a A_a u_a + u_{e_1} \rho_{e_1} A_{e_1} u_{e_1} + u_{e_2} \rho_{e_2} A_{e_2} u_{e_2}$$

$$- u_a \rho_a A_1 u_a + u_a \rho_a A_2 u_a - \dot{m}_f u_{fx} + \dot{m}_r u_a$$

now if the exhaust is fully mixed

$u_{e_1} = u_{e_2} = u_e$ $\quad$ $\rho_{e_2} = \rho_{e_1} = \rho_e$

$\rho_{e_1} = \rho_{e_2} = \rho_e$ $\quad$ Also $A_e = A_{e_1} + A_{e_2}$

so above becomes with no body Force

$$F + p_a A_e + p_a - \rho_e A_e - p_a A_2 + F_{bx} =$$

$$- u_a \rho_a A_a u_a + u_e \rho_e A_e u_e - u_a \rho_a A_1 u_{a_1} +$$

$$u_a \rho_a A_2 u_a + \dot{m}_r u_a - \dot{m}_f u_{fx}$$

This is the same as eq. 1.F.2 (exactly)! One can proceed exactly as in Text.

1.7 Exhausted Fan $u_a = M_a\sqrt{\gamma RT_a} = 748$

$p_a = 4.36$ psia @ 30000 ft

(a) $\dot{m}_{fuel}$ small

$$F = \dot{m}(u_B - u_a) + A_B(p_B - p_a) + \dot{m}_s(u_9 - u_a) + A_9(p_9 - p_a)$$

$$= \frac{190}{32.17}(1880 - 748) + 901(9.42 - 4.36) + 1.2\times\frac{190}{32.17}(1350 - 748) + 936(4.36 - 4.36)$$

$$= 6686 + 4561 + 4267 + 0$$

$$= \underbrace{11247}_{\text{core}} + \underbrace{4267}_{\text{Fan}}$$

$$= 15514 \text{ lbf}$$

(b) $\dot{m}_{fuel}$ not small

$$F = \dot{m}_B u_B - \dot{m} u_a + A_B(p_B - p_a) + \dot{m}_s(u_9 - u_a) + A_9(p_9 - p_a)$$

$$= \left(\frac{190 + 4.66}{32.17}\right)(1880) - \frac{190}{32.17}(748) + 901(9.42 - 4.36) + 1.2\times\frac{190}{32.2}(1350 - 748) + 936(4.36 - 4.36)$$

$$= 6955 + 4561 + 4267 + 0$$

$$= \underbrace{11516}_{\text{core}} + \underbrace{4267}_{\text{Fan}} = 15783 \text{ lbf}$$

1.7 cont

or.

$$F = \dot{m}_e(u_e - u_a) + \dot{m}_s(u_s - u_a) + \dot{m}_f(u_a) + A_e(p_e - p_a) + A_c(p_s - p_a)$$

$$= \frac{(190 + 4.66)}{32.17}(1880 - 748) + \frac{1.2 \times 190}{32.2}(1350 - 748)$$

$$+ \frac{4.66}{32.17} \times 748 + 901(9.42 - 4.36) + 936(4.36 - 7.31)$$

$$= 15783$$

1.8 18000 ft $M=0.8$ $A_e = 361\ in^2$ $p_e = 22.4\ psia$

$V_e = 1938$ $F = 11666\ lb_f$ $\dot{m}_f = 4.04\ \frac{lb_m}{s}$

$\dot{m} = ?$ (a) $\dot{m}_f \ll \dot{m}$ (b) $\dot{m}_f$ included.

$$F = \dot{m}_f u_a + \dot{m}_e (u_e - u_a) + A_e (p_e - p_a) - \dot{m}_f u_{f_x}$$

