CHAPTER 1

1.1. (a) Total distance $= 1 + \frac{1}{2} + \frac{1}{4} + \cdots = \frac{1}{1-\frac{1}{2}} = 2$ m

(b) Distance north $= 1 - \frac{1}{4} + \frac{1}{16} - \cdots = \frac{1}{1+\frac{1}{4}} = 0.8$ m

Distance east $= \frac{1}{2} - \frac{1}{8} + \frac{1}{32} - \cdots = \frac{1}{2}\left(1 - \frac{1}{4} + \frac{1}{16} - \cdots\right) = 0.4$ m

$\therefore$ Final position is (0.8, 0.4)

(c) Straight line distance $= \sqrt{(0.8)^2 + (0.4)^2} = 0.8944$ m

1.2. $\mathbf{A} + \mathbf{B} + \mathbf{C} = 2\mathbf{a}_1 + 3\mathbf{a}_2 + 2\mathbf{a}_3$ — (1)

$2\mathbf{A} + \mathbf{B} - \mathbf{C} = \mathbf{a}_1 + 3\mathbf{a}_2$ — (2)

$\mathbf{A} - 2\mathbf{B} + 3\mathbf{C} = 4\mathbf{a}_1 + 5\mathbf{a}_2 + \mathbf{a}_3$ — (3)

$(1) + (2) \rightarrow 3\mathbf{A} + 2\mathbf{B} = 3\mathbf{a}_1 + 16\mathbf{a}_2 + 2\mathbf{a}_3$ — (4)

$(2) \times 3 + (3) \rightarrow 7\mathbf{A} + \mathbf{B} = 7\mathbf{a}_1 + 14\mathbf{a}_2 + \mathbf{a}_3$ — (5)

$[(5) \times 2 - (4)] \div 11 \rightarrow \mathbf{A} = \mathbf{a}_1 + 2\mathbf{a}_2$ — (6)

$(5) - (6) \times 7 \rightarrow \quad \mathbf{B} = \mathbf{a}_3$ — (7)

$(1) - (6) - (7) \rightarrow \quad \mathbf{C} = \mathbf{a}_1 + \mathbf{a}_2 + \mathbf{a}_3$ — (8)

1.3. $(\mathbf{A} + \mathbf{B}) \bullet (\mathbf{A} - \mathbf{B}) = \mathbf{A} \bullet \mathbf{A} - \mathbf{A} \bullet \mathbf{B} + \mathbf{B} \bullet \mathbf{A} - \mathbf{B} \bullet \mathbf{B} = A^2 - B^2$

$(\mathbf{A} + \mathbf{B}) \times (\mathbf{A} - \mathbf{B}) = \mathbf{A} \times \mathbf{A} - \mathbf{A} \times \mathbf{B} + \mathbf{B} \times \mathbf{A} - \mathbf{B} \times \mathbf{B} = 2\mathbf{B} \times \mathbf{A}$

For $\mathbf{A} = 3\mathbf{a}_1 - 5\mathbf{a}_2 + 4\mathbf{a}_3$ and $\mathbf{B} = \mathbf{a}_1 + \mathbf{a}_2 - 2\mathbf{a}_3$,

$\mathbf{A} + \mathbf{B} = 4\mathbf{a}_1 - 4\mathbf{a}_2 + 2\mathbf{a}_3$, $\mathbf{A} - \mathbf{B} = 2\mathbf{a}_1 - 6\mathbf{a}_2 + 6\mathbf{a}_3$,

$A^2 = 9 + 25 + 16 = 50$, and $B^2 = 1 + 1 + 4 = 6$

$(\mathbf{A} + \mathbf{B}) \bullet (\mathbf{A} - \mathbf{B}) = 8 + 24 + 12 = 44 = A^2 - B^2$

$$(\mathbf{A}+\mathbf{B})\times(\mathbf{A}-\mathbf{B})=\begin{vmatrix}\mathbf{a}_x & \mathbf{a}_y & \mathbf{a}_z\\ 4 & -4 & 2\\ 2 & -6 & 6\end{vmatrix}=-12\mathbf{a}_x-20\mathbf{a}_y-16\mathbf{a}_z$$

$$=2\begin{vmatrix}\mathbf{a}_x & \mathbf{a}_y & \mathbf{a}_z\\ 1 & 1 & -2\\ 3 & -5 & 4\end{vmatrix}=2\mathbf{B}\times\mathbf{A}$$

1.4. $\mathbf{B}\times\mathbf{C}=-4\mathbf{a}_x+2\mathbf{a}_y+8\mathbf{a}_z$, $\mathbf{A}\times(\mathbf{B}\times\mathbf{C})=8\mathbf{a}_x+16\mathbf{a}_y$

$\mathbf{C}\times\mathbf{A}=-\mathbf{a}_x-2\mathbf{a}_y+7\mathbf{a}_z$, $\mathbf{B}\times(\mathbf{C}\times\mathbf{A})=-12\mathbf{a}_x-8\mathbf{a}_y-4\mathbf{a}_z$

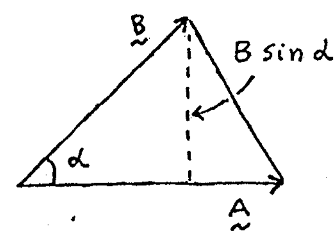
$\mathbf{A}\times\mathbf{B}=\mathbf{a}_x+2\mathbf{a}_y+3\mathbf{a}_z$, $\mathbf{C}\times(\mathbf{A}\times\mathbf{B})=4\mathbf{a}_x-8\mathbf{a}_y+4\mathbf{a}_z$

$\mathbf{A}\times(\mathbf{B}\times\mathbf{C})+\mathbf{B}\times(\mathbf{C}\times\mathbf{A})+\mathbf{C}\times(\mathbf{A}\times\mathbf{B})=0$

In fact, this quantity is zero for any **A**, **B**, and **C**.

