

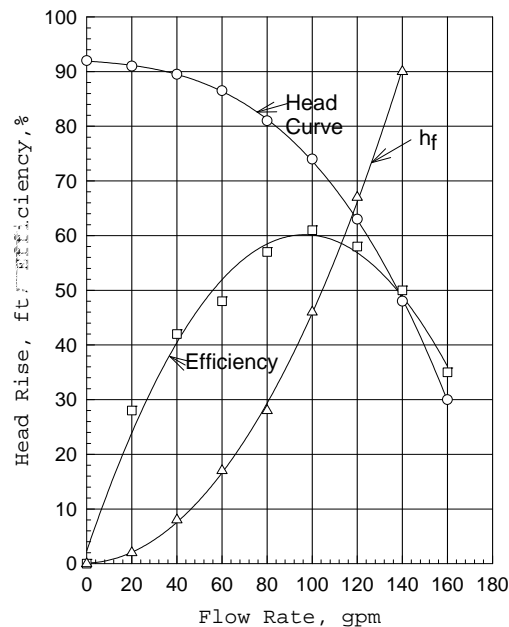
## SOLUTIONS

### CHAPTER 1

1.1. If the pump in Figure P1.1 is operated at 1750 rpm and is connected to a 2 inch i.d. galvanized iron pipe 175 ft long, what will be the flow rate and bhp?

Solution: We have a pump and a system which interact with each other to determine a statically stable operating condition. This operating point requires that the system resistance match the pump head at the same flow for both pump and system. The  $h_f$  curve for the pump may be plotted as a function of flow rate and superposed on the head flow curve of the pump. The intersect of the resistance curve and head flow curve is the operating point ( i.e., 2 equations in 2 unknowns).

$h_f$  for this system is given by  $h_f = f(L/D)(V^2/2g)$ , where  $V = Q/A = 10.2(Q_{\text{gpm}}/100)$  in ft/s.  $(L/D) = 1,050$  and  $f = f(\epsilon/D, Re_D)$ . Here  $\epsilon/D = 0.003$  and for sufficiently large



$Re_D$  (i.e.,  $>4 \times 10^5$ ),  $f = 0.027$ . Combining the numbers yields  $h_f = 45.8 (Q_{\text{gpm}}/100)^2$  in feet of head. Superposing on the pump curve shows the intersect occurring at about 118 gpm.

Checking the pipe  $Re_D$  gives  $2 \times 10^5$  and  $f = 0.0275$  (a negligible change). Here then we have 118 gpm and  $\eta = 0.58$ . Thus the power required is given by  $P = \rho g H Q / \eta = \rho g h_f Q / \eta = 3.26 \text{ hp}$ .

1.2. The pump of Figure P1.1 is installed in the system of Figure P1.2 as sketched. What flow rate, power, and efficiency result? All pipe is 2 inch i.d. commercial steel.

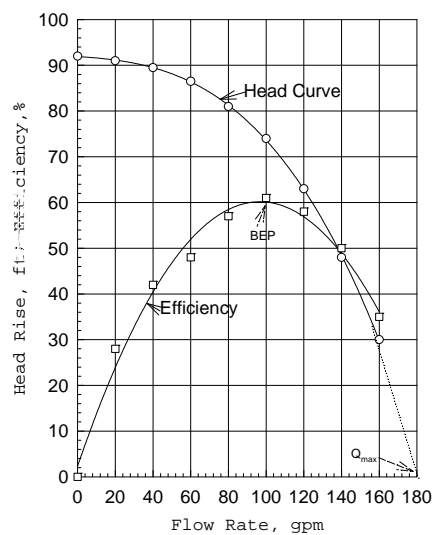
Solution: We write the resistance head as  $h_f = f(L/D)(V^2/2g) + \sum K_m(V^2/2g) + \Delta z$  where  $\Delta z = 50 \text{ ft}$ ,  $L/D = 350 \times 12/2 = 2100$ , and  $\epsilon/D = 0.00015 \times 12/2$ . The  $\sum K_m$  term consists of 2 screwed elbows ( $K = 0.95 \times 2 = 1.90$ ), a sharp entry and exit ( $K = 0.5$  and  $K = 1.0$ ) so that  $\sum K_m = 3.40$ . If we assume a fully rough turbulent flow, then  $f = 0.0192$  and  $h_f = 0.0192 \times 2100(V^2/2g) + 3.40(V^2/2g) + 50 = 43.72(V^2/2g) + 50$ . The table below summarizes the comparison of  $H$  and  $h_f$  as functions of  $Q$  ( $V = Q/A$ ).