(a) $$F = \dot{m} (u_e - u_a) + A_e (p_e - p_a)$$

18000 ft $\Rightarrow T_a = 454.6\ R$ $p_a = 7.347$

$$a_a = \sqrt{\gamma R T} = 1045\ ft/s$$

$$u_a = M_a a_a = 836.1$$

$$11666 = \dot{m} \frac{(1938 - 836)}{32.17} + 361 (22.4 - 7.35)$$

$$\boxed{\dot{m} = 181.9\ lb_m/s}$$

(b)

$$11666 = 4.04 \frac{(836.1)}{32.17} + \dot{m}_e \frac{(1938 - 836)}{32.17} + 361 (22.4 - 7.35)$$

$$\dot{m}_e - 178.9 = \dot{m} + \dot{m}_f$$

$$\boxed{\dot{m} = 174.8\ lb_m/s}$$

1.9 Turbofan 24000 ft $p_a = 5.69$ psia

$M_a = 0.70$ $T_a = 433.1$ R

$\alpha = 3.0$ $u_a = M_a\sqrt{\gamma RT_a} = 716$ ft/s

$\dot{m}_{fuel} = 3.77$ lbm/s

$A_{fan\,ex} = 1259$ m^2 $p_e = 5.69$ psia (core)

$A_{core\,ex} = 1302$ m^2 $p_s = 9.74$ psia (fan)

Fan exit $V_{e1} = 1146$ ft/s

Core exit $V_{e1} = 1489$ ft/s

Thrust $F = 15536$ lbf

(a) $F = A_c(p_e - p_a) + A_s(p_s - p_a) + \dot{m}_e(u_e - u_a) + \dot{m}_s(u_s - u_a)$

ignoring $\dot{m}_f$ $\dot{m}_e = \dot{m}_{core} = \dot{m}$

$\dot{m}_s = \alpha\dot{m} = 3\dot{m}$

$F - A_c(p_e - p_a) - A_s(p_s - p_a) = \dot{m}(u_e - u_a) + 3\dot{m}(u_s - u_a)$

or

$$\dot{m} = \frac{F - A_c(p_e - p_a) - A_s(p_s - p_a)}{u_e + 3u_s - 4u_a}$$

$$= \frac{15536 - 1302(5.69 - 5.69) - 1259(9.74 - 5.69)}{1489 + 3\times 1146 - 4\times 716} \times 32.17$$

$$\boxed{\dot{m} = 162.9 \text{ lbm/s}}$$

(b) include $\dot{m}_f$

$$F = A_e(p_e - p_a) + A_s(p_s - p_a) + \dot{m}_e(u_e - u_a) + \dot{m}_s(u_s - u_a) + \dot{m}_f u_a$$

$$\dot{m}_e = \dot{m} + \dot{m}_f$$

$$\dot{m}_s = \alpha \dot{m} = 3\dot{m}$$

$$\dot{m} = \frac{F - A_e(p_e - p_a) - A_s(p_s - p_a) - \dot{m}_f u_e}{u_e + 3u_s - 4u_a}$$

$$= \frac{15536 - 1302(5.69 - 5.69) - 1259(9.74 - 5.69) - \frac{3.77 \times 1489}{3.22}}{1489 + 3(1146) - 4 \times 7.16} \times 32.17$$

$$\boxed{\dot{m} = 160.18 \ \frac{lbm}{s}}$$

$$\dot{m}_e = 160.18 + 3.77$$

$$\boxed{\dot{m}_e = 163.95 \ lbm/s}$$

7.11 Turbo prop

9000 ft alt ⇒ $p_a = 10.5$ psia $\quad p_e = 10.5$ psia

$V_a = 487$ ft/s $\quad V_e = 1137$ ft/s

$\rho_e = 0.000453$ sl/ft^3 $\quad \dot{m}_{fuel} = 0.95$ lbm/s

$F_{prop} = 4243$ lbf $\quad F_{tot} = 4883$ lbf

$\dot{m}_a = \dot{m}_e - \dot{m}_f = \rho_e V_e A_e - \dot{m}_f$

$F_{tot} = F_{prop} + F_{core}$

$F_{core} = F_{tot} - F_{prop} = 4883 - 4243 = 640$ lbf

$F_{core} = \dot{m}_f V_a + \dot{m}_e (u_e - u_a) + A_e (p_e - p_a)$ (0)

$A_e \rho_e u_e (u_e - u_a) = F - \dot{m}_f u_a$

$= 640 - 0.95 \frac{\text{lbm}}{\text{s}} \cdot 487 \text{ ft/s} / 32.2 \frac{\text{lbm}}{\text{s}}$

