Sections 1.2-1.3: Charge, Current, and Kirchhoff's Current Law; Voltage and Kirchhoff's Voltage Law

Problem 1.1

A free electron has an initial potential energy per unit charge (voltage) of 17 kJ/C and a velocity of 93 Mm/s. Later, its potential energy per unit charge is 6 kJ/C. Determine the change in velocity of the electron.

Solution:

Known quantities:

Initial Coulombic potential energy, $V_i = 17kJ/C$; initial velocity, $U_i = 93M\frac{m}{s}$; final Coulombic potential energy, $V_f = 6kJ/C$.

Find:

The change in velocity of the electron.

Assumptions:

$\Delta PE_g << \Delta PE_c$

Analysis:

Using the first law of thermodynamics, we obtain the final velocity of the electron:

$Q_{heat} - W = \Delta KE + \Delta PE_c + \Delta PE_g + \ldots$

Heat is not applicable to a single particle. W=0 since no external forces are applied.

$\Delta KE = -\Delta PE_c$

$$\frac{1}{2}m_e(U_f^2 - U_i^2) = -Q_e(V_f - V_i)$$

$$U_f^2 = U_i^2 - \frac{2Q_e}{m_e}(V_f - V_i)$$

$$= \left(93\ M\ \frac{m}{s}\right)^2 - \frac{2\left(-1.6\times10^{-19}\ C\right)}{9.11\times10^{-37}\ g}\left(6kV - 17kV\right)$$

$$= 8.649\times10^{15}\ \frac{m^2}{s^2} - 3.864\times10^{15}\ \frac{m^2}{s^2}$$

$$U_f = 6.917\times10^7\ \frac{m}{s}$$

$$\left|U_f - U_i\right| = 93\ M\ \frac{m}{s} - 69.17\ M\ \frac{m}{s} = 23.83\ M\ \frac{m}{s}.$$

Problem 1.2

The units for voltage, current, and resistance are the volt (V), the ampere (A), and the ohm (Ω), respectively. Express each unit in fundamental MKS units.

Solution:

Known quantities:

MKSQ units.

Find:

Equivalent units of volt, ampere and ohm.

Analysis:

$$\text{Voltage} = \text{Volt} = \frac{\text{Joule}}{\text{Coulomb}} \quad V = \frac{J}{C}$$

$$\text{Current} = \text{Ampere} = \frac{\text{Coulomb}}{\text{second}} \quad a = \frac{C}{s}$$

$$\text{Resistance} = \text{Ohm} = \frac{\text{Volt}}{\text{Ampere}} = \frac{\text{Joule} \times \text{second}}{\text{Coulomb}^2} \quad \Omega = \frac{J \cdot s}{C^2}$$

$$\text{Conductance} = \text{Siemens or Mho} = \frac{\text{Ampere}}{\text{Volt}} = \frac{C^2}{J \cdot s}$$

Problem 1.3

A particular fully charged battery can deliver 2.7 x 10^6 coulombs of charge.

a. What is the capacity of the battery in ampere-hours?
b. How many electrons can be delivered?

Solution:

Known quantities:

$q_{Battery} = 2.7 \cdot 10^6$ C.

Find:

The current capacity of the battery in ampere-hours
The number of electrons that can be delivered.

Analysis:

There are 3600 seconds in one hour. Amperage is defined as 1 Coulomb per second and is directly proportional to ampere-hours.

$$2.7 \cdot 10^6 C \cdot \frac{1hr}{3600s} = 750\ AH$$

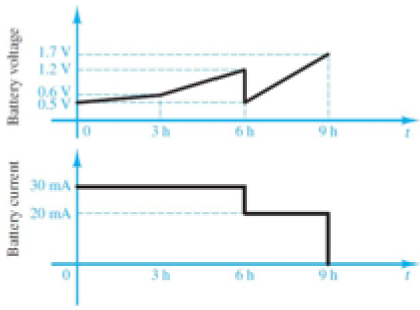
a) The charge of a single electron is $-1.602 \cdot 10^{-19}$ C. The negative sign is negligible. Simple division gives the solution:

$$2.7 \cdot 10^6 C \Big/ \frac{1.602 \cdot 10^{-19} C}{1\ electron} = 1.685 \cdot 10^{25}\ electrons$$

Problem 1.4

The charge cycle shown in Figure P1.4 is an example of a three-rate charge. The current is held constant at 30 mA for 6 h. Then it is switched to 20 mA for the next 3 h. Find:

a. The total charge transferred to the battery.
b. The energy transferred to the battery.
Hint: Recall that energy w is the integral of power, or P = dw/dt.



Solution:

Known quantities:

See Figure P1.4

Find:

a) The total charge transferred to the battery.
b) The energy transferred to the battery.

Analysis:

Current is equal to $\frac{Coulombs}{Second}$, therefore given the current and a duration of that current, the transferred charge can be calculated by the following equation:

$$A \cdot t = C$$

The two durations should be calculated independently and then added together.

$$0.030A \cdot 21600s = 648C$$
$$0.020A \cdot 10800s = 216C$$
$$648C + 216C = \mathbf{864C}$$

P=V·I, therefore, an equation for power can be found by multiplying the two graphs together. First separate the voltage graph into three equations:

0 h → 3 h :$V = 9.26 \cdot 10^{-6}t + 0.5$

3 h → 6 h :$V = 5.55 \cdot 10^{-5}t$

6 h → 9 h :$V = 1.11 \cdot 10^{-4}t - 1.6$

Next, multiply the first two equations by 0.03A and the third by 0.02A.

0 h → 3 h :$P = 2.77 \cdot 10^{-7}t + 0.015$

3 h → 6 h :$P = 1.66 \cdot 10^{-6}t$

6 h → 9 h :$P = 2.22 \cdot 10^{-6}t - 0.032$

Finally, since Energy is equal to the integral of power, take the integral of each of the equations for their specified times and add them together.

0 h → 3 h :$E = \left[\frac{2.77 \cdot 10^{-7}t^2}{2} + 0.015t\right]\Big|_{0}^{10800}$= 178.2 J

3 h → 6 h :$E = \left[\frac{1.66 \cdot 10^{-6}t^2}{2}\right]\Big|_{10800}^{21600}$= 290.43 J

6 h → 9 h :$E = \left[\frac{2.22 \cdot 10^{-6}t^2}{2} + 0.032t\right]\Big|_{21600}^{32400}$ = 992.95 J

$$\boldsymbol{E_{Total} = 1462\,J}$$

Problem 1.5

Batteries (e.g., lead-acid batteries) store chemical energy and convert it to electric energy on demand. Batteries do not store electric charge or charge carriers. Charge carriers (electrons) enter one terminal of the battery, acquire electrical potential energy, and exit from the other terminal at a lower voltage. Remember the electron has a negative charge! It is convenient to think of positive carriers flowing in the opposite direction, that is, conventional current, and exiting at a higher voltage. All currents in this course, unless otherwise stated, are conventional current. (Benjamin Franklin caused this mess!) For a battery with a rated voltage = 12 V and a rated capacity = 350 A-h,
determine
a. The rated chemical energy stored in the battery.
b. The total charge that can be supplied at the rated

Solution:

Known quantities:

Rated voltage of the battery; rated capacity of the battery.

Find:

The rated chemical energy stored in the battery
The total charge that can be supplied at the rated voltage.

Analysis:

a)

$$\Delta V \equiv \frac{\Delta PE_c}{\Delta Q} \quad I = \frac{\Delta Q}{\Delta t}$$

$$\text{Chemical energy} = \Delta PE_c = \Delta V \cdot \Delta Q = \Delta V \cdot (I \cdot \Delta t)$$

$$= 12\ V\ 350\ \text{A} - \text{hr}\ 3600\ \frac{\text{s}}{\text{hr}} = 15.12\ \text{MJ}.$$

As the battery discharges, the voltage will decrease below the rated voltage. The remaining chemical energy stored in the battery is less useful or not useful.

b) ΔQ is the total charge passing through the battery and gaining 12 J/C of electrical energy.

$$\Delta Q = I \cdot \Delta t = 350\ a\ hr = 350\ \frac{\text{C}}{\text{s}}\ \text{hr} \cdot 3600\ \frac{\text{s}}{\text{hr}} = 1.26\ \text{MC}.$$

Problem 1.6

What determines:
a. The current through an ideal voltage source?
b. The voltage across an ideal current source?