Q, gpm	0	20	40	60	80	100	120
H, ft	92	91	90	87	81	74	63
$h_f$ , ft	50	52.8	61.3	75.5	95.4	120.9	152.1

The resistance values become greater than the pump head curve values between 60 and 80 gpm. By interpolation ( or a graph), the intersect can be seen to occur at about 70 gpm, with  $H = 83 \text{ ft}$  and , from the curve,  $\eta_T = 0.57$ . So the Power is  $P = Q \rho g H / \eta_T = 2.64 \text{ hp}$ .

1.3. An Allis-Chalmers pump has the performance data given by Figure P1.1. At the pump's best efficiency point, what values of  $Q$ ,  $H$ , and  $bhp$  occur? Estimate maximum or free-delivery flow rate.

Solution: The BEP location is found by visual inspection of the performance graph. The value of the highest efficiency is about 60%, occurring at  $Q = 100 \text{ gpm}$  and  $H = 74 \text{ ft}$ . Thus



the Power is given as  $P = \rho g H Q / \eta = 1.94 \times 32.2 \times 74 \times 100 \times 0.002228 / (.60 \times 550) = 3.12$  hp.

The maximum flow or free-delivery flow rate is estimated by extrapolating the performance curve down to the x-axis -- to zero head rise. as shown on the curve the result is approximately 180 gpm.

1.4. The pump of Figure P1.1 is used to pump water 800 feet uphill through 1000 feet of 2 inch i.d. steel pipe. You can do this if you connect a number of these pumps in series. How many pumps will it take? (HINT: fix the pump flow at BEP.)

Solution: If  $Q = Q_{BEP} = 100$  gpm and  $A = \pi D^2 / 4 = 0.028$  ft<sup>2</sup>, we get  $V = 10.2$  ft/s. For commercial steel pipe,  $\epsilon/D = 0.0009$  and  $f = 0.022$ . Then  $h_f = 0.022(1000 \times 12/2)(10.22/(2 \times 32.2)) + 800 = 1,014$  ft. Also at the BEP point we have  $H_{BEP} = 74.0$  ft. The number of pumps needed is  $(h_f/H_{BEP}) = 1,014/74 = 13.70$  pumps. We can't use a fraction of a pump so the requirement is for 14 pumps staged in a series configuration.

1.5. Two pumps of the type of Figure P1.1 may be used to pump flows with a vertical head rise of 130 ft.

(a) Should they be in series or parallel?

(b) What size steel pipe would you use to keep both pumps operating at BEP.

Solution: First check the extremes of performance for 2 pumps in series and 2 pumps in parallel. In parallel we have  $H$  remaining the same and  $Q$  doubling to form a new performance curve. That is flow is additive and the head rise is the same for each pump. In series the flow is the same through each pump while the head rise is additive. So, flow is the same while the head doubles. In parallel arrangement the max head rise is still  $H_{max} = 92$  ft, while in series the max head rise is  $H_{max} = 2 \times 92 \text{ ft} = 184$  ft. The parallel value is less than 130 ft and can't make the duty, while the series value is clearly adequate. Use the series arrangement.

Since we want to run the pumps at their BEP values, we need to size the pipe to fit the curve. That is  $Q = 100$  gpm and  $H = 2 \times 74 \text{ ft} = 148$  ft (as read off the graph of Figure P1.1 where efficiency peaks at 60% at 100 gpm). The head from the pump must be  $H = h_f + \Delta z = f(L/D)(V^2/2g) + \Delta z$  so that  $h_f = H - \Delta z = f(L/D)(V^2/2g) = 148 \text{ ft} - 130 \text{ ft} = 18$  ft. We can solve for  $V$  so that at  $Q = 100$  gpm  $= 0.22$  ft<sup>3</sup>/s  $= V \times A = V \times (\pi D^2/4)$

This gives  $D = (0.445f)^{1/3}$ . The calculation neglects any dependence of  $f$  on  $D$  but  $f$  depends on  $\epsilon/d$  and  $Re_D$  in a complex way. Assume  $f = 0.02$  (a typical fully rough value for steel pipe) and  $D = 0.208$  ft, calculation of  $Re$  yields 188,000 and the relative roughness value is about 0.0007, which indicates a value for  $f$  of about .019. A few iterations drive to  $D = 0.219$  ft = 2.62 inches with  $f = 0.024$ .

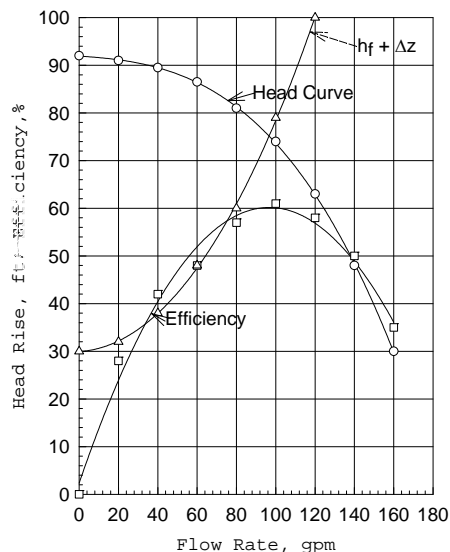
1.6. A fuel transfer pump delivers 400 gpm of 20° C gasoline. The pump has an efficiency of 80% and is driven by a 20-bhp motor. Calculate the pressure rise and head rise.