1.5. Area $=\dfrac{1}{2}AB\sin\alpha=\dfrac{1}{2}|\mathbf{A}\times\mathbf{B}|$

For the points (1, 2, 1), (–3, –4, 5), and (2, –1, –3),



$\mathbf{A}=4\mathbf{a}_x+6\mathbf{a}_y-4\mathbf{a}_z$

$\mathbf{B}=5\mathbf{a}_x+3\mathbf{a}_y-8\mathbf{a}_z$

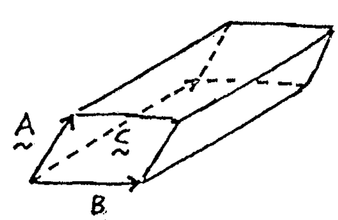
$\mathbf{A}\times\mathbf{B}=-36\mathbf{a}_x+12\mathbf{a}_y-18\mathbf{a}_z$

$\therefore$ Area $=\dfrac{1}{2}\sqrt{(-36)^2+(12)^2+(-18)^2}=21$ units.

1.6. Area of the base $=|\mathbf{B}\times\mathbf{C}|$

Height of parallelepiped = Projection of **A** onto the normal to the base

$=\mathbf{A}\bullet\dfrac{\mathbf{B}\times\mathbf{C}}{|\mathbf{B}\times\mathbf{C}|}$



$\therefore$ Volume of parallelepiped = Area of base × height $=\mathbf{A}\bullet\mathbf{B}\times\mathbf{C}$

For $\mathbf{A}=4\mathbf{a}_x$, $\mathbf{B}=2\mathbf{a}_x+\mathbf{a}_y+3\mathbf{a}_z$, and $\mathbf{C}=2\mathbf{a}_y+6\mathbf{a}_z$, $\mathbf{A}\bullet\mathbf{B}\times\mathbf{C}=0$.

Hence, volume of the parallelepiped is zero. The three vectors lie in a plane.

1.7. The vector **A** must be perpendicular to both $(-\mathbf{a}_y + 2\mathbf{a}_z)$ and $(\mathbf{a}_x - 2\mathbf{a}_z)$.

Hence $\mathbf{A} = C(-\mathbf{a}_y + 2\mathbf{a}_z) \times (\mathbf{a}_x - 2\mathbf{a}_z) = C(2\mathbf{a}_x + 2\mathbf{a}_y + \mathbf{a}_z)$ where C is a constant. To find C,

we note that $\mathbf{a}_x \times \mathbf{A} = \mathbf{a}_x \times C(2\mathbf{a}_x + 2\mathbf{a}_y + \mathbf{a}_z) = 2\mathbf{a}_z - \mathbf{a}_y$

$\therefore$ $C = 1$ and $\mathbf{A} = 2\mathbf{a}_x + 2\mathbf{a}_y + \mathbf{a}_z$.

Verification: $\mathbf{a}_y \times \mathbf{A} = \mathbf{a}_y \times (2\mathbf{a}_x + 2\mathbf{a}_y + \mathbf{a}_z) = \mathbf{a}_x - 2\mathbf{a}_z$.

1.8. Vector from $A(5, 0, 3)$ to $B(3, 3, 2) = -2\mathbf{a}_x + 3\mathbf{a}_y - \mathbf{a}_z$

Vector from $C(6, 2, 4)$ to $D(3, 3, 6) = -3\mathbf{a}_x + \mathbf{a}_y + 2\mathbf{a}_z$

Component of **AB** along **CD** = $\mathbf{AB} \cdot \dfrac{\mathbf{CD}}{CD} = \dfrac{6+3-2}{\sqrt{9+1+4}} = 1.8708$

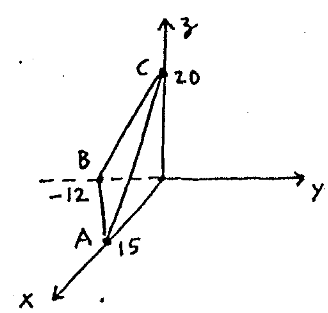
1.9. Writing the equation for the plane as $\dfrac{x}{15} - \dfrac{y}{12} + \dfrac{z}{20} = 1$, we find the intercepts on the x, y, and z-axes to be at 15, –12, and 20, respectively. Thus

$\mathbf{R}_{AB} = -15\mathbf{a}_x - 12\mathbf{a}_y$

$\mathbf{R}_{AC} = -15\mathbf{a}_x + 20\mathbf{a}_z$

$\mathbf{R}_{AC} \times \mathbf{R}_{AB} = 240\mathbf{a}_x - 300\mathbf{a}_y + 180\mathbf{a}_z$

$$\mathbf{a}_n = \frac{\mathbf{R}_{AC} \times \mathbf{R}_{AB}}{|\mathbf{R}_{AC} \times \mathbf{R}_{AB}|} = \frac{4\mathbf{a}_x - 5\mathbf{a}_y + 3\mathbf{a}_z}{5\sqrt{2}}$$



Distance from origin to the plane = $15\mathbf{a}_x \cdot \mathbf{a}_n = 6\sqrt{2}$.

1.10. For $y = 2x$, $z = 4y$, we have $dy = 2\,dx$, $dz = 4\,dy = 8\,dx$.

$\therefore$ $d\mathbf{l} = dx\,\mathbf{a}_x + dy\,\mathbf{a}_y + dz\,\mathbf{a}_z = dx\,\mathbf{a}_x + 2\,dx\,\mathbf{a}_y + 8\,dx\,\mathbf{a}_z$

$= (\mathbf{a}_x + 2\mathbf{a}_y + 8\mathbf{a}_z)\,dx$, independent of the point.

