Chapter 1

Introduction

1.1 Describe at least three applications for each of the four lubrication regimes.

Solution:

(a) Examples of hydrodynamic lubrication include: journal bearing for a crankshaft in a car engine, tilting pad bearings in hydroelectric power plants, magnetic head separated from a computer hard disk, water skis, air hockey pucks, etc.

(b) Examples of elastohydrodynamic lubrication include ball bearings, gear contacts, human hip joints, bowling balls on lubricated lanes, etc.

(c) Examples of boundary lubrication include rubbing bearings, door hinges, MEMS devices, metal forming operations such as coining, etc.

(d) Examples of partial lubrication include most metal forming operations, wet clutches surfaces, artificial hips, lightly greased frying pan, etc.

1.2 Describe the differences between conformal and nonconformal surfaces.

Solution: Conformal surfaces fit snugly into each other, and they have very large contact areas. Nonconformal surfaces have very small contact areas. At zero load, the contact area in a conformal contact is still large, while in a non-conformal surface the contact at zero load is a single point. The load carrying area in a conformal surface is relatively constant, but it increases significantly with load for nonconformal surfaces.

1.3 Determine which of the following contact geometries is conformal and which is non-conformal:

(a) Meshing gear teeth

(b) Ball and inner race of a ball bearing

(c) Journal bearing

(d) Railway wheel and rail contact

(e) Car tire in contact with the road

(f) Egg and egg cup

(g) Eye and socket

(h) Golf ball and club

(i) Human hip joint

Solution: Conformal surfaces: journal bearing, egg and egg cup, eye and socket.

Nonconformal surfaces: meshing gear teeth, ball and inner race of a ball bearing, railway wheel and rail contact, car making contact with the road, golf ball and club.

The human hip joint is conformal if it is a natural hip because of the cartilage present; it is nonconformal if it is an artificial hip.

1.4 A hydrodynamic journal bearing is loaded with a normal load w_z and rotating with a surface velocity u_b. According to Eq. (1.1) find how much higher rotational speed the bearing needs to maintain the same minimum film thickness if the load w_z is doubled.

Solution: Equation (1.1) gives:

$$(h_{\min})_{\text{HL}} \propto \left(\frac{u_b}{W}\right)^{1/2}$$

Therefore, if the load is doubled, the speed must be doubled to maintain the same film thickness.

1.5 A ball bearing lubricated with a mineral oil runs at 3000 rpm. The bearing is made of AISI 52100 steel and is loaded to a medium-high level. Find how much thinner the oil film will be if the load is increased 10 times. How much must the speed be increased to compensate for the higher load while keeping the oil film constant?

Solution: If the load is increased by a factor of ten, then $W_2 = 10W_1$. Therefore,

$$\frac{h_{\min 1}}{h_{\min 2}} = \left(\frac{W_1}{W_2}\right)^{-0.073} = \left(\frac{1}{10}\right)^{-0.073} = 1.183$$

or $h_{\min 1} = 1.183 h_{\min 2}$. If the speed is to be increased, and the new film thickness is the same as the original case, then

$$\frac{h_{\min 2}}{h_{\min 1}} = \frac{1}{1.183} = \left(\frac{u_2}{u_3}\right)^{0.68} \quad \rightarrow \quad \frac{u_2}{u_3} = 0.781$$

or the new speed has to be (1/0.781)=1.280 or around a 28% increase.

Chapter 2

Bearing Classification and Selection

2.1 Figures 2.3 and 2.4 show the relationship between load and speed for four different types of bearings. How would you use these figures to help you select the appropriate bearing for your particular application?

Solution: Figure 2.3 shows that for low-speed applications, rolling-element bearings are the preferred type of bearing but at heavier loads the hydrodynamic oil film bearing is preferred. Rubbing or oil-impregnated porous metal bearings are not preferred because of the restrictions imposed in this considerations.

Figure 2.4 shows for thrust bearings there is a small region where rubbing bearings are preferred at light loads and low speeds, followed by rolling element bearings, and then hydrodynamic oil film bearings preferred at light loads and high speeds. The oil-impregnated porous metal bearing is not preferred because of the restrictions imposed in the consideration.