Solution:

Known quantities:

Resistance of external circuit.

Find:

Current supplied by an ideal voltage source
Voltage supplied by an ideal current source.

Assumptions:

Ideal voltage and current sources.

Analysis:

a) An ideal voltage source produces a constant voltage at or below its rated current. Current is determined by the power required by the external circuit (modeled as R).

$I = \frac{V_S}{R} P = V_S \cdot I$

b) An ideal current source produces a constant current at or below its rated voltage. Voltage is determined by the power demanded by the external circuit (modeled as R).

$V = I_S \cdot R \quad P = V \cdot I_S$

A real source will overheat and, perhaps, burn up if its rated power is exceeded.

Problem 1.7

An automotive battery is rated at 120 A-h. This means that under certain test conditions it can output 1 A at 12 V for 120 h (under other test conditions, the battery may have other ratings).
a. How much total energy is stored in the battery?
b. If the headlights are left on overnight (8 h), how much energy will still be stored in the battery in the morning? (Assume a 150-W total power rating for both headlights together.)

Solution:

Known quantities:

Rated discharge current of the battery; rated voltage of the battery; rated discharge time of the battery.

Find:

Energy stored in the battery when fully recharging
Energy stored in the battery after discharging

Analysis:

a) $\text{Energy} = \text{Power} \times \text{time} = (1A)(12V)(120\text{hr})\left(\frac{60\text{ min}}{\text{hr}}\right)\left(\frac{60\text{ sec}}{\text{min}}\right)$

$\boxed{\text{w} = 5.184 \times 10^6 \text{ J}}$

b) Assume that 150 W is the combined power rating of both lights; then,

$\text{w}_{\text{used}} = (150W)(8hrs)\left(\frac{3600\text{ sec}}{hr}\right) = 4.32 \times 10^6 \text{ J}$

$$\boxed{w_{stored} = w - w_{used} = 864 \times 10^3 \text{ J}}$$

Problem 1.8

A car battery kept in storage in the basement needs recharging. If the voltage and the current provided bythe charger during a charge cycle are shown in FigureP1.8,
a. Find the total charge transferred to the battery.
b. Find the total energy transferred to the battery.

Solution:

Known quantities:

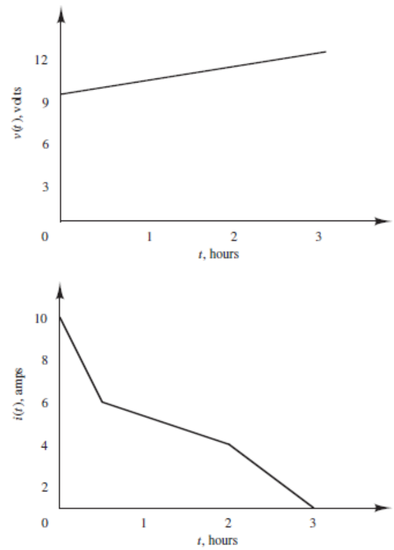
Recharging current and recharging voltage

Find:

Total transferred charge
Total transferred energy

Analysis:

a)



$$Q = \text{area under the current-time curve} = \int I dt$$

$$= \frac{1}{2}(4)(30)(60) + 6(30)(60) + \frac{1}{2}(2)(90)(60) + 4(90)(60) + \frac{1}{2}(4)(60)(60) = 48{,}600 \text{ C}$$

$$\boxed{Q = 48{,}600 \text{ C}}$$

b) $\frac{dw}{dt} = p$ so $w = \int pdt = \int vidt$

$$v = 9 + \frac{3}{10800} t \quad \text{V,} \quad 0 \le t \le 10800 \text{ s}$$

$$i_1 = 10 - \frac{4}{1800} t \quad \text{A,} \quad 0 \le t \le 1800 \text{ s}$$

$$i_2 = 6 - \frac{2}{5400} t \quad \text{A,} \quad 1800 \le t \le 7200 \text{ s}$$

$$i_3 = 12 - \frac{4}{3600} t \quad \text{A,} \quad 7200 \le t \le 10800 \text{ s}$$

where $i = i_1 + i_2 + i_3$
Therefore,

$$w = \int_0^{1800} v i_1 dt + \int_{1800}^{7200} v i_2 dt + \int_{7200}^{10800} v i_3 dt$$

$$= \left(90t + \frac{t^2}{720} - \frac{t^2}{100} - \frac{t^3}{4.86 \times 10^6} \right) \Bigg|_0^{1800}$$

$$+ \left(60t + \frac{t^2}{1080} - \frac{t^2}{600} - \frac{t^3}{29.16 \times 10^6} \right) \Bigg|_{1800}^{7200}$$

$$+ \left(108t + \frac{t^2}{600} - \frac{t^2}{200} - \frac{t^3}{9.72 \times 10^6} \right) \Bigg|_{7200}^{10800}$$

$$= 132.9 \times 10^3 + 380.8 \times 10^3 - 105.4 \times 10^3 + 648 \times 10^3 - 566.4 \times 10^3$$

$$\boxed{Energy = 489.9\ kJ}$$

Problem 1.9

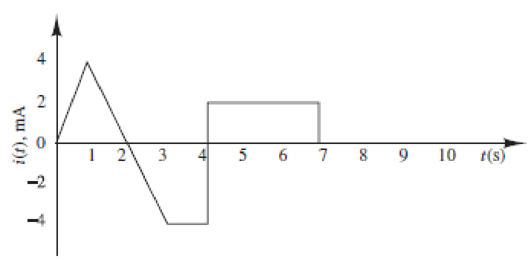
Suppose the current through a wire is given by the curve shown in Figure P1.9.

a. Find the amount of charge, q, that flows through the wire between t1 = 0 and t2 = 1 s.
b. Repeat part a for t2 = 2, 3, 4, 5, 6, 7, 8, 9, and 10 s.
c. Sketch q(t) for 0 ≤ t ≤ 10 s.

Solution:

Known quantities:

Current-time curve



Find:

Amount of charge during 1st second
Amount of charge for 2 to 10 seconds
Sketch charge-time curve

Analysis:

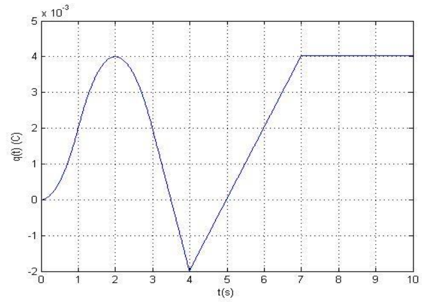
a) $i = \frac{4\times10^{-3} t}{1}$

$$Q_1 = \int_0^1 i dt = \int_0^1 4\times10^{-3} t dt = 4\times10^{-3} \frac{t^2}{2}\Bigg|_0^1 = 2\times10^{-3} \frac{\text{amp}}{\text{sec}} = 2\times10^{-3} \text{ Coulombs}$$

b) The charge transferred from $t=1$ to $t=2$ is the same as from $t=0$ to $t=1$.