Solution: We have  $400 \text{ gpm} = 0.002228 \times 400 \text{ ft}^3/\text{s} = 0.891 \text{ ft}^3/\text{s}$ . We can calculate the head or pressure from the power input to the fluid:  $P = Q \Delta p / \eta$  so that  $\Delta p = \eta P / Q$ .  $P = 20$

$h_p = 11,000 \text{ lbf ft/s}$ , So  $\Delta p = 0.8 \times 11,000 / 0.891 = 9,874 \text{ lbf/ft}^2 = 68.57 \text{ psi}$ . The head rise requires the fluid density. From Figure A2 (appendix A) we read  $\rho$  for  $20^\circ\text{C}$  gasoline as  $736 \text{ kg/m}^3$ . This converts to  $46.26 \text{ lbf/ft}^3$ . Then  $H = \Delta p / \rho g = 9874 / 46.3 = 213.3 \text{ ft}$ .

1.7. If the pump and pipe arrangement of Problem 1.1 includes an elevation increase of 30 feet, what flow will result?

Solution: In problem 1.1 we had 175 ft of 2 inch galvanized pipe, and we expressed the



frictional resistance as  $h_f = f(L/D)(V^2/2g) = 0.00459 Q^2 \text{ gpm}$ . In this problem, we must add  $\Delta z$  to  $h_f$  to equate to  $H_{\text{pump}}$ . The solution then becomes  $H_{\text{pump}} = 0.00459 Q^2 + 30$ . On the graph at right, the solution is  $Q = 100 \text{ gpm}$  at  $H = 74 \text{ ft}$  and  $\eta = 0.60$  (i.e., the BEP).

1.8. A small centrifugal fan is connected to a system of ductwork. The fan characteristic curve may be approximated by the equation  $\Delta p_s = a + bQ$  where  $a = 30 \text{ lbf/ft}^2$  and  $b = -0.40 \text{ (lbf/ft}^2\text{)/(ft}^3\text{/s)}$ . Duct resistance is given by  $h_f = (fL/d + \sum K_m)(V^2/2g)$ . The resistance factor,  $(fL/d + \sum K_m) = 10.0$  and the duct cross-sectional area is  $1 \text{ ft}^2$ .

- Calculate the volume flow rate of the fan-duct system.
- What is the static pressure of the fan?
- Estimate the horsepower required to run the fan (assume the static efficiency of the fan is  $\eta_s = 0.725$ ).

Solution: We have simply : for the fan,  $\Delta p_s = 30 - 0.4Q$  ; for the duct (assuming  $\rho = 0.00233 \text{ slugs/ft}^3$ )  $\Delta p_s = \rho g h_f = 0.07748 \times 10 (V^2/2g) = 0.0116 Q^2$ .

(a) Solve for  $Q$ :  $Q^2 + 344Q - 2583 = 0$  gives  $Q = 36.5 \text{ ft}^3/\text{s}$ .

(b) Static pressure is calculated from  $\Delta p_s = 30 - 0.4Q = 15.42 \text{ lbf/ft}^2$ .

(c) Power is  $P = Q \Delta p_T / \eta_T = \Delta p_s Q / \eta_s = 36.5 \times 15.42 / 0.725 = 776.3 \text{ ft lbf/s} = 1.41$

hp.

1.9. A slurry pump operates with a mixture of sand and water, which has a density of  $1200 \text{ kg/m}^3$  and an equivalent kinematic viscosity of  $5 \times 10^{-5} \text{ m}^2/\text{s}$ . The pump, when tested in pure water, generated a head rise of 15 meters at a flow rate of  $3 \text{ m}^3/\text{s}$ . Estimate the head rise in the sand slurry with a flow rate of  $3 \text{ m}^3/\text{s}$ .

Solution: The head rise of the pump is defined as  $H = \Delta p / \rho g$  and is given in feet or meters of the fluid. Hence the head rise is not a function of the density of the fluid, although clearly the pressure rise is.