It should be emphasized that if a different criterion than

(a) Good engineering practice
(b) Commercially available parts
(c) Standard materials
(d) Bearings with widths equal to their diameters
(e) Medium-viscosity mineral oil

is used, different bearings may appear in Figs. 2.3 and 2.4.

2.2 Suggest suitable types of bearing to meet the following situations: (a) High load, very low speed, very low friction (b) Light load, very high speed, no liquid lubricant (c) Light load, low speed, no liquid lubricant

Solution:

(a) For high load, very low speed and very low friction a hydrodynamic bearing would be recommended.
(b) For light load, very high speed, and no liquid lubricant, a self-acting gas-lubricated bearing would be recommended.
(c) For light load, low speed, and no liquid lubricant a rubbing bearing would be recommended.

2.3 Explain why gas-lubricated bearings are appealing. Describe the limiting features of this type of bearing.

Solution: Gas bearings are very appealing because of the following:

(a) Starting and running torque is very low.

(b) Running is silent.

(c) Operation is excellent at high temperatures, and in general performance parameters are only slightly affected by temperature whereas for oil-lubricated bearings the performance parameters are noticeably affected by changes in temperature.

Chapter 3

Surface Topography

3.1 Show that for a Gaussian distribution and a zero mean ($z^* = 0$) from the reference line (determined by the M system) the theoretical skewness is zero and the kurtosis is 3.

Solution: When all the moments of a distribution exist (i.e., when all moments are finite), it is possible to associate a moment-generating function with the distribution. This is defined as $E(e^{\theta z})$, where z is random height and θ is a dummy continuous variable; the expected value of $e^{\theta z}$ will be a function that can be denoted by

$$M = E(e^{\theta z}) = \int_{-\infty}^{\infty} e^{\theta z} \bar{\psi} \, dz$$

It can also be shown that

$$M_n = \int_{-\infty}^{\infty} z^n \bar{\psi} \, dz$$

and

$$M_n = \frac{d^n}{d\theta^n} (M)_{\theta=0}$$

where $\bar{\psi}$ is the probability density functioon of the distribution of random heights. The general expression for the probability density function while assuming a Gaussian distribution is

$$\bar{\psi} = \frac{1}{\bar{\sigma}(2\pi)^{1/2}} e^{-(z-z^*)^2/2\bar{\sigma}^2}$$

where z^* is the distance of the mean line from the value chosen as the origin, and $\bar{\sigma}$ is the standard deviation of heights. If the reference line is determined by the M system and the distribution is Gaussian, the following are true:

$$z^* = 0 \qquad \text{and} \qquad \bar{\sigma} = R_q$$

Thus, for a Guassian distribution with zero mean, the probability density function becomes

$$\bar{\psi} = \frac{1}{R_q \sqrt{2\pi}} e^{-z^2/2R_q^2}$$

Regardless of the distribution the total probability is

$$\int_{-\infty}^{\infty} \bar{\psi}(z) \, dz = 1$$

The mean value z^*, which is zero for a Gaussian distribution, may be associated with the *first moment* of the distribution as

$$z^* = 0 = \int_{-\infty}^{\infty} z \bar{\psi} \, dz = M_1$$

The *second moment* is associated with the root mean square R_q, which for a Gaussian distribution is equal to the standard deviation

$$R_q^2 = \bar{\sigma}^2 = \int_{-\infty}^{\infty} z^2 \bar{\psi}\, dz = M_2$$

The *third moment* is associated with the skewness; thus from Eq. (3.12),

$$\bar{\alpha} = \frac{1}{R_q^3} \int_{-\infty}^{\infty} z^3 \bar{\psi}\, dz = M_3/R_q^3$$

The *fourth moment* is associated with the kurtosis; thus from Eq. (3.13)

$$\bar{\beta} = \frac{1}{R_q^4} \int_{-\infty}^{\infty} z^4 \bar{\psi}\, dz = M_4/R_q^4$$