$Q_2 = 4 \times 10^{-3}$ Coulombs

The charge transferred from $t=2$ to $t=3$ is the same in magnitude and opposite in direction to that from $t=1$ to $t=2$. $Q_3 = 2\times10^{-3}$ Coulombs



$t=4$

$Q_4 = 2\times10^{-3} - \int_3^4 4\times10^{-3}dt = 2\times10^{-3} - 4\times10^{-3} = -2\times10^{-3}$ Coulombs

$t=5,6,7$

$Q_5 = -2\times10^{-3} + \int_4^5 2\times10^{-3}dt = 0$

$Q_6 = 0 + \int_5^6 2\times10^{-3}dt = 2\times10^{-3}$ Coulombs

$Q_7 = 2\times10^{-3} + \int_6^7 2\times10^{-3}dt = 4\times10^{-3}$ Coulombs

$t=8,9,10s$

$Q = 4\times10^{-3}$Coulombs

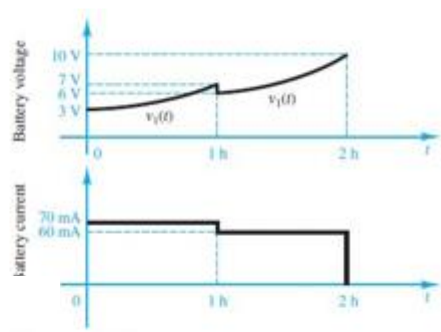
Problem 1.10

The charge cycle shown in Figure P2.10 is an example of a two-rate charge. The current is held constant at 70 mA for 1 h. Then it is switched to 60 mA for the next 1 h. Find:

a. The total charge transferred to the battery.
b. The total energy transferred to the battery.
Hint: Recall that energy w is the integral of power, or P = dw/dt. Let:

$$v_1 = 5 + e^{t/5194.8}\ V$$

$$v_2 = \left(6 - \frac{4}{e^1 - 1}\right) + \frac{4}{e^2 - e^1} * e^t\ V$$



Solution:

Known quantities:

See Figure P1.10

Find:

a) The total charge transferred to the battery.
b) The energy transferred to the battery.

Analysis:

Current is equal to $\frac{Coulombs}{Second}$, therefore given the current and a duration of that current, the transferred charge can be calculated by the following equation:

$$A \cdot t = C$$

The two durations should be calculated independently and then added together.

$$0.070A \cdot 3600s = 252C$$
$$0.060A \cdot 3600s = 216C$$
$$648C + 216C = \mathbf{468C}$$

P=V·I, therefore, an equation for power can be found by multiplying the two graphs together. First separate the voltage graph into three equations:

0 h → 1 h : $V = 5 + e^{t/5194.8}V$

1 h → 2 h : $V = \left(6 - \frac{4}{e^1-1}\right) + \frac{4}{e^2-e^1} * e^t V$

Next, multiply the first equation by 0.07A and the second by 0.06A.

0 h → 1 h :$P = 0.35 + 0.07e^{t/5194.8}$

1 h → 2 h :$P = 0.06\left(6 - \frac{4}{e^{1h}-1}\right) + 0.06\frac{4}{e^{2h}-e^{1h}} * e^{t}V$

Finally, since Energy is equal to the integral of power, take the integral of each of the equations for their specified times and add them together.

0 h → 1 h :$E = \left[0.35t + 363.64e^{t/5194.8}\right]|_{0}^{3600}$= 1623.53 J

1 h → 2 h :$E = [0.36t + 2.88 * 10^{-3128} * 2.72^{t}]|_{3600}^{7200}$= 1296.24 J

$$\boldsymbol{E_{Total} = 2919.77\,J}$$

Problem 1.11

The charging scheme used in Figure P1.11 is an example of a constant-current charge cycle. The charger voltage is controlled such that the current into the battery is held constant at 40 mA, as shown in Figure P1.11. The battery is charged for 6 h. Find:

a. The total charge delivered to the battery.

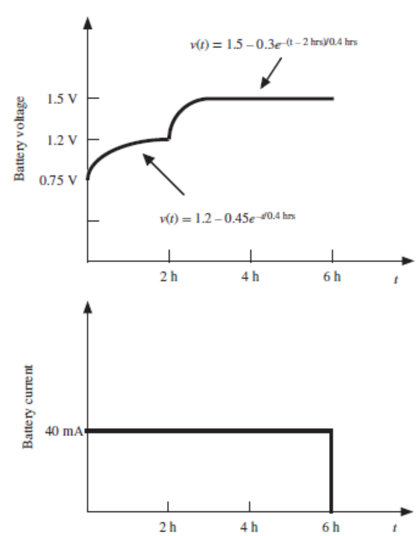
b. The energy transferred to the battery during the charging cycle.

Hint: Recall that the energy, w, is the integral of power, or P = dw/dt.

Solution:

Known quantities:

Current-time curve and voltage-time curve of battery recharging



Find:

Total transferred charge

Total transferred energy

Analysis:

a) $40mA = 0.04A$

$Q = \text{area under the current - time curve} = \int Idt = (0.04)(6)(3600) = 864\ \text{C}$

$\boxed{Q = 864\ \text{C}}$

b) $\frac{dw}{dt} = P$ so

$$w = \int Pdt = \int vidt = (3600)\int_0^2 vidt + (3600)\int_2^4 vidt$$

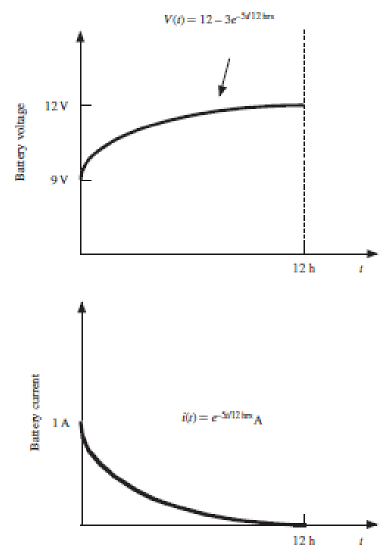
$$= (3600)\int_0^2 \left(1.2 - 0.45e^{-t/0.4}\right)(0.04)dt + (3600)\int_2^4 \left(1.5 - 0.3e^{-(t-2)/0.4}\right)(0.04)dt$$

$$= 1{,}167\,J$$

$\boxed{Energy = 1{,}167\,J}$

Problem 1.12

The charging scheme used in Figure P1.12 is called a tapered-current charge cycle. The current starts at the highest level and then decreases with time for the entire charge cycle, as shown. The battery is charged for 12 h. Find:
a. The total charge delivered to the battery.
b. The energy transferred to the battery during the charging cycle.
Hint: Recall that the energy, w, is the integral of power, or P = dw/dt.



Solution:

Known quantities:

Current-time curve and voltage-time curve of battery recharging

Find:

Total transferred charge
Total transferred energy

Analysis:

$$Q = \text{area under the current - time curve} = \int Idt = (3600)\int_0^{12} e^{-5t/12}dt = 8{,}564\text{ C}$$

a)
$\boxed{Q = 8{,}564\text{ C}}$

b) $\frac{dw}{dt} = P$ so

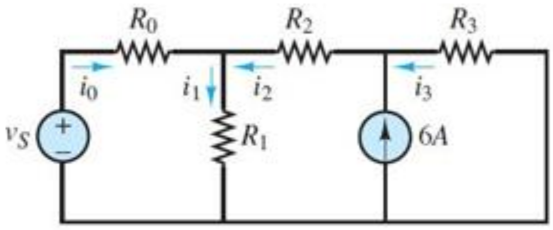
$$w = \int Pdt = \int vidt = (3600)\int_0^2 \left(12 - 3e^{-5t/12}\right)\left(e^{-5t/12}\right)dt$$

$$= 8{,}986\text{J}$$

$\boxed{\text{Energy} = 8{,}986\text{J}}$

Problem 1.13

Use KCL to determine the unknown currents in Figure P1.13.



Solution:

Known quantities:

i_o = 2 A, i_2 = -7 A

Find:

i_1
i_3

Analysis:

a) Use KCL at the node between R_o, R_1, and R_2.

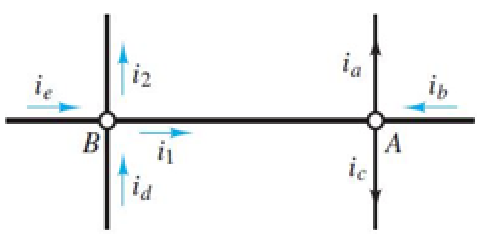
$$i_0 - i_1 + i_2 = 0$$
$$i_1 = i_0 + i_2$$
$$\boldsymbol{i_1 = -5A}$$

b) Use KCL at the node between R_2, R_3, and the current source.

$$6A + i_3 - i_2 = 0$$
$$i_3 = i_2 - 6A$$
$$\boldsymbol{i_3 = -13A}$$

Problem 1.14

Use KCL to find the currents i_1 and i_2 in Figure P1.14.



Solution:

Known quantities:

i_a = 3 A, i_b = -2 A, i_c = 1 A, i_d = 6 A, i_e = -4 A

Find:

i_1
i_2

Analysis:

a) Use KCL at Node A.