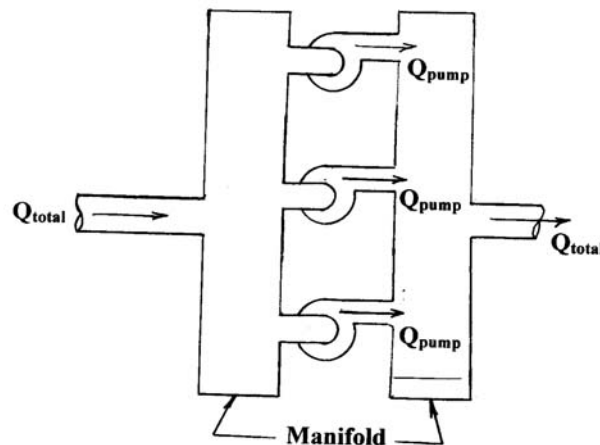
In water  $\Delta p = \rho g H = 998 \times 9.81 \times 15 \text{ N/m}^2 = 146,855 \text{ Pa}$ . In the slurry this changes to  $\Delta p = \rho g H = 1200 \times 9.81 \times 15 \text{ Pa} = 176,580 \text{ Pa}$ .

1.10. A three stage axial fan has a diameter of 0.5 m and runs at 1485 rpm. When operating in a system consisting of a 300 m long duct with a 0.5 m diameter ( $f = 0.040$ ) the fan generates a flow rate of  $5.75 \text{ m}^3/\text{s}$ . Estimate the static pressure rise required for each of the three stages.

Solution: The fans are arranged in series so that each handles the same volume of flow and hence supplies a pressure rise equal to that of the other fans. Denoting this rise as  $H_{\text{stage}}$ , the total generated is  $H = 3 H_{\text{stage}} = h_f = f(L/D)(V^2/2g)$ .  $V = Q/A = 5.75 \text{ m}^3/\text{s} / ((\pi 0.5^2)/4) = 29.3 \text{ m/s}$ . Then  $h_f = 0.04 \times (300/0.5) (29.3^2 / 2 \times 9.81) = 1,049 \text{ m}$  (of air) and  $H_{\text{stage}} = 1,049/3 = 350 \text{ m}$ . The pressure rise is  $\rho g H$  so  $\Delta p_{\text{s-stage}} = \rho g H_{\text{stage}} = 4,151 \text{ Pa}$  (assuming  $\rho = 1.21 \text{ kg/m}^3$ ).

1.11. In order to supply 375 gpm, a group of pumps identical to the pump of Figure P1.1 are operated in parallel. How many pumps are needed to operate the group at  $Q = 375 \text{ gpm}$  at the BEP point of operation? Develop a graph for the combined performance of this group of pumps.

Solution: Here we have several pumps side-by-side (as sketched) so that each pump



contributes to the overall flow while each operates at the same head rise. With "n" pumps

the total flow is  $nQ_{\text{pump}}$  and  $H=H_{\text{pump}}$ . At the BEP we have  $Q=100$  gpm at  $H=74$  ft. To get 375 gpm requires  $Q_{\text{total}}=nQ_{\text{pump}}=n \times 100$  gpm or  $n=375/100 = 3.75$  pumps. You can't have a fraction of a pump so we would require 4 pumps in parallel to supply the needed flow.

1.12. A double width centrifugal blower supplies flow to a system whose resistance is dominated by the pressure drop through a heat exchanger. The heat exchanger has a  $K$  - factor of 20, and a cross-sectional area of  $1 \text{ m}^2$ . If the blower is generating a pressure rise of 1 kPa, what is the volume flow rate ( $\text{m}^3/\text{s}$ ) for each side of the double-sided impeller?

Solution: The pressure rise for the fan of 1 kPa must be balanced by the system resistance, so that  $\Delta p_s = \rho g h_f = \rho g K (V^2/2g)$ . Here,  $V$  is the only unknown variable. We can solve for  $V$  and convert it to  $Q = VA$ . Assuming a standard density of  $1.21 \text{ kg/m}^3$ .  $V = (2\Delta p_s / \rho K)^{1/2} = (2,000 / 1.21 \times 20)^{1/2} = 9.091 \text{ m/s}$ . With  $1 \text{ m}^2$  flow area then  $Q = VA = 9.091 \text{ m}^3/\text{s}$ . The flow per side of the double width fan is then  $9.091/2 = 4.546 \text{ m}^3/\text{s}$ .

1.13. A medium sized pump delivers  $1.50 \text{ m}^3/\text{s}$  of SAE 50 engine oil ( $SG = 0.9351$ ). The pump has a total efficiency of 0.679 and is direct connected to a motor which can supply a shaft power of 25 kW. What head rise can the pump generate at this flow and power?

Solution: The given information allows us to use the expression for fluid power and shaft power to back out the head rise. That is,  $P_{\text{fl}} = P_{\text{sh}} \eta = \rho g H Q = \rho g h_f Q$ . Note that the fluid density is  $\rho = S.G. \times \rho_{\text{water}} = 998 \times 0.9351 = 933 \text{ kg/m}^3$ . We solve for the head rise as  $H = \eta P_{\text{sh}} / \rho g Q = 1.162 \text{ m}$ .