1.11. For $x = y = z^2$, we have $dx = dy = 2z\,dz$.

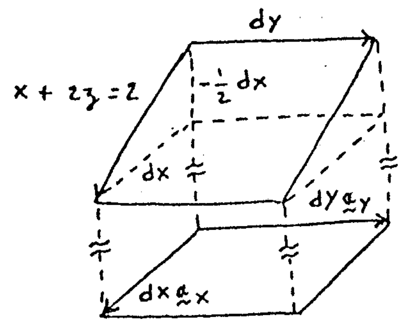
At the point (4, 4, 2), $dx = dy = 4\,dz$

$\therefore$ $d\mathbf{l} = dx\,\mathbf{a}_x + dy\,\mathbf{a}_y + dz\,\mathbf{a}_z = 4\,dz\,\mathbf{a}_x + 4\,dz\,\mathbf{a}_y + dz\,\mathbf{a}_z$

$= (4\mathbf{a}_x + 4\mathbf{a}_y + \mathbf{a}_z)\,dz$

1.12. Differential length vector having projection $dy\,\mathbf{a}_y = dy\,\mathbf{a}_y$

Differential length vector having projection $dx\,\mathbf{a}_x$ is



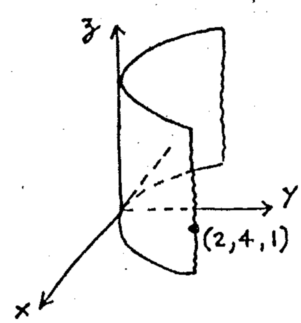
$$dx\,\mathbf{a}_x + dz\,\mathbf{a}_z = dx\,\mathbf{a}_x - \frac{1}{2}\,dx\,\mathbf{a}_z$$

$$= \left(\mathbf{a}_x - \frac{1}{2}\mathbf{a}_z\right) dx,$$

since for $x + 2z = 2$, $dz = -\dfrac{1}{2}\,dx$, independent of the point.

$$\therefore\ d\mathbf{S} = \left(\mathbf{a}_x - \frac{1}{2}\mathbf{a}_z\right) dx \times dy\,\mathbf{a}_y = \left(\frac{1}{2}\mathbf{a}_x + \mathbf{a}_z\right) dx\,dy\,.$$

1.13. One vector tangential to the surface is $dz\,\mathbf{a}_z$. Another tangential vector is given by



$$d\mathbf{l} = dx\,\mathbf{a}_x + dy\,\mathbf{a}_y$$

$$= dx\,\mathbf{a}_x + 2x\,dx\,\mathbf{a}_y$$

$$= (\mathbf{a}_x + 4\mathbf{a}_y)\,dx$$

$\therefore$ Vector normal to the plane $= (\mathbf{a}_x + 4\mathbf{a}_y)\,dx \times dz\,\mathbf{a}_z$

$$= (4\mathbf{a}_x - \mathbf{a}_y)\,dx\,dz$$

Unit vector normal to the plane $= \dfrac{4\mathbf{a}_x - \mathbf{a}_y}{\sqrt{17}}$.

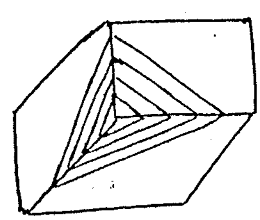
1.14. Denoting $h(x, y)$ to be the height field, we have

$x^2 + y^2 + h^2 = 4,\ x^2 + y^2 \le 4$

or, $h = \sqrt{4 - x^2 - y^2}\ ,\ x^2 + y^2 \le 4$.

1.15. The number field is $x + y + z$.

$\therefore$ Constant magnitude surfaces are the planes $x + y + z =$ constant.

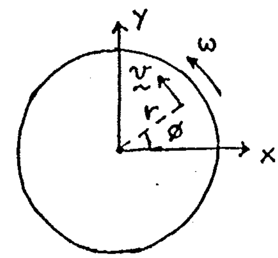


1.16. $\mathbf{d}(x, y, z) = x\mathbf{a}_x + y\mathbf{a}_y + z\mathbf{a}_z$

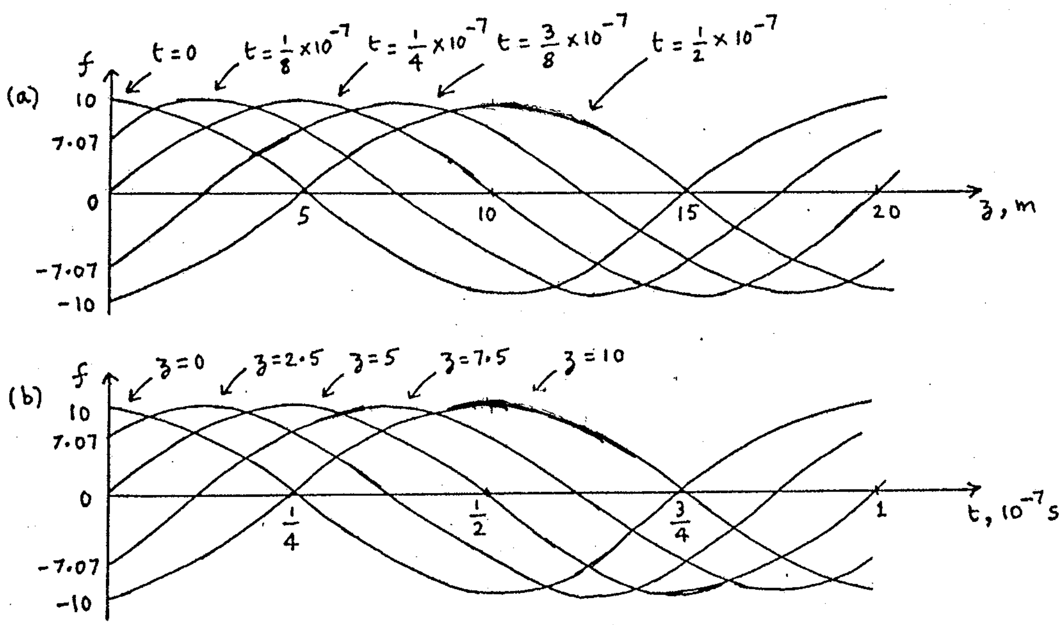
Constant magnitude surfaces are $x^2 + y^2 + z^2 =$ constant, and hence are spherical surfaces centered at the corner. Direction lines are radial lines emanating from the corner.