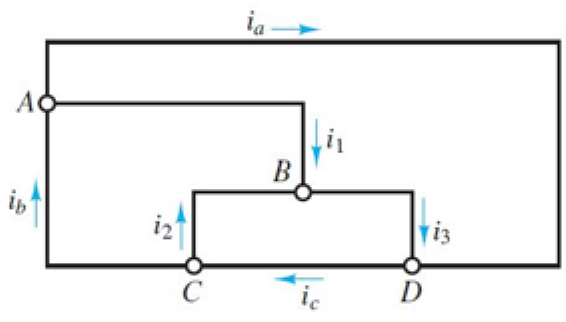
$$i_1 + i_b - i_a - i_c = 0$$
$$i_1 = i_a - i_b + i_c$$
$$\boldsymbol{i_1 = 6A}$$

b) Use KCL at Node B.

$$i_e + i_d - i_1 - i_2 = 0$$
$$i_2 = i_e + i_d - i_1$$
$$\boldsymbol{i_2 = -4A}$$

Problem 1.15

Use KCL to find the current i_1, i_2, and i_3 in the circuit of Figure P1.15.



Solution:

Known quantities:

i_a = 2 mA, i_b = 7 mA, i_c = 4 mA

Find:

i_1
i_2
i_3

Analysis:

a) Use KCL at Node A.

$$i_b - i_a - i_1 = 0$$
$$i_1 = i_b - i_a$$
$$\boldsymbol{i_1 = 5mA}$$

b) Use KCL at Node C.

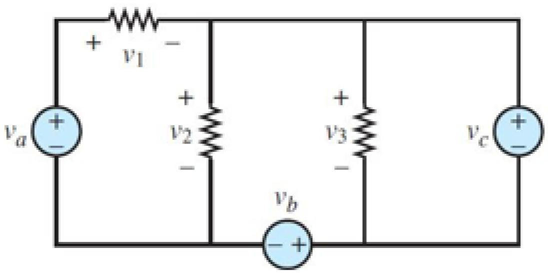
$$i_c - i_2 - i_b = 0$$
$$i_2 = i_c - i_b$$
$$\boldsymbol{i_2 = -3mA}$$

c) Use KCL at Node D.

$$i_3 + i_a - i_c = 0$$
$$i_3 = i_2 - i_a$$
$$\boldsymbol{i_3 = 2mA}$$

Problem 1.16

Use KVL to find the voltages v_1, v_2, and v_3 in Figure P1.16.



Solution:

Known quantities:

$V_a = 2$ V, $V_b = 4$ V, $V_c = 5$ V

Find:

V_1
V_2
V_3

Analysis:

a) Use KVL at the third loop.

$$V_3 - V_c = 0$$
$$V_3 = V_c$$
$$\boldsymbol{V_3 = 5V}$$

b) Use KVL at the second loop.

$$V_2 - V_3 - V_b = 0$$
$$V_2 = V_3 + V_b$$

$$\boldsymbol{V_2 = 9V}$$

c) Use KCL at the first loop.

$$V_a - V_1 - V_2 = 0$$
$$V_1 = V_a - V_2$$
$$\boldsymbol{V_1 = -7V}$$

Problem 1.17

Use KCL to determine the current i_1, i_2, i_3, and i_4 in the circuit of Figure P1.17.

Solution:

Known quantities:

i_a = -2 A, i_b = 6 A, i_c = 1 A, i_d = -4 A

Find:

i_1
i_2
i_3
i_4

Analysis:

a) Use KCL at Node A.

$$i_1 - i_a - i_c = 0$$
$$i_1 = i_a + i_c$$
$$\boldsymbol{i_1 = -1A}$$

b) Use KCL at Node B.

$$i_2 - i_1 - i_b = 0$$
$$i_2 = i_1 + i_b$$
$$\boldsymbol{i_2 = 5A}$$

c) Use KCL at Node C.

$$i_3 - i_2 - i_d = 0$$
$$i_3 = i_2 + i_d$$
$$\boldsymbol{i_3 = 1A}$$

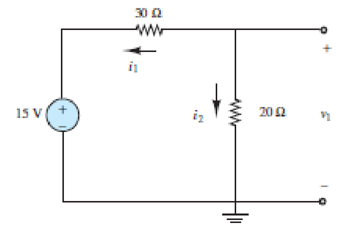
d) Use KCL at Node D.

$$i_c + i_4 - i_3 = 0$$
$$i_4 = i_3 - i_c$$
$$\boldsymbol{i_4 = 0A}$$

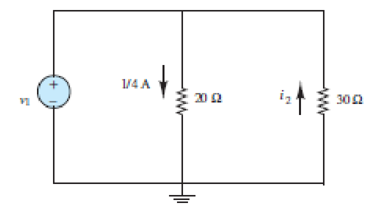
Section 1.4 Power and the Passive Sign Convention

Problem 1.18

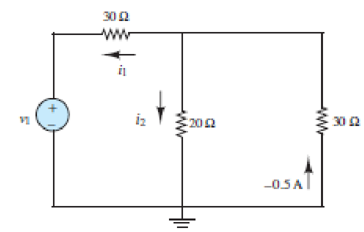
In the circuits of Figure P1.18, the directions of current and polarities of voltage have already been defined. Find the actual values of the indicated currents and voltages.



(a)



(b)



(c)

Solution:

Known quantities:

Circuit shown in Figure P1.18.

Find:

Voltages and currents in every figure.

Analysis:

(a) Using $I = \frac{15}{30+20}$ (clockwise current) : $I_1 = -0.3A; I_2 = 0.3A; V_1 = 6V$

(b) The voltage across the 20 Ω resistor is $\frac{20}{4} = 5\text{V}$; since the current flows from top to bottom, the polarity of this voltage is positive on top. Then it follows that $V_1 = 5V$ and $\text{I}_2 = \frac{5}{30} = \text{-}0.167\text{A}$

(the negative sign follows from the direction of I_2 in the drawing).

(c) Since -0.5A pointing upward is the same current as 0.5A pointing downward, the voltage across the 30 Ω resistor is

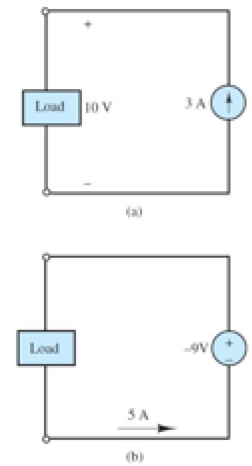
$V_{30\Omega} = 15V$ (positive on top); and $\text{I}_2 = \frac{15}{20} = 0.75\text{A}$,

since $V_{30\Omega}$ is also the voltage across the 20 Ω resistor. Finally,

$I_1 = \text{-}(I_2 + 0.5) = \text{-}1.25A$,and $V_1 = \text{-}30\ I_1 + 15 = 52.5V$

Problem 1.19

Find the power delivered by each source in Figure P1.19.



and

Solution:

Known quantities:

Circuit shown in Figure P1.19.

a) Power delivered by the 3A Current Source
b) Power delivered by the -9V Voltage Source

Analysis:

a) Follow the counterclockwise current:

$$P = (+3A) \cdot (+10V)$$

$$\boldsymbol{P = +30W \text{supplied}}$$

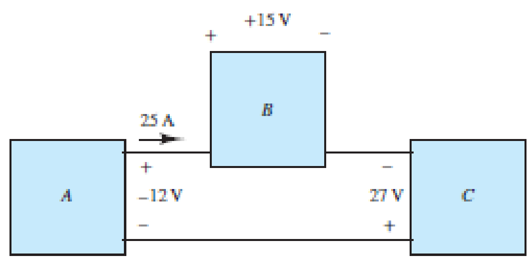
b) Follow the counterclockwise current:

$$P = (+5A) \cdot (-9V)$$

$$\boldsymbol{P = -45W \text{supplied}}$$

Problem 1.20

Determine whether each element in Figure P2.20 is supplying or dissipating power, and how much.



Solution:

Known quantities:

Circuit shown in Figure P1.20.

Find:

Determine power dissipated or supplied for each power source.