1.14. The centrifugal fan described in Problem 1.8 must be used to meet a flow-pressure rise specification in metric units. Convert the equation for its pumping performance into SI units and:

(a) Estimate the flow rate achievable through a 15 cm diameter duct for which the resistance is characterized by  $K_{\text{res}} = (fL/D + \sum K_m) = 1.0$ .

(b) Develop a curve of  $Q$  versus diameter for  $15 \text{ cm} < D < 45 \text{ cm}$  with  $K_{\text{res}} = 1.0$ .

(c) Develop a curve of  $Q$  versus  $K_{\text{res}}$  with a diameter of 10 cm and  $1 < K_{\text{res}} < 5$ .

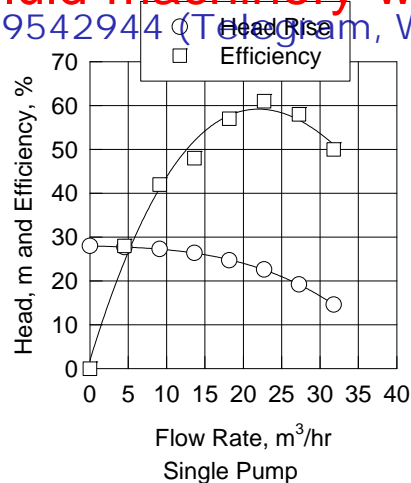
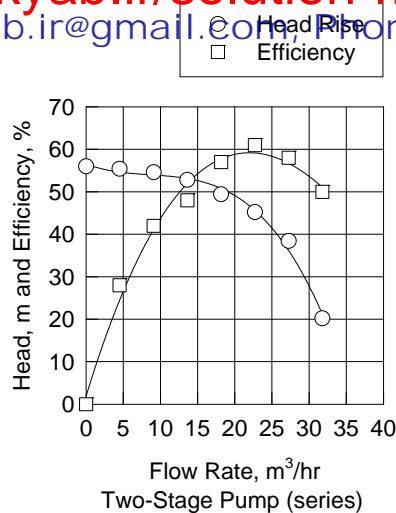
Solution: We had the form  $\Delta p_s = a + b Q = 30 (\text{lbf/ft}^2) - 0.4 ((\text{lbf/ft}^2)/(\text{ft}^3/\text{s})) Q$  with  $Q$  in  $\text{ft}^3/\text{s}$ . all in BG units. To convert we use  $\Delta p (\text{kPa}) = \Delta p (\text{lbf/ft}^2) \times (101.3 \text{ kPa} / 211.6 \text{ lbf/ft}^2) = 0.04787 \Delta p (\text{lbf/ft}^2)$ .

So  $\Delta p_s = 1436 - 676 Q$  in kPa with  $Q$  in  $\text{m}^3/\text{s}$ .

(a) Here,  $\Delta p_s = \rho g h_f = 0.605 V^2 (\text{N/m}^2)$ , Convert to  $Q$  as  $Q = VA = 0.0176 V$ , so  $V^2 = (Q/A)^2 = 3,202 Q^2$  and  $\Delta p_s = 1436 - 676 Q = 1937 Q^2$  or  $Q = 0.704 \text{ m}^3/\text{s}$ .

(b) At  $D = 30 \text{ cm}$ ,  $A = 0.0768 \text{ m}^2$ ,  $V^2 = 200 Q^2$  and  $\Delta p_s = 121 Q^2$ , so we have  $Q^2 + 5.59 Q - 11.87 = 0$ . Thus  $Q = 1.641 \text{ m}^3/\text{s}$ . With  $D = 45 \text{ cm}$ ,  $Q = 1.945 \text{ m}^3/\text{s}$ .

(c) Here,  $K=1$  gives  $Q = 0.971 \text{ m}^3/\text{s}$ ;  $K = 2$  gives  $Q = 0.750 \text{ m}^3/\text{s}$ ;  $K = 3$  gives  $Q = 0.637 \text{ m}^3/\text{s}$ . Plot as desired.



1.15. Develop a set of performance curves, in SI units, for the pump data given in Figure P1.1 ( $Q$  in  $\text{m}^3/\text{hr}$ ,  $H$  in  $\text{m}$  and  $P$  in  $\text{kW}$ ). Use these curves to predict flow rate, head rise and required power if two of these pumps are connected in series to a filter system with a resistance coefficient of  $K_m = 50.0$ . Assume the diameter of the flow system is 5 cm and neglect other resistances.

Solution: To develop performance curves from Figure P1.1 into SI units we need to convert according to  $Q_{\text{SI}} = 3.2808^3 \times 0.002228 \times 3600 Q_{\text{gpm}} = 0.2271 Q_{\text{gpm}}$ . Head is simply  $H_{\text{SI}} = 0.3048 H_{\text{ft}}$ . Efficiency is unchanged and the data of Figure P1.1 is converted.