$= 625.6$ lbf

so $A_e = \dfrac{625.6 \text{ lbf}}{0.000453 \frac{\text{sl}}{\text{ft}^3} \cdot 1137 \text{ ft/s} (1137 - 487) \text{ ft/s}}$

$= 1.869 \text{ ft}^2$

if Round $\quad A_e = \pi \frac{D^2}{4} \Rightarrow \boxed{D = 1.54 \text{ ft}}$

$\dot{m}_a = \dot{m}_e - \dot{m}_f = \rho_e u_e A_e - \dot{m}_f$

$= 0.000453 \frac{\text{sl}}{\text{ft}^3} \cdot 1137 \frac{\text{ft}}{\text{s}} \cdot 1.87 \text{ ft}^2 \times 32.2 \frac{\text{lbm}}{\text{s}} - 0.95 \frac{\text{lbm}}{\text{s}}$

$\boxed{\dot{m}_a = 30. \text{ lbm/s}}$

1.11 Turbo jet @ 20000 ft @ $M = 0.75$

$p_a = 6.75$ psia $\quad T_a = 447.4°R \quad \gamma = 1.40$

$p_{exit} = 19.7$ psia

$u_{exit} = 1965$ ft/s

$\rho_{exit} = 0.000979$ slug/ft^3

$\dot{m}_f = 3.16$ lbm/s

$F = 8995$ lbf

$u_a = 0.75\sqrt{\gamma R T_a} = 779.5$ ft/s

(a) Find injected mass flow rate

$$F = \dot{m}_f u_a + \dot{m}_e (u_e - u_a) + A_e (p_e - p_a)$$

$$A_e = \dot{m}_e / \rho_e u_e \qquad \dot{m}_{core} = \dot{m}_e - \dot{m}_f$$

$$F = \dot{m}_f u_a + \dot{m}_e (u_e - u_a) + \dot{m}_e (p_e - p_a)/\rho_e u_e$$

$$\dot{m}_e = \left[F - \dot{m}_f u_a\right] \Big/ \left[(u_e - u_a) + \frac{p_e - p_a}{\rho_e u_e}\right]$$

$$= \frac{(8995)(32.2) - (3.16)(779.5)}{(1965 - 779.5) + \frac{144(19.7 - 6.75)}{(0.000979)(1965)}}$$

$$= 133.27 \text{ lbm/s}$$

$\therefore \dot{m}_{core} = 133.27 - 3.16 = \boxed{130.1 \text{ lbm/s}}$

$$TSFC = \frac{\dot{m}_f}{F} = \frac{3.16 \times 3600}{8995}$$

$$= \boxed{1.26 \frac{lbm}{hr\,lbf}}$$

(b) Find Diameter

$$A_e = \frac{\pi}{4} D_e^2 = \frac{\dot{m}_e}{\rho_e u_e}$$

$$D_e = \sqrt{\frac{4\dot{m}_e}{\pi \rho_e u_e}}$$

$$= \sqrt{\frac{(4)(133.27)}{(3.1416)(0.000979)(1965)(32.2)}}$$

$$= \boxed{1.655\ ft} \text{ or } 19.9\ in$$

1.12 50000 ft $\Rightarrow$ $p_a = 1.69$ psia

$T_a = 390.0\ ^\circ R$

$a_a = \sqrt{\gamma R T} = \sqrt{(1.4)(53.35)(32.17)(390)} = 968$ ft/sec

$u_a = M_a a_a = 3872$ ft/sec $\qquad A_e = \frac{\pi D_e^2}{4} = 789\ in^2$

$\dot{m}_e = \dot{m} + \dot{m}_f = 9043.74 = 937$

$$F = \dot{m}_f u_a + \dot{m}_e (u_e - u_a) + A_e (p_e - p_a)$$

$$= \frac{3.74}{32.17}(3872) + \frac{937}{32.17}(5726 - 3872) - 789(1.60 - 1.69)$$

$$= 450 + 5402 - 71 = \boxed{5781\ lb_f}$$

$$TSFC = \frac{3.74 \times 3600}{5781} = \boxed{2.33\ \frac{lb_m}{hr\ lb_f}}$$

10.13 $F = A_e(p_e - p_a) + A_f(p_f - p_a) + \dot{m}_e(u_e - u_a) + \dot{m}_{se}(u_s - u_a) + \dot{m}_f u_a$