Analysis:

Element A:

$P = -vi = -(-12V)(25A) = 300W$ (dissipating)

Element B:

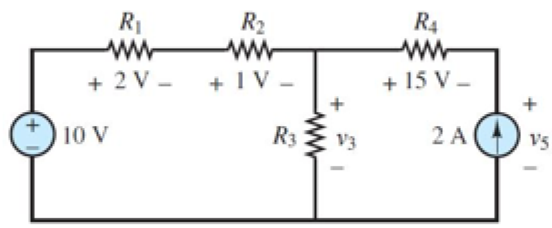
$P = vi = (15V)(25A) = 375W$ (dissipating)

Element C:

$P = vi = (27V)(25A) = 675W$ (supplying)

Problem 1.21

In the circuit of Figure P1.21, find the power absorbed by R_4 and the power delivered by the current source.



Solution:

Known quantities:

Circuit shown in Figure P1.21.

Find:

a) Power absorbed by R_4
b) Power delivered by the current source

Analysis:

a) Follow the counterclockwise current in the rightmost loop:

$$P = (2A) \cdot (-15V)$$

$$\boldsymbol{P = -30W \text{ absorbed}}$$

$$\boldsymbol{P = +30W \text{ supplied}}$$

b) Use KVL at the leftmost loop to find V_3:

$$10V - 2V - 1V - V_3 = 0$$

$$V_3 = 7V$$

Use KVL at the rightmost loop to find V_5:

$$7V - 15V - V_5 = 0$$

$$V_5 = -8V$$

The current source has a -8V drop across it. Use this to calculate the power dissipated using the proper sign convention.

$$\boldsymbol{(+2A) \cdot (-8V) = -16W \text{ supplied}}$$

Problem 1.22

For the circuit shown in Figure P1.22:
a. Determine whether each component is absorbing or delivering power.
b. Is conservation of power satisfied? Explain your answer.

Solution:

Known quantities:

Circuit shown in Figure P1.22.

Find:

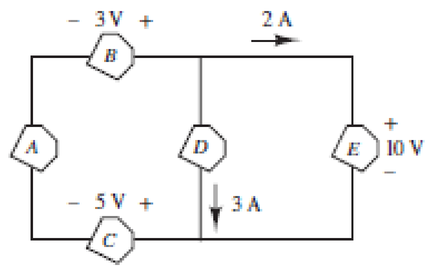
Determine power absorbed or power delivered
Testify power conservation

Analysis:

By KCL, the current through element B is 5A, to the right.

By KVL, $-v_a - 3 + 10 + 5 = 0$.

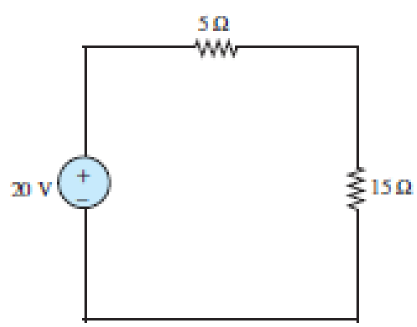
Therefore, the voltage across element A is
$v_a = 12V$ (positive at the top).



A supplies $(12V)(5A) = 60W$
B supplies $(3V)(5A) = 15W$
C absorbs $(5V)(5A) = 25W$
D absorbs $(10V)(3A) = 30W$
E absorbs $(10V)(2A) = 20W$
Total power supplied = $60W + 15W = 75W$
Total power absorbed = $25W + 30W + 20W = 75W$
Tot. power supplied = Tot. power absorbed
$\therefore$ conservation of power is satisfied.

Problem 1.23

For the circuit shown in Figure P1.23, determine the power absorbed by the 5 Ω resistor.



Solution:

Known quantities:

Circuit shown in Figure P1.23.

Find:

Power absorbed by the 5Ω resistance.

Analysis:

The current flowing clockwise in the series circuit is $i = \frac{20V}{20\hat{o}} = 1A$

The voltage across the 5 □ resistor, positive on the left, is $v_{5\hat{o}} = (1A)(5\hat{o}) = 5V$

Therefore, $P_{5\hat{o}} = (5V)(1A) = 5\ W$

Problem 1.24

For the circuit shown in Figure P1.24, determine which components are supplying power and which are dissipating power. Also determine the amount of power dissipated and supplied.

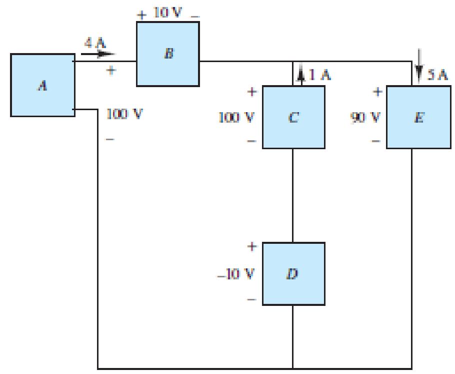
Solution:

Known quantities:

Circuit shown in Figure P1.24.

Find:

Determine power absorbed or power delivered and corresponding amount.



Analysis:

If current direction is out of power source, then power source is supplying, otherwise it is absorbing.

A supplies $(100V)(4A) = 400\ \text{W}$

B absorbs $(10V)(4A) = 40\ \text{W}$

C supplies $(100V)(1A) = 100W$

D supplies $(-10V)(1A) = -10W$, i.e absorbs $10W$

E absorbs $(90V)(5A) = 450W$

Problem 1.25

For the circuit shown in Figure P1.25.determine which components are supplying power and which are dissipating power. Also determine the amount of power dissipated and supplied.

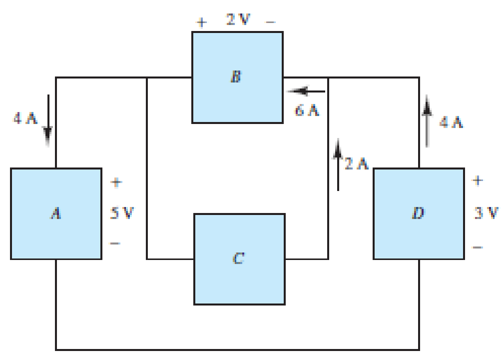
Solution:

Known quantities:

Circuit shown in Figure P1.25.

Find:

Determine power absorbed or power delivered and corresponding amount.



Analysis:

If current direction is out of power source, then power source is supplying, otherwise it is absorbing.

A absorbs $(5V)(4A) = 20W$

B supplies $(2V)(6A) = 12W$

D supplies $(3V)(4A) = 12W$

Since conservation of power is satisfied, Tot. power supplied = Tot. power absorbed

Total power supplied = $12\text{W} + 12\text{W} = 24\text{W}$

$\therefore$ C absorbs $24W - 20W = 4W$

Problem 1.26

If an electric heater requires 23 A at 110 V, determine

a. The power it dissipates as heat or other losses.

b. The energy dissipated by the heater in a 24-hperiod.

c. The cost of the energy if the power companycharges at the rate 6 cents/kWh.

Solution:

Known quantities:

Current absorbed by the heater; voltage at which the current is supplied; cost of the energy.

Find:

Power consumption
Energy dissipated in 24 hr.
Cost of the Energy

Assumptions:

The heater works for 24 hours continuously.

Analysis:

a) $P = VI = 110\ V\,(23\ A) = 2.53 \times 10^3 \frac{J}{A}\frac{A}{s} = 2.53\ \text{KW}$

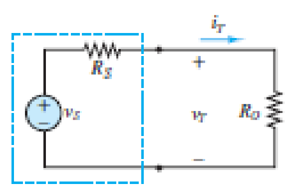
b) $W = Pt = 2.53 \times 10^3 \frac{\text{J}}{\text{s}} \times 24\ \text{hr} \times 3600 \frac{\text{s}}{\text{hr}} = 218.6\ \text{MJ}$

c) $\text{Cost} = (\text{Rate}) \times W = 6 \frac{\text{cents}}{\text{kW} - \text{hr}} (2.53\ \text{kW})(24\ \text{hr}) = 364.3\ \text{cents} = \$3.64$

Sections 1.5-1.6: *i-v* Characteristics and Sources; Resistance and Ohm's Law

Problem 1.27

In the circuit shown in Figure P1.27, determine the terminal voltage v_T of the source, the power absorbed by $R_o = R_L$ and the efficiency of the circuit. Efficiency is defined as the ratio of load power to source power.