For two pumps in series, we have  $h_f = KV^2/2g = k(Q/A)^2/2g = 0.510 Q^2$ , with  $D = 0.05\text{m}$  and  $Q$  in  $\text{m}^3/\text{s}$ . The intersect of the resistance curve with the 2-pump curve yields  $Q = 29 \text{ m}^3/\text{hr}$ ,  $H = 38\text{m}$ ,  $\eta = 0.57$ , and  $P = 5.26 \text{ kW}$ .

Q	Q	H	$\eta$	P
gpm	$\text{m}^3/\text{hr}$	m		kW
0	0	28	0	
20	4.50	27.7	0.28	1.211
40	9.09	27.3	0.42	1.607
60	13.63	26.4	0.48	2.039
80	18.17	24.7	0.57	2.141
100	22.71	22.6	0.60	2.288
120	27.26	19.2	0.58	2.454
140	31.79	14.6	0.50	2.524

For two pumps in series:

Q	Q	H	$\eta$	P
gpm	$\text{m}^3/\text{hr}$	m		kW

0	0	56	0	
20	4.50	55.4	0.28	2.422
40	9.09	54.6	0.42	3.214
60	13.63	52.8	0.48	4.078
80	18.17	49.4	0.57	4.282
100	22.71	45.2	0.60	4.576
120	27.26	38.4	0.58	4.908
140	31.79	29.2	0.50	5.048

1.16. Estimate the pressure rise of the slurry pump of Problem 1.9 at the stated conditions. Compare this result to the pressure rise with water as the working fluid.

Solution: The pressure, being proportional to the density, will scale directly as the ratio of the densities of water and the sand-water slurry -- that is with the specific gravity of the slurry. So  $\Delta p_{s\text{-slurry}} = SG_{\text{slurry}} \times \Delta p_{s\text{-water}} = (1200/998) (\rho g H)_{\text{water}} = 1.2024 \times 9790 \times 15 = 176.6 \text{ kPa}$ .

1.17. Use the pump performance curves developed in Problem 1.15 to estimate the BEP power and efficiency for the SI pump. Estimate the shut-off and free-delivery performance

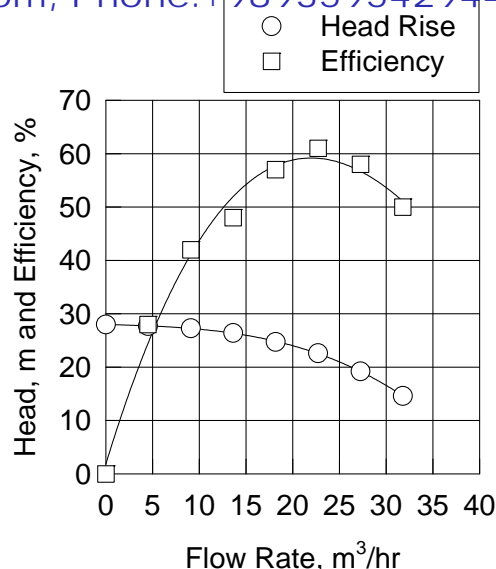
Solution: From the curves developed for Problem 1.15 the results are  $Q_{\text{max}} = 41 \text{ m}^3/\text{hr}$ ,  $H_{\text{max}} = 56 \text{ m}$ . The BEP has  $Q_{\text{BEP}} = 22.7 \text{ m}^3/\text{hr}$ ,  $H_{\text{BEP}} = 22.6 \text{ m}$  and  $\eta_{\text{BEP}} = 0.60$ .

1.18. Estimate the flow delivery of the fan of Problem 1.12 if the fan is a single width version of the given fan. Assume that  $\Delta p_{s\text{-max}} = 1.50 \times \Delta p_{s\text{-BEP}}$  ( at  $Q=0$ ).

Solution: You can assume that the fan curves may be approximated as a polynomial of some order -- perhaps parabolic as in a previous problem, or perhaps linear. For a linear fit, we use half of the flow rate of the DWDI fan at its operating point, so  $Q_{\text{BEP}} = 4.54 \text{ m}^3/\text{s}$  while  $\Delta p_{s\text{BEP}} = 1 \text{ kPa}$ . That makes  $\Delta p_{s\text{-max}} = 1.5 \text{ kPa}$ , so the linear fit for a SWSI version is  $\Delta p_s = 1.5 - (0.5/4.54)Q$  or  $\Delta p_s = 1.5 - 0.110Q$  in kPa, with  $Q$  in  $\text{m}^3/\text{s}$ . Recall that the resistance to the flow of the fan was a heat exchanger with a resistance factor of  $K = 20$ , and a cross-sectional area of  $1 \text{ m}^2$ . The resistance curve then becomes  $\Delta p_{s\text{-res.}} = \rho(Q/A)^2 K/2$ . Equate to  $\Delta p_s$  for the SWSI fan and  $1.5 - 0.110Q = 0.0121Q^2$ , for which  $Q = 7.48 \text{ m}^3/\text{s}$ .