Therefore, we can write

$$M = \int_{-\infty}^{\infty} \frac{1}{R_q\sqrt{2\pi}} e^{\theta z - z^2/2R_q^2}\, dz$$

or

$$M = \frac{1}{R_q\sqrt{2\pi}} \int_{-infty}^{\infty} e^{-(az^2+bz)}\, dz$$

where $a = 1/2R_q^2$ and $b = -\theta$. From Schaum's Outline Series - Mathematical Handbook of Formulas and Tables, p. 98, Eq. (15.75),

$$\int_{-\infty}^{\infty} e^{-(az^2+bz)}\, dz = \sqrt{2\pi R_q^2} e^{2\theta^2/R_1^2}$$

Therefore

$$M = e^{\theta^2 R_q^2/2}$$

Using the equation for M_1 above yields

$$M_1 = \left[\frac{d}{d\theta}(M)\right]_{\theta=0} = \theta R_q^2 e^{\theta^2 R_q^2/2} = 0$$

Also,

$$M_2 = \left[\frac{d^2}{d\theta^2}(M)\right]_{\theta=0} = \left(R_q^2 e^{\theta^2 R_q^2/2} + \theta^2 R_q^4 e^{\theta^2 R_q^2/2}\right)_{\theta=0} = R_q^2$$

and

$$M_3 = \left[\frac{d^3}{d\theta^3}(M)\right]_{\theta=0} = \left(\theta R_q^4 e^{\theta^2 R_q^2/2} + 2\theta R_q^4 e^{\theta^2 R_q^2/2} + \theta^3 R_q^6 e^{\theta^2 R_q^2/2}\right)_{\theta=0} = 0$$

So that $\bar{\alpha} = 0$. Also,

$$M_3 = \left[\frac{d^3}{d\theta^3}(M)\right]_{\theta=0} = \left(3R_q^4 e^{\theta^2 R_q^2/2} + 3\theta^2 R_q^6 e^{\theta^2 R_q^2/2} + 3\theta^2 R_q^6 e^{\theta^2 R_q^2/2} + \theta^4 R_q^8 e^{\theta^2 R_q^2/2}\right)_{\theta=0} = 3R_q^4$$

Therefore, $\bar{\beta} = 3$. Therefore, for a Gaussian distribution, the skewness $\bar{\alpha}$ is zero and the kurtosis $\bar{\beta}$ is three.

3.2 Show that the skewness for a deeply pitted surface is less than zero ($\bar{\alpha} < 0$) and that the kurtosis for a relatively flat height distribution approaches zero $\left(\bar{\beta} \to 0\right)$.

Solution: A deeply pitted surface has large negative values of z for any odd power of z (z^3, $z^5, \ldots$). The distribution is not Gaussian but is such that the mean value is less than zero ($z^* < 0$). From Problem 3.1,

$$M_3 = (z^*)^3 + 3\bar{\sigma}^2 z^*$$

Also, the skewness can be written as

$$\bar{\alpha} = \frac{M_3}{\bar{\sigma}^3} = \frac{M_3}{R_q^3}$$

Thus, if $z^* < 0$, then $\bar{\alpha} < 0$. Regarding the kurtosis, the following is obtained from Problem 3.1:

$$M_4 = (z^*)^4 + 6\bar{\sigma}^2 (z^*)^2 + 3\bar{\sigma}^4$$

Also, the kurtosis can be expressed as

$$\bar{\beta} = \frac{M_4}{R_q^4} = \frac{M_4}{\bar{\sigma}^4}$$

For a flat height distribution, $z^* \to 0$ and $\bar{\sigma} = R_q \to 0$. Therefore, as can be seen from the equations for M_4 and $\bar{\beta}$, if $z^* \to 0$ and $\bar{\sigma} \to 0$, then $\bar{\beta} \to 0$.