Solution:

Known quantities:

Circuit shown in Figure P1.27 with voltage source, $V_s = 12V$; internal resistance of the source, $R_s = 5k\Omega$; and resistance of the load, $R_L = 7k\Omega$.

Find:

The terminal voltage of the source; the power supplied to the circuit, the efficiency of the circuit.

Assumptions:

Assume that the only loss is due to the internal resistance of the source.

Analysis:

$$KVL: \quad -V_S + I_T R_S + V_T = 0 \qquad OL: \quad V_T = I_T R_L \quad \therefore \quad I_T = \frac{V_T}{R_L}$$

$$-V_S + \frac{V_T}{R_L} R_S + V_T = 0$$

$$V_T = \frac{V_S}{1+\frac{R_S}{R_L}} = \frac{12\ V}{1+\frac{5\ k\Omega}{7\ k\Omega}} = 7\ V \quad or \quad VD: V_T = \frac{V_S R_L}{R_S + R_L} = \frac{12\ V\ 7\ k\Omega}{5\ k\Omega + 7\ k\Omega} = 7\ V.$$

$$P_L = \frac{V_R^2}{R_L} = \frac{V_T^2}{R_L} = \frac{(7\ V)^2}{7\times 10^3\ \frac{\text{V}}{\text{A}}} = 7\ \text{mW}$$

$$\eta = \frac{P_{out}}{P_{in}} = \frac{P_{R_L}}{P_{R_S} + P_{R_L}} = \frac{I_T^2 R_L}{I_T^2 R_S + I_T^2 R_L} = \frac{7\ k\Omega}{5\ k\Omega + 7\ k\Omega} = 0.5833 \quad or \quad 58.33\%$$

.

Problem 1.28

A 24-V automotive battery is connected to two headlights that are in parallel, similar to that shown in Figure 1.11. Each headlight is intended to be a 75-W load; however, one 100-W headlight is mistakenly installed. What is the resistance of each headlight? What is the total resistance seen by the battery?

Solution:

Known quantities:

Headlights connected in parallel to a 24-V automotive battery; power absorbed by each headlight.

Find:

Resistance of each headlight; total resistance seen by the battery.

Analysis:

Headlight no. 1:

$$P = v \times i = 100\ \text{W} = \frac{v^2}{R} \quad \text{or}$$

$$R = \frac{v^2}{100} = \frac{576}{100} = 5.76\ \Omega$$

Headlight no. 2:

$$P = v \times i = 75\ \text{W} = \frac{v^2}{R} \quad \text{or}$$

$$R = \frac{v^2}{75} = \frac{576}{75} = 7.68\ \Omega$$

The total resistance is given by the parallel combination:

$$\frac{1}{R_{TOTAL}} = \frac{1}{5.76\ \Omega} + \frac{1}{7.68\ \Omega}$$ or $R_{TOTAL} = 3.29\ \Omega$

Problem 1.29

What is the equivalent resistance seen by the battery of Problem 1.28 if two 15-W taillights are added (in parallel) to the two 75-W (each) headlights?

Solution:

Known quantities:

Headlights and 24-V automotive battery of problem 2.13 with 2 15-W taillights added in parallel; power absorbed by each headlight; power absorbed by each taillight.

Find:

Equivalent resistance seen by the battery.

Analysis:

The resistance corresponding to a 75-W headlight is:

$$R_{75W} = \frac{v^2}{75} = \frac{576}{75} = 7.68\ \Omega$$

For each 15-W tail light we compute the resistance:

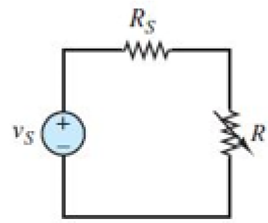
$$R_{15W} = \frac{v^2}{15} = \frac{576}{15} = 38.4\ \Omega$$

Therefore, the total resistance is computed as:

$$\frac{1}{R_{TOTAL}} = \frac{1}{7.68\Omega} + \frac{1}{7.68\Omega} + \frac{1}{38.4\Omega} + \frac{1}{38.4\Omega} \text{ or } R_{TOTAL} = 3.2\ \Omega$$

Problem 1.30

For the circuit shown in Figure P1.30, determine the power absorbed by the variable resistor R, ranging from 0 to 30 Ω. Plot the power absorption as a function of R. Assume that v_S = 15 V, R_S = 10 Ω.



Solution:

Known quantities:

v_s=15V, Rs=10 Ohms, and the circuit in Figure P1.30.

Find:

R

Analysis:

Use ohms law to find an equation for P as a function of R:

$$P_R = V_R * I_R$$

The voltage across R is equal to the source voltage minus the voltage across R_s:

$$V_R = 15V - V_{Rs}$$

V_{RS} is determined by the current through the loop which can be found by adding the resistors in series:

$$I_R = \frac{15V}{(R_s + R)}$$

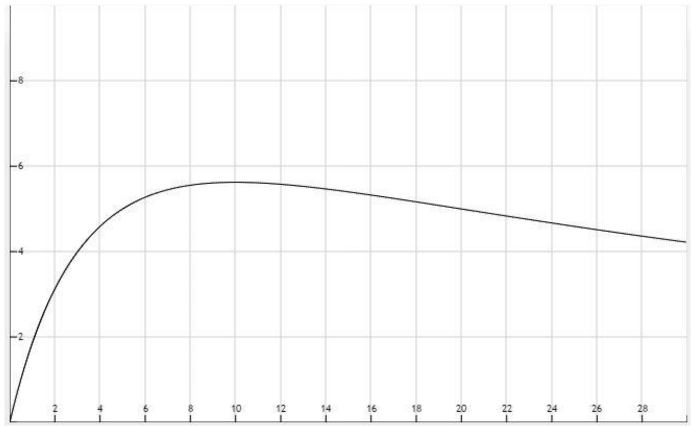
$$V_{RS} = 10\Omega * I_R$$

Simplify:

$$P_R = \left[15V - \left(10\Omega * \frac{15V}{10\Omega + \mathrm{R}}\right)\right] * \left[\frac{15V}{10\Omega + \mathrm{R}}\right]$$

Plot:

P_R

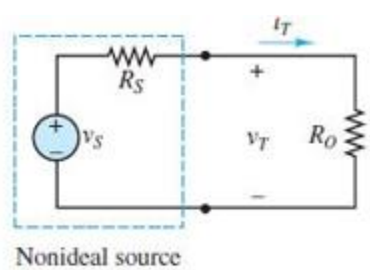


R

Problem 1.31

Refer to Figure P1.27 and assume that v_S = 15 V and R_S = 100 _. For i_T = 0, 10, 20, 30, 80, and 100 mA:

a. Find the total power supplied by the ideal source.
b. Find the power dissipated within the non-ideal source.
c. How much power is supplied to the load resistor?
d. Plot the terminal voltage v_T and power supplied to the load resistor as a function of terminal current i_T .



Solution:

Known quantities:

v_S=15V, Rs=100 Ohms, i_T= 0, 10, 20, 30, 80, 100 mA.
The circuit in Figure P1.27.

Find:

a) The total power supplied by the ideal source
b) The power dissipated within the non-ideal source
c) How much power is supplied to the load resistor
d) Plot v_T and power supplied to R_0 as a function of i_T.