1.17. $\mathbf{v} = -r\omega \sin \phi\, \mathbf{a}_x + r\omega \cos \phi\, \mathbf{a}_y$

$= \omega(-y\mathbf{a}_x + x\mathbf{a}_y)$

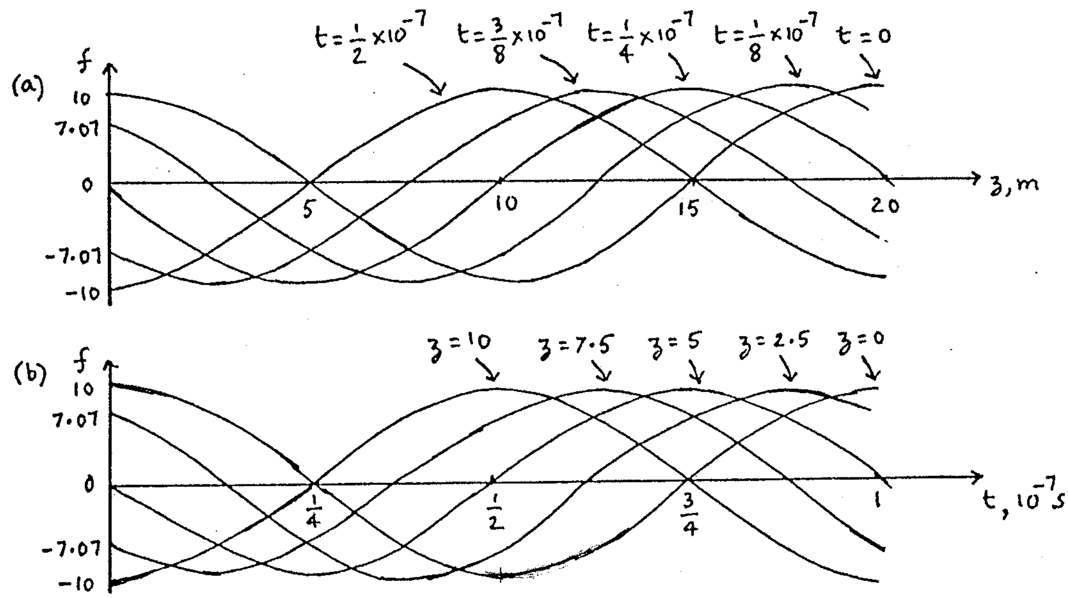


1.18. $f(z, t) = 10 \cos (2\pi \times 10^7 t - 0.1\, \pi z)$



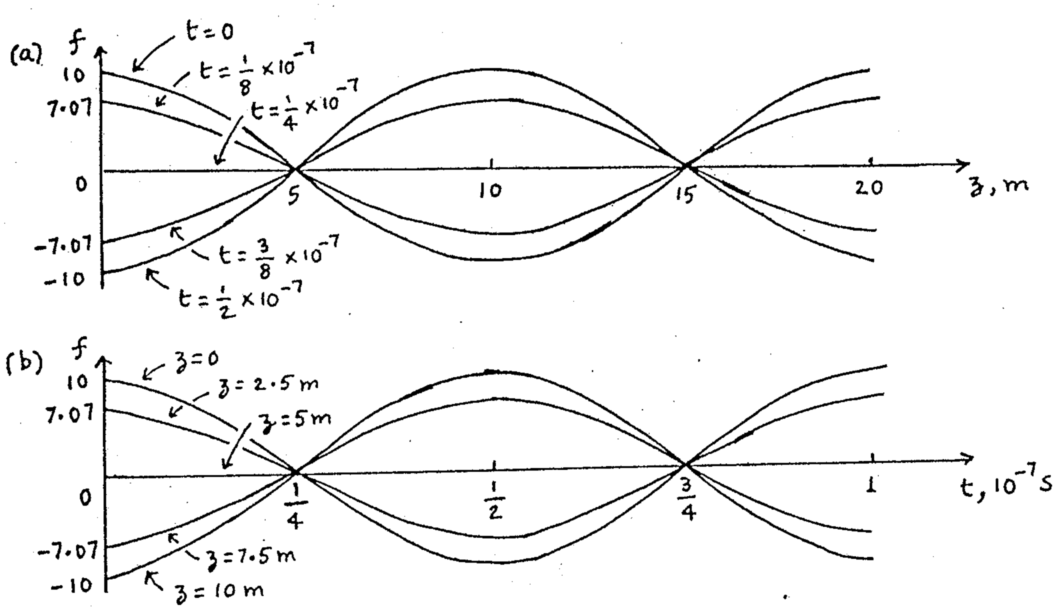
$f(z, t)$ represents a traveling wave progressing with time in the positive z-direction.

1.19. $f(z, t) = 10\cos(2\pi \times 10^7 t + 0.1\pi z)$



$f(z, t)$ represents a traveling wave progressing with time in the negative z-direction.

1.20. $f(z, t) = 10\cos 2\pi \times 10^7 t \cos 0.1\pi z$



$f(z, t)$ represents a standing wave.

1.21. (a) The two components are in phase; hence, linear polarization.

(b) The two components are perpendicular in direction, differ in phase by 90° and equal in amplitude; hence, circular polarization.

(c) The two components are perpendicular in direction, differ in phase by 90° but unequal in amplitude; hence elliptical polarization.

1.22. $\mathbf{F}_1$ and $\mathbf{F}_2$ differ in phase by 90°.

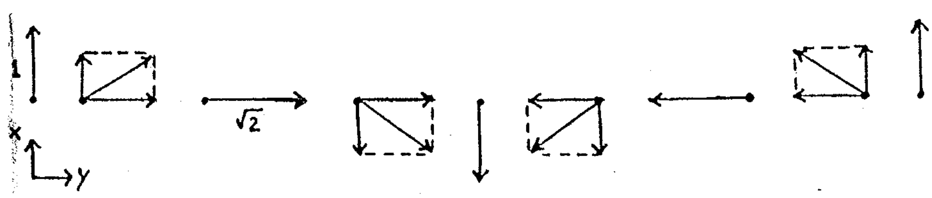
$$|\mathbf{F}_1| = \sqrt{3+1}\,\cos\omega t = 2\cos\omega t;\ |\mathbf{F}_2| = \sqrt{\frac{1}{4}+\frac{3}{4}+3}\,\sin\omega t = 2\sin\omega t\,.$$

$\therefore$ $\mathbf{F}_1$ and $\mathbf{F}_2$ are equal in amplitude.

$$\mathbf{F}_1\cdot\mathbf{F}_2 = -\frac{\sqrt{3}}{2}+\frac{\sqrt{3}}{2} = 0\,.\ \ \therefore\ \mathbf{F}_1 \text{ is perpendicular to } \mathbf{F}_2.$$

Thus $\mathbf{F}_1 + \mathbf{F}_2$ is circularly polarized.