25000 ft $\Rightarrow$ $T_a = 429.6^\circ R$ $p_a = 5.46$ psia

$a_a = \sqrt{\gamma R T_a} = 1016$ ft/sec

$\dot{m}_{se} = \rho_{se} u_s A_f$ $\dot{m}_e = \rho_e u_e A_e$ $A_e = \dot{m}_e/\rho_e u_e$ etc.

f = fuel ratio $= \dot{m}_f/\dot{m}$

$\dot{m}_e = \dot{m} + \dot{m}_f = \dot{m}(1+f)$

(a)

$$F = \frac{\dot{m}(1+f)}{\rho_e u_e}(p_e - p_a) + \frac{\dot{m}\alpha}{\rho_{se} A_f}(p_f - p_a) + \dot{m}(1+f)(u_e - u_a) + \alpha\dot{m}(u_s - u_a) + f\dot{m}u_a$$

$$10580 \frac{\text{sl ft}}{\text{sec}^2} = \dot{m}\left[\frac{1.0255(10.26 - 5.46)}{0.000578 \cdot 1852}\,44 + \frac{1.2(10.07 - 5.46)}{0.00154 \cdot 1147}\,\frac{\text{lb}_f\,\text{ft}^3}{\text{in}^2\,\text{sl}\,\text{ft}}\,\frac{\text{sl ft}}{\text{lb}_f\,\text{sec}^2}\,\frac{144\,\text{in}^2}{\text{ft}^2}\right.$$

$$\left. + \underset{\text{ft/sec}}{1.0255(1852 - 815)} + \underset{\text{ft/sec}}{1.2(1147 - 815)} + \underset{\frac{\text{ft}}{\text{sec}}}{0.0255 \cdot 815}\right]$$

$$= \dot{m}\left[662.1 + 451.0 + 1063.4 + 398.4 + 20.8\right]$$

$$= \dot{m}(2595.7)\ \text{ft/sec}$$

$$\dot{m} = 4.076\ \text{sl/sec} = \boxed{131\ \text{lb}_m/\text{sec}}$$

(b) $A_e = \dfrac{(1.0255)131/32.17}{0.000578 \cdot 1852} = \boxed{3.90\ \text{ft}^2}$ $\dfrac{\text{sl ft}^3}{\text{sec}^2\ \text{ft}}$

$A_f = \dfrac{(1.2)(131)/32.17}{0.00154 \cdot 1147} = \boxed{2.77\ \text{ft}^2}$

11.14 SLTO

$\dot{m}_s = 3000$ lbm/s
$\dot{m}_a = 375$ lbm/s
note $\dot{m}_s/\dot{m}_a = 8.0$
$\dot{m}_f = 9.0$ lbm/s
$u_s = 885$ ft/s
$u_e = 1255$ ft/s
$A_f = 6610$ in^2
$A_e = 1500$ in^2

$p_f = 14.69$
$p_0 = 14.69$
$p_a = 14.69$
$u_a = 0$

$$F = A_e(p_e - p_a) + A_f(p_f - p_a) + \dot{m}_e(u_e - u_a) + \dot{m}_s(u_s - u_a) + \dot{m}_f u_a$$

$$\dot{m}_e = 375 + 9 = 384 \text{ lbm/s}$$

$$F = 1500(0) + 6610(0) + \frac{384}{32.17}\frac{\text{lbm}}{\text{s}}\frac{\text{s}^2}{\text{lbm}}[1255 - 0] + \frac{3000}{32.17}[885 - 0] + 9(0)$$

$$= 0 + 0 + 14980 + 82530 + 0$$

$$\boxed{F = 97510 \text{ lbf}}$$

$$TSFC = \frac{\dot{m}_f}{F} = \frac{9.0}{97510}\frac{\text{lbm}}{\text{s}}\frac{3600 \text{ sec}}{\text{hr}}\frac{}{\text{lbf}} = \boxed{0.332 \frac{\text{lbm}}{\text{hr lbf}}}$$