Analysis:

a) The power supplied by the ideal source is equal to the current through the loop times the 15V of the supply. From current lowest to highest the power supplied would be:

$$0W \quad 0.15W \quad 0.3W \quad 0.45W \quad 1.2W \quad 1.5W$$

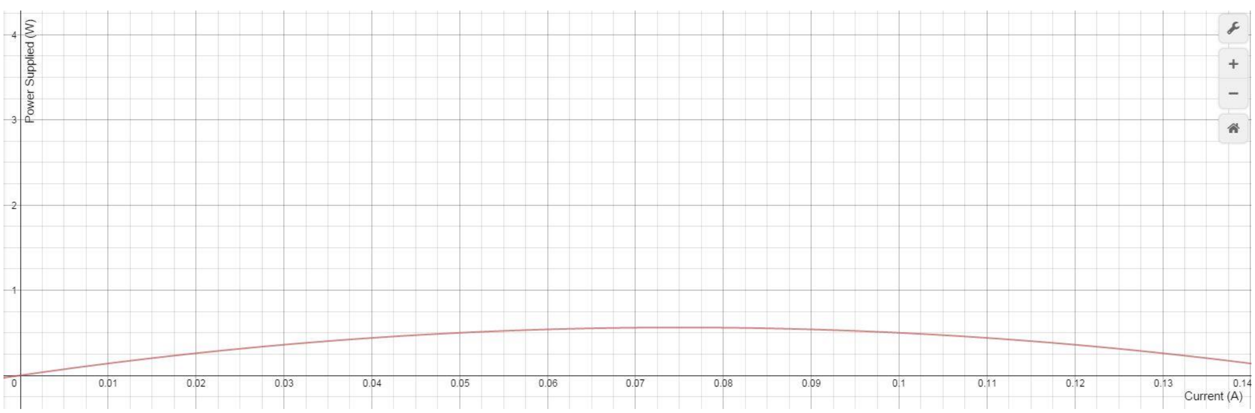
b) The power dissipated within the non-ideal source is the power dissipated by R_s which can be found using $P=i^2*r$. From current lowest to highest the power dissipated would be:

$$0W \quad 0.01W \quad 0.04W \quad 0.09W \quad 0.64W \quad 1W$$

c) The power supplied to the load resistor is equal to the total power supplied minus the power dissipated by the non-ideal source. From current lowest to highest the power supplied would be:

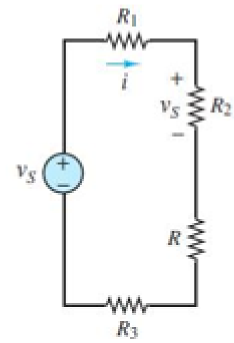
$0W \qquad 0.14W \qquad 0.26W \qquad 0.36W \qquad 0.56W \qquad 0.5W$

d) For the v_T plot Ohm's Law can be used to find the voltage drop across R_s which is equal to V_T. For the power plot, the data from part c can be used directly.



Problem 1.32

In the circuit in Figure P1.32, assume $v_2 = v_s/6$ and the power delivered by the source is 150 mW. Also assume that $R_1 = 8$ kΩ, $R_2 = 10$ kΩ, $R_3 = 12$ kΩ. Find R, v_s, v_2, and i.



Solution:

Known quantities:

R_1=8kΩ, R_2=10kΩ, R_3=12kΩ and the circuit in Figure P1.32.

Find:

R,v_s, v_2, and i.

Analysis:

Use ohms law to find v_s and i:

$$v_2 = \frac{v_s}{6} = R_2 * i$$

Also:

$$v_s * i = 150mW$$

So:

$$\boldsymbol{i = \sqrt[2]{\frac{150mW}{R_2 * 6}} = 1.6mA}$$

Since we know i, v_s can easily be found:

$$v_s * i = 150mW$$
$$\boldsymbol{v_s = 94.87V}$$

$$\boldsymbol{v_2 = \frac{v_s}{6} = 15.81V}$$

Use Ohm's law to find Req:

$$\frac{v_s}{i} = R_{eq} = 60k\Omega$$

Use Req to find R:

$$R_{eq} = R_1 + R + R_2 + R_3$$
$$R = R_{eq} - (R_1 + R_2 + R_3) = 30k\Omega$$

Problem 1.33

A GE SoftWhite Longlife lightbulb is rated as follows:
P_R = rated power = 60 W
P_{OR} = rated optical power = 820 lumens (lm) (average)
1 lumen = 1/680W
Operating life = 1,500 h (average)
V_R = rated operating voltage = 115 V
The resistance of the filament of the bulb, measured with a standard multimeter, is 16.7 Ω. Whenthe bulb is connected into a circuit and is operating atthe rated values given above, determine
a. The resistance of the filament.
b. The efficiency of the bulb.

Solution:

Known quantities:

Rated power; rated optical power; operating life; rated operating voltage; open-circuit resistance of the filament.

Find:

The resistance of the filament in operation
The efficiency of the bulb.

Analysis:

a)

$$P = VI \quad \therefore I = \frac{P_R}{V_R} = \frac{60 \text{ V}A}{115 \text{ V}} = 521.7 \text{ mA}$$

OL: $$R = \frac{V}{I} = \frac{V_R}{I} = \frac{115 \text{ V}}{521.7 \text{ mA}} = 220.4 \ \Omega$$

b)
Efficiency is defined as the ratio of the useful power dissipated by or supplied by the load to the total power supplied by the source. In this case, the useful power supplied by the load is the optical power. From any handbook containing equivalent units: 680 lumens=1 W

$$P_{o,out} = \text{ Optical Power Out } = 820 \text{ lum} \frac{\text{W}}{680 \text{ lum}} = 1.206 \text{ W}$$

$$\eta = \text{ efficiency } = \frac{P_{o,out}}{P_R} = \frac{1.206 \text{ W}}{60 \text{ W}} = 0.02009 = 2.009\ \%.$$

Problem 1.34

An incandescent lightbulb rated at 100 W will dissipate 100 W as heat and light when connected across a 110-V ideal voltage source. If three of these bulbs are connected in series across the same source, determine the power each bulb will dissipate.

Solution:

Known quantities:

Rated power; rated voltage of a light bulb.

Find:

The power dissipated by a series of three light bulbs connected to the nominal voltage.

Assumptions:

The resistance of each bulb doesn't vary when connected in series.

Analysis:

When connected in series, the voltage of the source will divide equally across the three bulbs. The across each bulb will be 1/3 what it was when the bulbs were connected individually across the source. Power dissipated in a resistance is a function of the voltage squared, so the power dissipated in each bulb when connected in series will be 1/9 what it was when the bulbs were connected individually, or 11.11 W:

Ohm's Law: $$P = IV_B = I^2 R_B = \frac{V_B^2}{R_B} \qquad V_B = V_S = 110 \text{ V} \qquad R_B = \frac{V_B^2}{P} = \frac{(110 \text{ V})^2}{100 \text{ V}A} = 121\ \Omega$$

Connected in series and assuming the resistance of each bulb remains the same as when connected individually:

KVL: $-V_S + V_{B1} + V_{B2} + V_{B3} = 0$ OL: $-V_S + IR_{B3} + IR_{B2} + IR_{B1} = 0$

$$I = \frac{V_S}{R_{B1} + R_{B2} + R_{B3}} = \frac{110 \text{ V}}{121 + 121 + 121 \text{ V}/A} = 303 \text{ mA}$$

$$P_{B1} = I^2 R_{B1} = (303 \text{ m}A)^2 (121 \text{ V}/A) = 11.11 \text{ W} = \frac{1}{9} 100 \text{ W.}$$

Problem 1.35

An incandescent lightbulb rated at 60 W will dissipate 60 W as heat and light when connected across a 100-V ideal voltage source. A 100-W bulb will dissipate 100 W when connected across the same source. If the bulbs are connected in series across the same source, determine the power that either one of the two bulbs will dissipate.

Solution:

Known quantities:

Rated power and rated voltage of the two light bulbs.

Find:

The power dissipated by the series of the two light bulbs.

Assumptions:

The resistance of each bulb doesn't vary when connected in series.