1.19. What flow rate will the pump of Problem 1.15 generate through 100 m of a pipe which is 5 cm in diameter with a friction factor of  $f = 0.030$ ?





Solution: In Problem 1.15 we developed the H-Q curve in  $\text{m}^3/\text{hr}$  and m of head. We show these curves here for reference. The only difference here is the coefficient of resistance for the system. Here  $K_{\text{res}} = f(L/D) + \sum K_m = f(L/D) = 0.030(100/.05) = 60$ . The  $h_f$  curve becomes  $0.0612Q^2$  with Q in  $\text{m}^3/\text{hr}$ . The intersect with the pump curve is at approximately  $Q = 19.5 \text{ m}^3/\text{hr}$  at 23.6 m of head.

1.20. We can arrange the three stages of the axial fan of Problem 1.10 to operate in a parallel, side by side arrangement, delivering their individual flow rates to a plenum which is connected to a duct. If this duct is to be 110 m long ( $f = 0.04$ ), what size must the duct be to ensure a total flow delivery of  $17.25 \text{ m}^3/\text{s}$ ? (Neglect any plenum losses.)

Solution: The original system had  $L = 110$ ,  $f = 0.04$  and we want to determine  $d$ , the duct diameter. In that, original system the flow rate was  $5.75 \text{ m}^3/\text{s}$  with  $h_f = 1,049 \text{ m}$ . So we use three fans, each operating at  $Q = Q_{\text{total}}/3 = 17.25/3 = 5.75 \text{ m}^3/\text{s}$ . The head rise of an individual stage was  $1,049/3 = 350 \text{ m}$ . We find a matched duct diameter such that  $h_f = 350 \text{ m} = f(L/d)(Q_{\text{total}}^2/A^2)/2g = 66.73/d^5$ , with  $A = \pi d^2/4$ . We solve for  $d$  as  $d = 0.718 \text{ m}$ .

1.21. An air compressor operates in ambient conditions such that  $p_{01} = 100 \text{ kPa}$ ,  $\rho_{01} = 1.2 \text{ kg/m}^3$ ,  $T_{01} = 280 \text{ K}$ . The compressor performance is  $Q = 1 \text{ m}^3/\text{s}$ ,  $\Delta p_T = 25 \text{ kPa}$ ,  $P_{\text{sh}} = 28 \text{ kW}$  and  $\rho_{02} = 1.4 \text{ kg/m}^3$ . Calculate the total efficiency  $\eta_T$  and the compression efficiency  $\eta_c$ .

Solution: Total efficiency is defined as  $\eta_T = Q\Delta p_T/P_{\text{sh}} = 1 \times 25,000/28,000 = 0.893$ . The compression efficiency is given by  $\eta_c = [(p_{01}/p_{02})^{(\gamma-1)/\gamma} - 1]/[(T_{02}/T_{01}) - 1]$ .  $p_{02} = p_{01} + \Delta p_T = 125 \text{ kPa}$ . Using the perfect gas law,  $T_{02} = [(p_{01}/p_{02})(\rho_{01}/\rho_{02})] T_{01} = (1.24/1.667)280 \text{ K} = 300 \text{ K}$ . Then  $\eta_c = [1.25^{0.2857} - 1]/[1.0714 - 1] = 0.922$ .

1.22. A compressor pumps natural gas ( $\gamma = 1.27$ ) with inlet flow conditions of 13.9 psia at a temperature of  $90^\circ\text{F}$  requiring 38 hp for an inlet flow rate of 1,800 cfm. The pressure ratio is 1.25 and the outlet temperature is  $135^\circ\text{F}$ . Calculate the total efficiency  $\eta_T$  and the compression efficiency  $\eta_c$ .

Solution: The total efficiency is defined as  $\eta_T = Q\Delta p_T/P_{sh}$  with  $\Delta p_T = p_{02} - p_{01} = 1.25 p_{01} - p_{01} = 0.25 \times 13.9 \times 144 \text{ lbf/ft}^2 = 500 \text{ psf}$ . Using  $Q = 1800/60 = 30 \text{ ft}^3/\text{s}$ ,  $\eta_T = (30 \times 500)/(38 \times 550) = 0.718$ . The compression efficiency is defined as  $\eta_c = [(p_{01}/p_{02})^{(\gamma-1)/\gamma} - 1]/[(T_{02}/T_{01}) - 1]$ . Here,  $(T_{02}/T_{01}) = [(p_{01}/p_{02})(\rho_{01}/\rho_{02})] = 1.25/1.18 = 1.059$ .  $\eta_c = [1.225^{0.2126} - 1]/[1.059 - 1] = 0.747$ .