1.23.



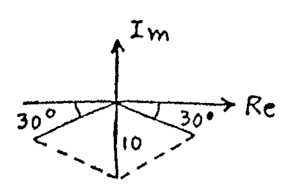
The polarization is elliptical with major axis in the y-direction, minor axis in the x-direction, and eccentricity equal to $\sqrt{2}$.

1.24. $10\cos(\omega t - 30°) + 10\cos(\omega t + 210°)$

$$\downarrow \qquad\qquad \downarrow$$

$$10\,e^{-j30°} \quad + \quad 10\,e^{j210°} \quad = 10\,e^{-j90°}$$

$\therefore$ The sum is $10\cos(\omega t - 90°) = 10\sin\omega t$.



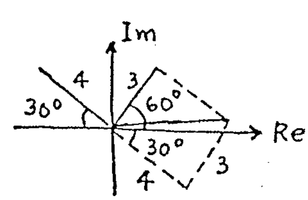
1.25. $3\cos(\omega t + 60°) - 4\cos(\omega t + 150°)$

$$\downarrow \qquad\qquad \downarrow$$

$$3\,e^{j60°} \quad - \quad 4\,e^{j150°}$$

$$= 5\,e^{j(60°-53.13°)} = 5e^{j6.87°}$$

$\rightarrow 5\cos(\omega t + 6.87°)$.



1.26. Replacing $\frac{di}{dt}$ by $j10^6\bar{I}$, i by $\bar{I}$, and 13 cos $10^6 t$ by $13e^{j0°}$, we have

$$5\times10^{-6}\times j10^6\bar{I}+12\bar{I} = 13e^{j0°}$$

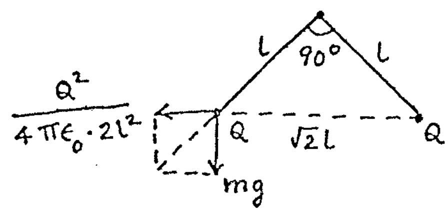
or, $(12+j5)\bar{I}=13, \bar{I}=\frac{13}{12+j5}=\frac{13}{13e^{j22.62°}}=1e^{-j22.62°}$

Thus $i = 1\cos(10^6 t - 22.62°) = 1\cos(10^6 t - 0.126\pi)$

1.27. From the construction shown,

$$\frac{Q^2}{4\pi\varepsilon_0\cdot 2l^2}\Big/ mg = \tan 45° = 1$$

or, $Q=\sqrt{8\pi\varepsilon_0 l^2 mg}$



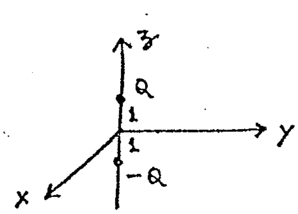
1.28. (a) At the point (0, 0, 100),

$$\mathbf{E}=\frac{Q}{4\pi\varepsilon_0(99)^2}\mathbf{a}_z+\frac{-Q}{4\pi\varepsilon_0(101)^2}\mathbf{a}_z$$

$$=\frac{Q}{4\pi\varepsilon_0}\frac{101^2-99^2}{99^2\times101^2}\mathbf{a}_z$$

$$=\frac{Q}{4\pi\varepsilon_0}\frac{(100+1)^2-(100-1)^2}{(100-1)^2\times(100+1)^2}\mathbf{a}_z=\frac{Q}{4\pi\varepsilon_0}\frac{400}{(100^2-1)^2}\mathbf{a}_z$$

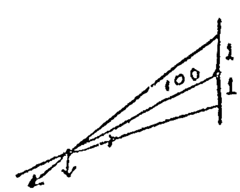
$$\approx\frac{Q}{4\pi\varepsilon_0}\frac{400}{100^4}\mathbf{a}_z=\frac{Q}{100^3\pi\varepsilon_0}\mathbf{a}_z$$



(b) At the point (100, 0, 0)

$$\mathbf{E}=-\frac{2Q}{4\pi\varepsilon_0(100^2+1^2)^{3/2}}\mathbf{a}_z$$

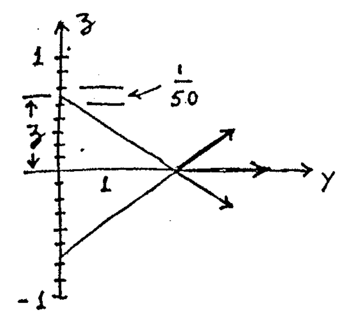
$$\approx-\frac{Q}{2\pi\varepsilon_0(100^3)}\mathbf{a}_z$$