3.3 Find R_a/R_q for a Gaussian distribution with zero mean ($z^* = 0$).

Solution: It is given that

$$R_a = \int_{-\infty}^{\infty} z\bar{\psi}\, dz$$

where

$$\bar{\psi} = \frac{1}{R_q\sqrt{2\pi}} e^{-z^2/2R_q^2}$$

Substituting the second equation into the first yields

$$R_a = \frac{2}{\sqrt{2\pi}R_q} \int_0^{\infty} z e^{-z^2/2R_q^2}\, dz$$

Use the variable substitution $u = -z^2/2R_q^2$, so that $du = -(z/R_q^2)\, dz$ and $z\, dz = -R_q^2\, du$. Therefore

$$R_a = -\sqrt{\frac{2}{\pi}} \frac{R_q^2}{R_1} \int_0^{-\infty} e^u\, du$$

or

$$R_a = \sqrt{\frac{2}{\pi}} R_q$$

and

$$\frac{R_q}{R_q} = \sqrt{\frac{\pi}{2}} = 1.25$$

3.4 Prove that $R_a < R_q$.

Solution: Let $z_1, z_2, z_3, \ldots, z_n$ represent a set of N sampled surface roughness profile ordinates and let $\vec{r}$ and $\vec{s}$ be defined as

$$\vec{r} = (|z_1|, |z_2|, |z_3|, \ldots, |z_n|)$$
$$\vec{s} = (1, 1, 1, \ldots, 1)$$

Using the general vector inequality

$$(\vec{r} \cdot \vec{s}) \leq |\vec{r}| \cdot |\vec{s}|$$

where the expression on the left side of the inequality denotes a scalar product and the "absolute value" symbol on the right denotes vector lengths, gives

$$\sum_{i=1}^{N} |z_i| \leq \sqrt{N \sum_{i=1}^{N} z_i^2}$$

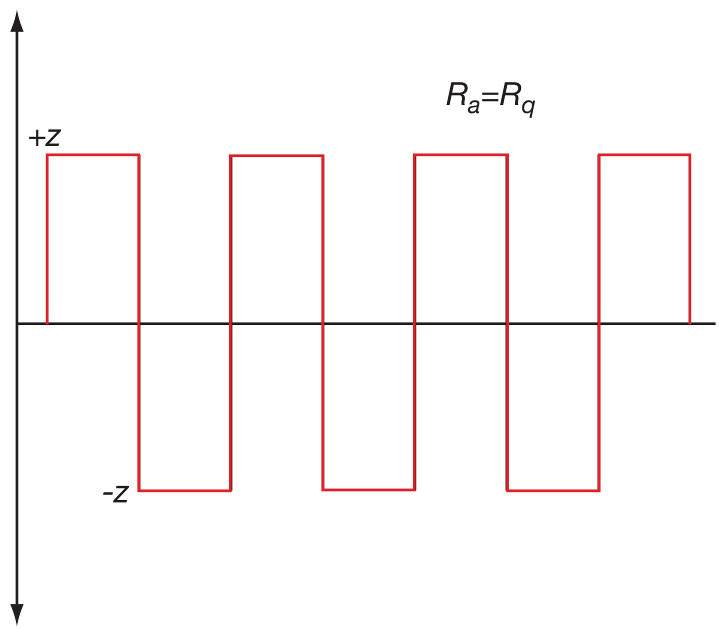
Dividing by N on both sides results in

$$\frac{1}{N} \sum_{i=1}^{N} |z_i| \leq \sqrt{\frac{1}{N} \sum_{i=1}^{N} z_i^2}$$

Therefore, from Eqs. (3.1) and (3.2), $R_a < R_q$.

3.5 What profile would produce an $R_a = R_q$?

Solution: Following the derivation in Problem 3.4, the equality $R_a = R_q$ holds when vectors $\vec{r}$ and $\vec{s}$ are parallel, which implies that $|z_i|$=constant. Also, since $\langle z_i \rangle = 0$, there must be equal numbers of positive and negative z_i. Putting this in a horizontal spacing, such values could be arranged in a *square wave profile* as sketched below.



3.6 Prove that the kurtosis is greater than or equal to 1.

Solution: This solution is similar to that for Problem 3.4. Let $z_1, z_2, z_3, \ldots, z_n$ represent a set of N sampled surface roughness profile ordinates, and let $\vec{r}$ and $\vec{s}$ be defined as

$$\vec{r} = \left(z_1^2, z_2^2, z_3^2, \ldots, z_n^2\right)$$