1.23. The performance of a water pump is to be measured in a test stand similar to the configuration shown in Figure 1.13. The pressure rise or pressure differential from “a” to “b” is inferred from a mercury manometer deflection of 25 cm. The flow meter is a sharp edged orifice with  $\beta = 0.6$  and  $d = 10 \text{ cm}$ . The pressure change across the meter from “c” to “d” is indicated by a column height change of 15 cm on a second mercury manometer. Find the pressure rise and volume flow rate of the pump (in kPa and  $\text{m}^3/\text{s}$ ).

Solution: The pressure rise is given by  $\Delta p_{\text{pump}} = \Delta p_{\text{man}} = \rho_{\text{Hg}} g H_{\text{Hg}} - \rho_{\text{H}_2\text{O}} g H_{\text{H}_2\text{O}}$   
 $= 13,546 \times 9.81 \times 0.25 - 998 \times 9.81 \times 0.25 = 33,222 - 2,448 = 30,774 \text{ kPa}$  (where the mercury deflection has been corrected by the opposite water deflection). The meter pressure change is  $\Delta p_{\text{meter}} = (\rho g H)_{\text{Hg}} - (\rho g H)_{\text{H}_2\text{O}} = 18.464 \text{ kPa}$ . Use this  $\Delta p$  in  $Q = C_d A_t [2\Delta p/\rho]^{1/2}$  where  $C_d$  is a function of  $Re_d$  and  $\beta$  for the orifice (see equations 1.31, 1.32 and 11.33). Assume initially that  $C_d \cong f(\beta) = 0.5959 + 0.0312\beta^{2.1} - 0.184\beta^8$  and recalculate for  $Re_d$  later. Here  $f(\beta) = 0.6035$  so  $Q \cong 0.6035 \times 0.00785 [2 \times 18,467/998]^{1/2} = 0.0278 \text{ m}^3/\text{s}$ . Then  $V_t = Q/A_t = 3.544 \text{ m/s}$ , and  $Re_d = 3.524 \times 0.1/10^{-5} = 35,444$ . Then  $C_d = 0.6035 + 97.1(0.6)^{2.5}(35,444)^{-0.75} = 0.6140$  and  $Q$  is corrected to  $0.0283 \text{ m}^3/\text{s}$ .

1.24. The pump of problem 1.23 is driven by an a/c induction motor with  $\eta_m = 0.85$ . The electrical input to the motor, as tested, was 1.20 kW. Calculate the efficiency of the pump at the test point. (The results for 1.23 were  $0.0283 \text{ m}^3/\text{s}$  and  $30.77 \text{ kPa}$ ).

Solution: The pump efficiency is given by  $\eta = Q\Delta p/P_{sh}$  where  $P_{sh} = \eta_m P_{\text{elec}}$ .  
 So  $\eta = (30,770 \times 0.0283)/(0.85 \times 1,200) = 0.854$ .

1.25. A three foot diameter axial flow blower was tested in a flow facility like the one shown in Figure 1.8. There were four one foot diameter nozzles flowing in the test. A performance point had  $\Delta p_s = 2 \text{ InWG}$ ,  $\Delta p_n = 2 \text{ InWG}$  and 7.6 hp supplied to the shaft of the blower. Calculate the values of  $Q$  (cfm),  $\Delta p_T$  (InWG) and  $\eta_s$  and  $\eta_T$  for the blower test point.

Solution: The nozzle velocity can be initially approximated with  $C_d = 1.0$  so  $V_n = [2\Delta p_n/\rho]^{1/2} = [2 \times 10.4/0.00235]^{1/2} = 94.1 \text{ ft/s}$ . Using  $v = 1.61 \times 10^{-4}$  gives a Reynolds number of  $Re_d = 94.1 \times 1/1.61 \times 10^{-4} = 5.84 \times 10^5$ .  $C_d$  can now be estimated as  $C_d = 0.9965 - 0.00653[10^6/Re_d]^{1/2} = 0.989$ , so the flow velocity corrects to  $93.1 \text{ ft/s}$  and the flow rate is  $Q = V_n \sum A_n = 93.1 \times \pi = 292.4 \text{ ft}^3/\text{s} = 17,545 \text{ cfm}$ . The blower discharge velocity is approximated as  $V_d = Q/A_{fan} = 292.4/[\pi D^2/4] = 41.4 \text{ ft/s}$ . Then the velocity pressure for the blower is  $(\rho/2)V_d^2 = 2.01 \text{ lbf/ft}^2 = 0.386 \text{ InWG}$ . So,  $\Delta p_T = 2.386 \text{ InWG}$ . Using  $\eta = Q\Delta p/P$  we calculate  $\eta_s = 0.707$  and  $\eta_T = 0.843$ .