1.29. $$\mathbf{E} = \frac{Q}{4\pi\varepsilon_0}\left[\frac{\mathbf{a}_x + 2\mathbf{a}_y + 2\mathbf{a}_z}{9^{3/2}} + \frac{2\mathbf{a}_x + \mathbf{a}_y + 2\mathbf{a}_z}{9^{3/2}} + \frac{2\mathbf{a}_x + 2\mathbf{a}_y + \mathbf{a}_z}{9^{3/2}}\right.$$
$$+ \frac{\mathbf{a}_x + \mathbf{a}_y + 2\mathbf{a}_z}{6^{3/2}} + \frac{2\mathbf{a}_x + \mathbf{a}_y + \mathbf{a}_z}{6^{3/2}} + \frac{\mathbf{a}_x + 2\mathbf{a}_y + \mathbf{a}_z}{6^{3/2}}$$
$$\left. + \frac{2\mathbf{a}_x + 2\mathbf{a}_y + 2\mathbf{a}_z}{12^{3/2}} + \frac{\mathbf{a}_x + \mathbf{a}_y + \mathbf{a}_z}{3^{3/2}}\right]$$
$$= \frac{Q}{4\pi\varepsilon_0}\left[\frac{5}{(3)^3} + \frac{4}{(\sqrt{6})^3} + \frac{2}{(\sqrt{12})^3} + \frac{1}{(\sqrt{3})^3}\right](\mathbf{a}_x + \mathbf{a}_y + \mathbf{a}_z)$$
$$= \frac{Q}{4\pi\varepsilon_0}(0.18519 + 0.27217 + 0.04811 + 0.19245)(\mathbf{a}_x + \mathbf{a}_y + \mathbf{a}_z)$$
$$= \frac{0.0555\,Q}{\varepsilon_0}(\mathbf{a}_x + \mathbf{a}_y + \mathbf{a}_z) \text{ N/C.}$$

1.30. For the ith segment,

$$z = \frac{2i-1}{100} \text{ and charge} = \frac{10^{-3}}{50} \text{ C.}$$

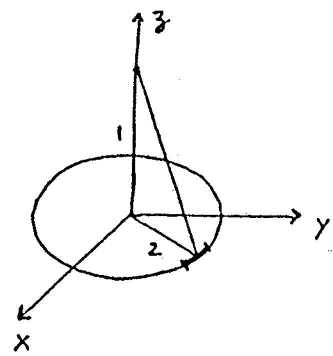


$$\therefore\ \mathbf{E} = \frac{2}{4\pi\varepsilon_0}\sum_{i=1}^{50}\frac{10^{-3}}{50}\frac{1}{\left(z^2+1\right)^{3/2}}\mathbf{a}_y$$
$$= \frac{10^{-5}}{\pi\varepsilon_0}\sum_{i=1}^{50}\left[10^{-4}(2i-1)^2 + 1\right]^{-3/2}\mathbf{a}_y$$

1.31. For the ith segment, $z = \frac{2i-1}{100}$, charge density $= 10^{-3}\,\frac{2i-1}{100}$ C/m,

$$\text{and charge} = 10^{-3}\,\frac{2i-1}{100}\cdot\frac{1}{50} = \frac{10^{-6}}{5}(2i-1) \text{ C.}$$
$$\therefore\ \mathbf{E} = \frac{2}{4\pi\varepsilon_0}\sum_{i=1}^{50}\frac{10^{-6}(2i-1)}{5}\frac{1}{\left(z^2+1\right)^{3/2}}\mathbf{a}_y$$
$$= \frac{10^{-7}}{\pi\varepsilon_0}\sum_{i=1}^{50}(2i-1)\left[10^{-4}(2i-1)^2 + 1\right]^{-3/2}\mathbf{a}_y$$

1.32. Dividing the circular ring into n segments and using the symmetry of the field about the z-axis, we obtain

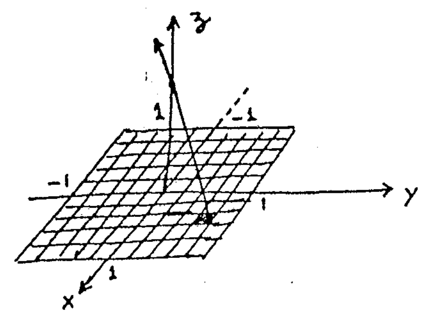


$$\mathbf{E} = \sum_n \frac{2\pi(2)\times 10^{-3}}{n\cdot 4\pi\varepsilon_0}\cdot\frac{1}{\left(2^2+1^2\right)^{3/2}}\mathbf{a}_z$$

$$= \frac{4\pi\times 10^{-3}}{4\pi\varepsilon_0\cdot 5^{3/2}}\mathbf{a}_z = \frac{0.08944\times 10^{-3}}{\varepsilon_0}\mathbf{a}_z$$

$$= 1.012\times 10^7\,\mathbf{a}_z \text{ N/C.}$$

1.33. For the (ij)th area,



$$x = \frac{2i-1}{100},\ y = \frac{2j-1}{100}, \text{ and}$$

$$\text{charge} = \frac{4}{10000}\times 10^{-3} = 4\times 10^{-7}\,\text{C}$$

$$\therefore\ \mathbf{E} = \frac{4}{4\pi\varepsilon_0}\sum_{i=1}^{50}\sum_{j=1}^{50}\frac{4\times 10^{-7}}{(x^2+y^2+1)^{3/2}}\mathbf{a}_z$$

$$= \frac{4\times 10^{-7}}{\pi\varepsilon_0}\sum_{i=1}^{50}\sum_{j=1}^{50}\left[10^{-4}(2i-1)^2+10^{-4}(2j-1)^2+1\right]^{-3/2}\mathbf{a}_z$$

1.34. For the (ij)th area, $x = \dfrac{2i-1}{100}$, $y = \dfrac{2j-1}{100}$,

$$\text{charge density} = 10^{-3}\left(\frac{2i-1}{100}\right)\left(\frac{2j-1}{100}\right)^2 = 10^{-9}(2i-1)(2j-1)^2\ \text{C/m}^2$$

$$\text{charge} = \frac{4}{10000}\times 10^{-9}(2i-1)(2j-1)^2 = 4\times 10^{-13}(2i-1)(2j-1)^2\ \text{C}$$

$$\therefore\ \mathbf{E} = \frac{4}{4\pi\varepsilon_0}\sum_{i=1}^{50}\sum_{j=1}^{50}\frac{4\times 10^{-13}(2i-1)(2j-1)^2}{(x^2+y^2+1)^{3/2}}\mathbf{a}_z$$

$$= \frac{4\times 10^{-13}}{\pi\varepsilon_0}\sum_{i=1}^{50}\sum_{j=1}^{50}\frac{(2i-1)(2j-1)^2}{\left[10^{-4}(2i-1)^2+10^{-4}(2j-1)^2+1\right]^{3/2}}\mathbf{a}_z$$

1.35. (a) $\mathbf{J} = Ne\mathbf{v} = \dfrac{Ne^2}{m\omega} E_0 \sin \omega t \, \mathbf{a}_x$

$$= \frac{10^{12} \times 1.6021^2 \times 10^{-38}}{9.1083 \times 10^{-31} \times 2\pi \times 10^7} \times 10^{-3} \sin 2\pi \times 10^7 t \, \mathbf{a}_x$$

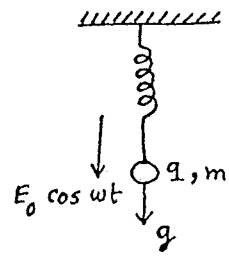
$$= 0.4485 \times 10^{-6} \sin 2\pi \times 10^7 t \, \mathbf{a}_x \ \text{A/m}^2$$

(b) $\Delta I = \mathbf{J} \bullet \Delta \mathbf{S} = 0.4485 \times 10^{-6} \sin 2\pi \times 10^7 t \, \mathbf{a}_x \bullet 0.01 \, (\mathbf{a}_x + \mathbf{a}_y)$

$$= 0.4485 \times 10^{-8} \sin 2\pi \times 10^7 t \ \text{A}$$

1.36. Denoting x to be the displacement,

we write the equation of motion to be



$$m\frac{d^2x}{dt^2} = mg - kx + qE_0 \cos \omega t$$

or, $m\dfrac{d^2x}{dt^2} + kx = mg + qE_0 \cos \omega t$

The steady state solution consists of two parts.

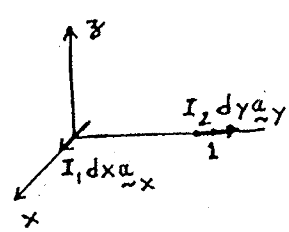
One is $x_1 = \dfrac{mg}{k}$ due to mg. To find the second part x_2, we write

$(j\omega)^2 m\bar{x}_2 + k\bar{x}_2 = qE_0 e^{j0}$, or, $(k - \omega^2 m)\bar{x}_2 = qE_0$

$\bar{x}_2 = \dfrac{qE_0}{k - \omega^2 m}$. Thus $x = x_1 + x_2 = \dfrac{mg}{k} + \dfrac{qE_0}{k - \omega^2 m} \cos \omega t$

$\therefore$ Velocity $= \dfrac{dx}{dt} = -\dfrac{qE_0 \omega}{k - \omega^2 m} \sin \omega t$

1.37. $d\mathbf{F}_1 = I_1 \, dx \, \mathbf{a}_x \times \left[\dfrac{\mu_0 I_2 \, dy \, \mathbf{a}_y \times (-\mathbf{a}_y)}{4\pi (1)^2} \right]$



$$= 0$$

$$d\mathbf{F}_2 = I_2 \, dy \, \mathbf{a}_y \times \left[\frac{\mu_0 I_1 \, dx \, \mathbf{a}_x \times \mathbf{a}_y}{4\pi (1)^2} \right]$$

$$= I_2 \, dy \, \mathbf{a}_y \times \frac{\mu_0}{4\pi} I_1 \, dx \, \mathbf{a}_z = \frac{\mu_0}{4\pi} I_1 I_2 \, dx \, dy \, \mathbf{a}_x$$

1.38. (a) For (0, 1, 1), $\mathbf{a}_R = \dfrac{-\mathbf{a}_x + \mathbf{a}_y + \mathbf{a}_z}{\sqrt{3}}$, $R = \sqrt{3}$, and

$$\mathbf{B} = \frac{\mu_0}{4\pi}\frac{I\,dx\,(\mathbf{a}_x + 2\mathbf{a}_y + 2\mathbf{a}_z)}{3} \times \frac{(-\mathbf{a}_x + \mathbf{a}_y + \mathbf{a}_z)}{\sqrt{3}}$$

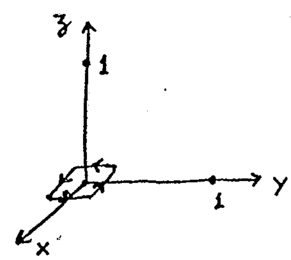
$$= \frac{\mu_0 I\,dx}{4\sqrt{3}\pi}(-\mathbf{a}_y + \mathbf{a}_z)$$

(b) For (2, 2, 2), $\mathbf{a}_R = \dfrac{\mathbf{a}_x + 2\mathbf{a}_y + 2\mathbf{a}_z}{\sqrt{3}}$, $R = 3$, and

$$\mathbf{B} = \frac{\mu_0}{4\pi}\frac{I\,dx\,(\mathbf{a}_x + 2\mathbf{a}_y + 2\mathbf{a}_z)}{9} \times \frac{(\mathbf{a}_x + 2\mathbf{a}_y + 2\mathbf{a}_z)}{3}$$

$$= 0$$

1.39. (a) At (0, 0, 1), the components of **B** perpendicular to the z-axis cancel, whereas the z components add. Thus



$$\mathbf{B} = 4\left[\frac{\mu_0}{4\pi}\frac{0.01\mathbf{a}_y \times (-0.005\mathbf{a}_x + \mathbf{a}_z)}{(1 + 0.005^2)^{3/2}} \bullet \mathbf{a}_z\right]\mathbf{a}_z$$

$$\approx \left[\frac{\mu_0}{\pi}(0.00005\mathbf{a}_z + 0.01\mathbf{a}_x) \bullet \mathbf{a}_z\right]\mathbf{a}_z = \frac{5 \times 10^{-5}\mu_0}{\pi}\mathbf{a}_z$$

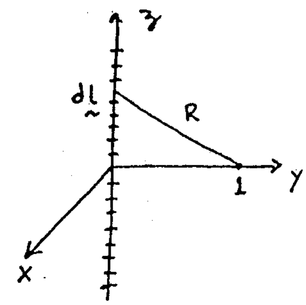
(b) At the point (0, 1, 0),

$$\mathbf{B} = \frac{\mu_0}{4\pi}\left[-\frac{0.01}{(1 - 0.005)^2}\mathbf{a}_z + \frac{0.01}{(1 + 0.005)^2}\mathbf{a}_z + \frac{0.01\mathbf{a}_y \times (-0.005\mathbf{a}_x + \mathbf{a}_y)}{(1 + 0.005^2)^{3/2}} - \frac{0.01\mathbf{a}_y \times (0.005\mathbf{a}_x + \mathbf{a}_y)}{(1 + 0.005^2)^{3/2}}\right]$$

$$= \frac{\mu_0}{4\pi}\left[-\frac{0.01 \times 4 \times 1 \times 0.005}{(1 - 0.005^2)^2}\mathbf{a}_z + \frac{2 \times 0.01 \times 0.005}{(1 + 0.005^2)^{3/2}}\mathbf{a}_z\right]$$

$$\approx -\frac{10^{-4}\mu_0}{4\pi}\mathbf{a}_z$$

1.40. For the ith element, $d\mathbf{l} = \frac{1}{50}\mathbf{a}_z$,



$$R = \sqrt{1 + \left(\frac{2i-1}{100}\right)^2},$$

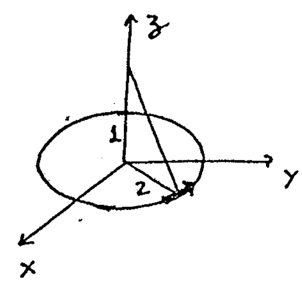
$$\mathbf{a}_R = \frac{1}{R}\left[\mathbf{a}_y - \left(\frac{2i-1}{100}\right)\mathbf{a}_z\right], \text{ and}$$

$$d\mathbf{B} = \frac{\mu_0 I}{4\pi}\frac{1}{50}\mathbf{a}_z \times \left[\mathbf{a}_y - \left(\frac{2i-1}{100}\right)\mathbf{a}_z\right]\frac{1}{R^3}$$

$$= -\frac{\mu_0 I}{200\pi R^3}\mathbf{a}_x$$

$$\mathbf{B} = 2\sum_{i=1}^{50} d\mathbf{B} = -\frac{\mu_0 I}{100\pi}\sum_{i=1}^{50}\left[1 + 10^{-4}(2i-1)^2\right]^{-3/2}\mathbf{a}_x$$

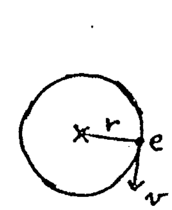
1.41. Dividing the loop into n segments and using the symmetry of the field about the z-axis, we obtain



$$\mathbf{B} = \sum_n \frac{\mu_0 I}{4\pi}\frac{2\pi(2)}{n}\frac{2}{\left(2^2+1^2\right)^{3/2}}\mathbf{a}_z$$

$$= \frac{8\pi\mu_0 I}{4\pi \times 5^{3/2}}\mathbf{a}_z = 0.179\mu_0 I\,\mathbf{a}_z$$

1.42. Equating the magnetic force to the centripetal force, we have



$$evB_0 = \frac{mv^2}{r}, \text{ or, } r = \frac{mv}{eB_0}$$

$$\omega = \frac{v}{r} = \frac{eB_0}{m}$$

$$\text{Orbital frequency} = \frac{\omega}{2\pi} = \frac{eB_0}{2\pi m}$$

For $B_0 = 5 \times 10^{-5}$,

$$\text{orbital frequency} = \frac{1.7578\times 10^{11}}{2\pi}\times 5\times 10^{-5} = 1.3988\times 10^{6}\ \text{Hz}$$

$$= 1.3988\ \text{MHz.}$$

1.43. $\mathbf{F} = q(\mathbf{E} + \mathbf{v}\times\mathbf{B}) = 0$

$$\therefore\ \mathbf{E} = -\mathbf{v}\times\mathbf{B}$$

$$= -v_0(3\mathbf{a}_x - \mathbf{a}_y + 2\mathbf{a}_z)\times B_0(\mathbf{a}_x + 2\mathbf{a}_y - 4\mathbf{a}_z)$$

$$= -v_0B_0(14\mathbf{a}_y + 7\mathbf{a}_z)$$

1.44. (a) $q\mathbf{E} + qv_0\mathbf{a}_x\times\mathbf{B} = 0 \qquad$ — (1)

$q\mathbf{E} + qv_0\mathbf{a}_y\times\mathbf{B} = 0 \qquad$ — (2)

$q\mathbf{E} + qv_0(\mathbf{a}_x + \mathbf{a}_y)\times\mathbf{B} = -qE_0\mathbf{a}_z \qquad$ — (3)

$(1) + (2) - (3) \rightarrow q\mathbf{E} = qE_0\mathbf{a}_z$

$\therefore\ \mathbf{E} = E_0\mathbf{a}_z$

$(1) - (2) \rightarrow (\mathbf{a}_x - \mathbf{a}_y)\times\mathbf{B} = 0$

$\therefore\ \mathbf{B} = C(\mathbf{a}_x - \mathbf{a}_y)$ where C is a constant

To find C, we use (1). Thus,

$qE_0\mathbf{a}_z + qv_0\mathbf{a}_z\times C(\mathbf{a}_x - \mathbf{a}_y) = 0$

or, $qE_0\mathbf{a}_z - qv_0C\mathbf{a}_z = 0$

$$C = \frac{E_0}{v_0}$$

Thus, $\mathbf{E} = E_0\mathbf{a}_z$ and $\mathbf{B} = \dfrac{E_0}{v_0}(\mathbf{a}_x - \mathbf{a}_y)$

(b) For $\mathbf{v} = v_0(\mathbf{a}_x - \mathbf{a}_y)$,

$$\mathbf{F} = qE_0\mathbf{a}_z + qv_0(\mathbf{a}_x - \mathbf{a}_y)\times\frac{E_0}{v_0}(\mathbf{a}_x - \mathbf{a}_y)$$

$$= qE_0\mathbf{a}_z$$