Analysis:

For the two bulbs in series KVL and KCL require

$$100\text{V} = V_{100} + V_{60} \quad \text{and} \quad I_{100} = I_{60}$$

The resistance of each bulb when connected individually across a 100V source is

$$R_{100} = \frac{(100V)^2}{100W} = 100\Omega \quad \text{and} \quad R_{60} = \frac{(100V)^2}{60W} \cong 167\Omega$$

Assume that the resistance of each bulb is the same when operated in series as when operated alone. Then

$$V_{100} = I_{100}R_{100} = I_{60}(100) \quad \text{and} \quad V_{60} = I_{60}R_{60} = I_{60}(167)$$

Plug into the KVL equation to find

$$100\text{V} \cong I_{60}(100 + 167) \quad \rightarrow \quad I_{60} \cong \frac{100}{267}\text{A} \cong I_{100}$$

The power absorbed by each bulb is

$$P_{100} = I_{100}^2 R_{100} \cong 14.0\text{W}$$

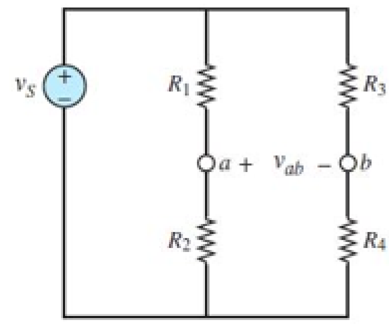
And

$$P_{60} = I_{60}^2 R_{60} \cong 23.4\text{W}$$

Notes: 1. It's strange but it's true that a 60 W bulb connected in series with a 100 W bulb will dissipate more power than the 100 W bulb. 2. If the power dissipated by the filament in a bulb decreases, the temperature at which the filament operates and therefore its resistance will decrease. This fact made the assumption about the resistance necessary.

Problem 1.36

Refer to Figure P1.36, and assume that v_S = 12 V, R_1 = 5 Ω, R_2 = 3 Ω, R_3 = 4 Ω, and R_4 = 5 Ω. Find:
a. The voltage v_{ab}.
b. The power dissipated in R_2.



Solution:

Known quantities:

Circuit of Figure P2.36. R_1=5Ω, R_2=3Ω, R_3=4Ω, R_4=5Ω, v_s=12V.

Find:

v_{ab} and the power dissipated in R_2.

Analysis:

Find the current through each resistor pair by combining them in series:

$$R_1 + R_2 = 8\Omega$$
$$R_3 + R_4 = 9\Omega$$

Since there is 12V across both of them Ohm's law is used to find the currents:

$$12V = 8\Omega * I_a$$
$$I_a = 1.5A$$
$$12V = 9\Omega * I_b$$
$$I_b = 1.33A$$

Now find the voltage drop across R_1 and R_3:

$$V_a = 12V - (I_a * R_1) = 4.5V$$
$$V_b = 12V - (I_b * R_3) = 6.66V$$
$$\boldsymbol{V_{ab} = -2.17V}$$

An alternate and more efficient approach is to apply voltage division to find the voltage across R_2 and R_4, respectively:

$$v_2 = v_s \frac{R_2}{R_1 + R_2} = 4.5V$$

$$v_4 = v_s \frac{R_4}{R_3 + R_4} = \frac{20}{3} V$$

It is implied in the calculations that the polarities of v_2 and v_4 are high (+) to low (-) from above to below each resistor in the figure. Apply KVL to find:

$$v_{ab} = v_2 - v_4 = \frac{13.5}{3} - \frac{20}{3} = -\frac{6.5}{3} V \cong -2.16V$$

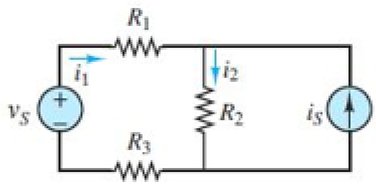
P_{R2} is equal to I_b times V_a:

$$\boldsymbol{P_{R2} = v_2 i_2 = \frac{v_2^2}{R_2} = 6.75W}$$

Problem 1.37

Refer to Figure P1.37, and assume that V_S = 7 V, I_S = 3A, R_1 = 20 Ω, R_2 = 12 Ω, and R_3 = 10 Ω.

Find:
a. The currents i_1 and i_2.
b. The power supplied by the voltage source v_S.



Solution:

Known quantities:

Circuit of Figure P2.37. R_1=20Ω, R_2=12Ω, R_3=10Ω,

I_s=3A, v_s=7V.

Find:

i_1, i_2, and P_v.

Analysis:

Use KVL of the rightmost loop:

$$v_s - R_1 * i_1 - R_2 * i_2 - R_3 * i_1 = 0$$
$$v_s = R_1 * i_1 + R_2 * i_2 + R_3 * i_1$$

Use KCL where i_1 meets i_s:

$$i_1 + i_s - i_2 = 0$$
$$i_1 + i_s = i_2$$

Combine the two equations to solve for i_1:

$$v_s = R_1 * i_1 + R_2 * (i_1 + i_s) + R_3 * i_1$$
$$\boldsymbol{i_1 = -0.69A}$$

Solve for i_2:

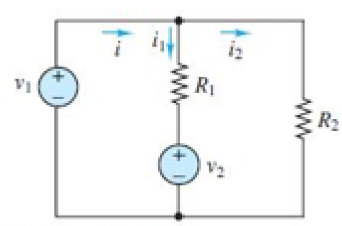
$$i_1 + i_s = i_2$$
$$\boldsymbol{i_2 = 2.31A}$$

Power delivered equals the $v_s$times i_1:

$$\boldsymbol{P_v = v_s * i_1 = -4.83W}$$

Problem 1.38

Refer to Figure P1.38, and assume v1 = 15 V, v2 = 6 V, R1 = 18Ω, R2 = 10Ω. Find:
a. The currents i_1 and i_2.
b. The power delivered by the sources v_1 and v_2.



Solution:

Known quantities:

Circuit of Figure P2.38. R_1=18Ω, R_2=10Ω, v_1=15V, v_2=6V.

Find:

i_1, i_2, P_{v1},P_{v2}.

Analysis:

Use KVL of the rightmost loop:

$$v_2 + R_1 * (-i_1) - R_2 * i_2 = 0$$
$$v_2 = R_1 * i_1 + R_2 * i_2$$

Use KCL where i_1 meets i_s:

$$i - i_1 - i_2 = 0$$
$$i_1 + i_2 = i$$

The assumption can be made that v_1 is equal to the voltage across R_2.

$$v_2 = R_2 * i_2$$
$$\boldsymbol{i_2 = 0.6A}$$

The assumption can be made that the voltage drop across R_1 is equal to the difference between v_1 and v_2.

$$v_1 - v_2 = R_1 * i_1$$
$$\boldsymbol{i_1 = 0.5A}$$

Solve for i:

$$i_1 + i_2 = i$$
$$i = 1.1A$$

Power delivered equals the voltage source times the current through it:

$$\boldsymbol{P_1 = v_1 * i = 16.5W}$$
$$\boldsymbol{P_2 = v_2 * (-i_1) = -3W}$$

Problem 1.39

Consider NiMH hobbyist batteries depicted in Figure P1.39.
a. If V_1 = 12.0 V, R_1 = 0.15 Ω and R_o = 2.55Ω find the load current I_o and the power dissipated by the load.
b. If battery 2 with V_2 = 12 V and R_2 = 0.28 Ω is placed in parallel with battery 1, will the load current I_o increase or decrease? Will the power dissipated by the load increase or decrease? By how much?

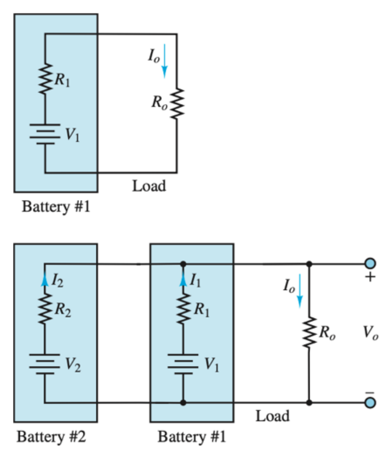


Figure P1.39

Solution:

Known quantities:

Schematic of the circuit in Figure P1.39.

Find:

If $V_1 = 12.0V, R_1 = 0.15\Omega, R_L = 2.55\Omega$, the load current and the power dissipated by the load

If a second battery is connected in parallel with battery 1with $V_2 = 12.0V, R_2 = 0.28\Omega$, determine the variations in the load current and in the power dissipated by the load due to the parallel connection with a second battery.

Analysis:

a) $I_L = \dfrac{V_1}{R_1 + R_L} = \dfrac{12}{0.15+2.55} = \dfrac{12}{2.7}$ = 4.44 A

$P_{Load} = I_L^2 R_L$ = 50.4 W.

b) with another source in the circuit we must find the new power dissipated by the load. To do so, we write KVL twice using mesh currents to obtain 2 equations in 2 unknowns: