

Chapter - 1

1.1 (a) Unsteady flow during the operation of the gate. Steady flow afterwards. Rapidly varied flow in the immediate vicinity downstream of the gate.

- (b) Normally steady , uniform flow
- (c) Gradually varied unsteady flow
- (d) Rapidly varied unsteady flow
- (e) Steady rapidly varied flow
- (f) Unsteady flow on both upstream and downstream of the gate
- (g) Usually spatially varied steady flow. It can be SVUF also.

1.2 $V = \frac{u_m}{2}, \quad \frac{u}{y} = \frac{u_m}{D} \text{ and hence } u = \frac{y}{D} u_m$

$$\alpha = \frac{\int_0^D u^3 dy}{V^3 D} = \frac{\int_0^D u^3 dy}{\frac{u_m^3}{8} D} = \frac{\frac{u_m^3}{D^3} \int_0^D y^3 dy}{\frac{u_m^3}{8} D}$$

$$= 2.0$$

$$\beta = \frac{\int_0^D u^2 dy}{V^2 D} = \frac{\frac{u_m^2}{D^2} \int_0^D y^2 dy}{\frac{u_m^2}{4} D} = \frac{4}{3} = 1.33$$

1.3 $\bar{V} = \frac{1}{y_0} \int_0^{y_0} v dy = \frac{v_m}{y_0} \frac{1}{y_0^{1/n}} \int_0^{y_0} y^{1/n} dy = \frac{n}{(n+1)} v_m$

$$\beta = \frac{\int_0^{y_0} v^2 dy}{\bar{V}^2 y_0} = \frac{v_m^2}{\left(\frac{n}{n+1}\right)^2 v_m^2} \cdot \frac{1}{y_0} \int_0^{y_0} \frac{1}{y_0^{2/n}} \cdot y^{2/n} dy$$

$$\beta = \frac{(n+1)^2}{n(n+2)}$$

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$$\alpha = \frac{\int_0^{y_0} v^3 dy}{\bar{V}^3 y_0} = \frac{v_m^3}{\left(\frac{n}{n+1}\right)^3 v_m^3} \cdot \frac{1}{y_0} \int_0^{y_0} \frac{1}{y_0^{3/n}} \cdot y^{3/n} dy$$

$$\alpha = \frac{(n+1)^3}{n^2(n+3)}$$

$$1.4 \quad \beta = \frac{\int u^2 dA}{\bar{u}^2 A} = \frac{\int (\bar{u}^2 + (\delta u)^2 + 2\bar{u} \delta u) dA}{\bar{u}^2 A}$$

Since $\int \delta u \cdot dA = 0$, $\beta = 1 + \eta$ where $\eta = \frac{\int (\delta u)^2 dA}{\bar{u}^2 A}$

$$\alpha = \frac{\int u^3 dA}{\bar{u}^3 A} = \frac{\int (\bar{u}^3 + (\delta u)^3 + 3\bar{u} (\delta u)^2 + 3\bar{u}^2 (\delta u)) dA}{\bar{u}^3 A}$$

Since $\int \delta u \cdot dA = 0$ and $\int (\delta u)^3 dA$ is negligibly small ,

$$\alpha = 1 + 3\eta$$

$$1.5 \quad v = C/r \quad r_1 = 4.25 \text{ m} \quad r_2 = 5.75 \text{ m}$$

$$Q = \int_{r_1}^{r_2} v B dr = 6.0$$

$$\int_{r_1}^{r_2} \frac{C}{r} dr = \frac{6.0}{2.0} = 3.0$$

$$C = \frac{3.0}{\ln \frac{r_2}{r_1}} = \frac{3.0}{\ln \frac{5.75}{4.25}} = 9.9245$$

$$(a) \quad v = \frac{9.9245}{r}$$

$$v_1 = \frac{C}{4.25} = 2.335 \text{ m/s}$$

$$v_2 = \frac{C}{5.75} = 1.726 \text{ m/s}$$

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$$(b) \quad v_{av} = \frac{Q}{B y_0} = \frac{6.0}{2.0 \times 1.5} = \mathbf{2.0 \text{ m/s}}$$

$$(c) \quad \beta = \frac{\int v^2 dy}{v_{av}^2 y_0} = \frac{\int_{r_1}^{r_2} \frac{C^2}{r^2} dr}{(2)^2 \times 1.5} = \left[\frac{C^2}{6.0} \left(-\frac{1}{r} \right) \right]_{4.25}^{5.75}$$

$$= 16.415 \left[-\frac{1}{5.75} + \frac{1}{4.25} \right] = \mathbf{1.008}$$

$$\alpha = \frac{\int v^3 dy}{v_{av}^3 y_0} = \frac{\int_{r_1}^{r_2} \frac{C^3}{r^3} dr}{(2)^3 \times 1.5} = \left[\frac{C^3}{12.0} \left(-\frac{1}{2r^2} \right) \right]_{4.25}^{5.75}$$

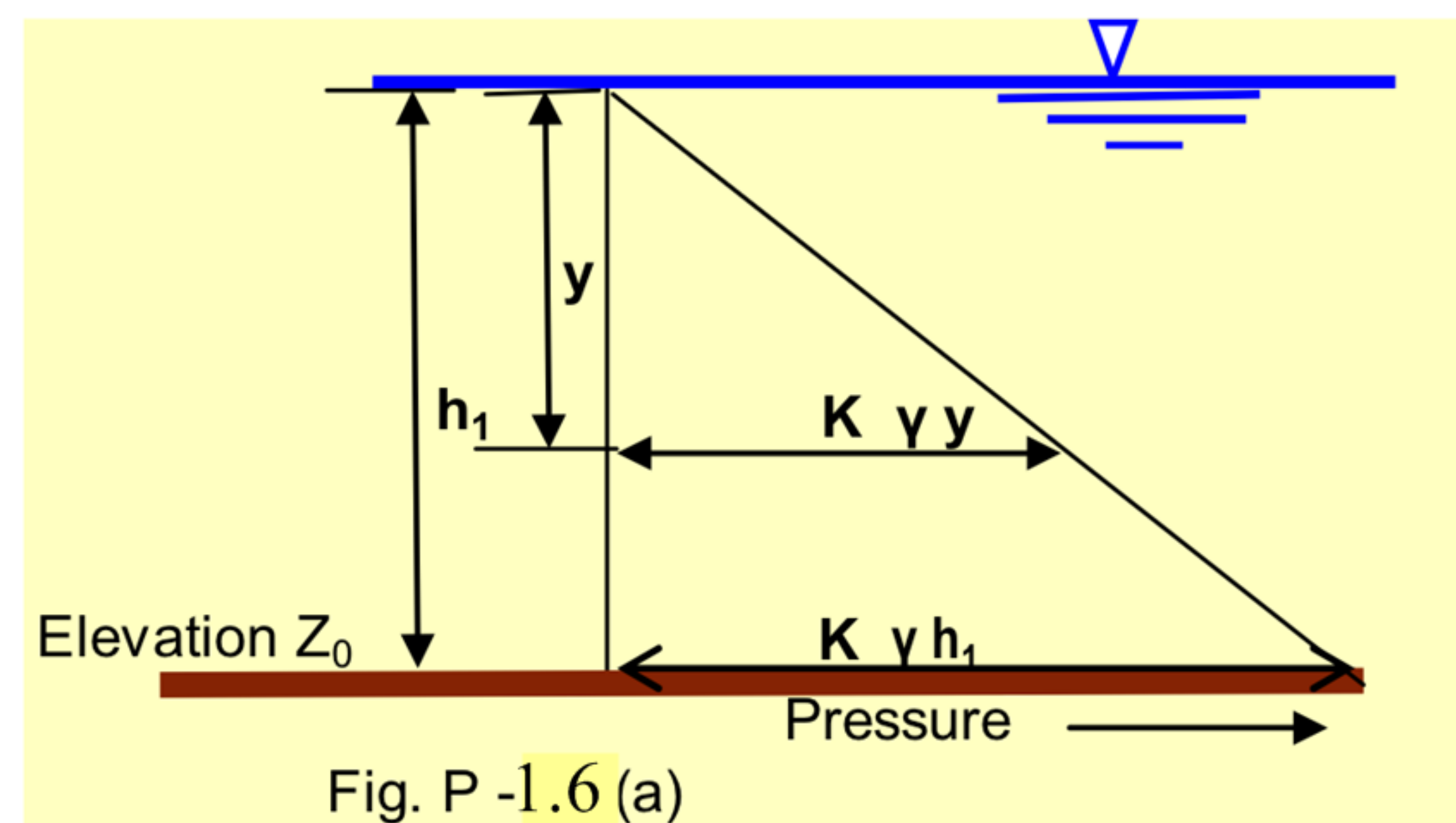
$$= 40.73 \left[-\frac{1}{(5.75)^2} + \frac{1}{(4.25)^2} \right] = \mathbf{1.023}$$

1.6 (a) Refer to Fig. P-1.6 (a)

$$Z_0 + h_1 + \Delta h = Z_0 + (h_1 - y) + k y$$

$$\Delta h = (k - 1)y$$

$$h_{ep} = Z_0 + h_1 + \frac{1}{h_1} \int_0^{h_1} (k - 1)y dy$$



$$= Z_0 + h_1 + (k - 1) \frac{h_1^2}{2h_1} = Z_0 + h_1 + \frac{(k - 1)}{2} h_1$$

$$= Z_0 + \left(\frac{k + 1}{2} \right) h_1$$

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(b) Refer to Fig. P-1.6(b)

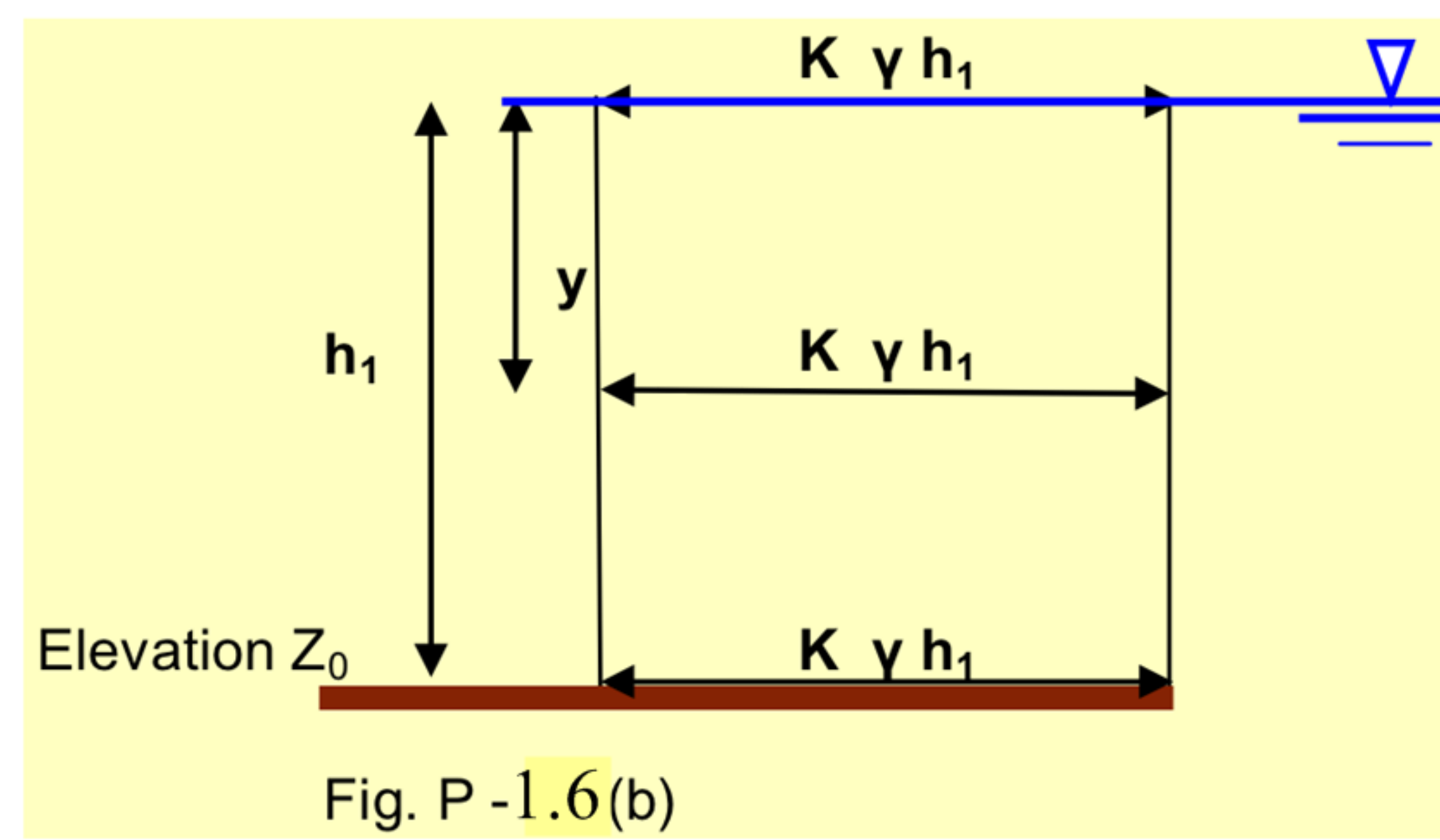
$$Z_0 + h_1 + \Delta h = Z_0 + (h_1 - y) + k h_1$$

$$\Delta h = k h_1 - y$$

$$h_{ep} = Z_0 + h_1 + \frac{1}{h_1} \int_0^{h_1} (k h_1 - y) dy$$

$$= Z_0 + h_1 + \frac{1}{h_1} \left[k h_1^2 - \frac{h_1^2}{2} \right]$$

$$= Z_0 + \left(\frac{2k+1}{2} \right) h_1$$



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1.7 Refer to Fig. P-1.7(a) and (b)

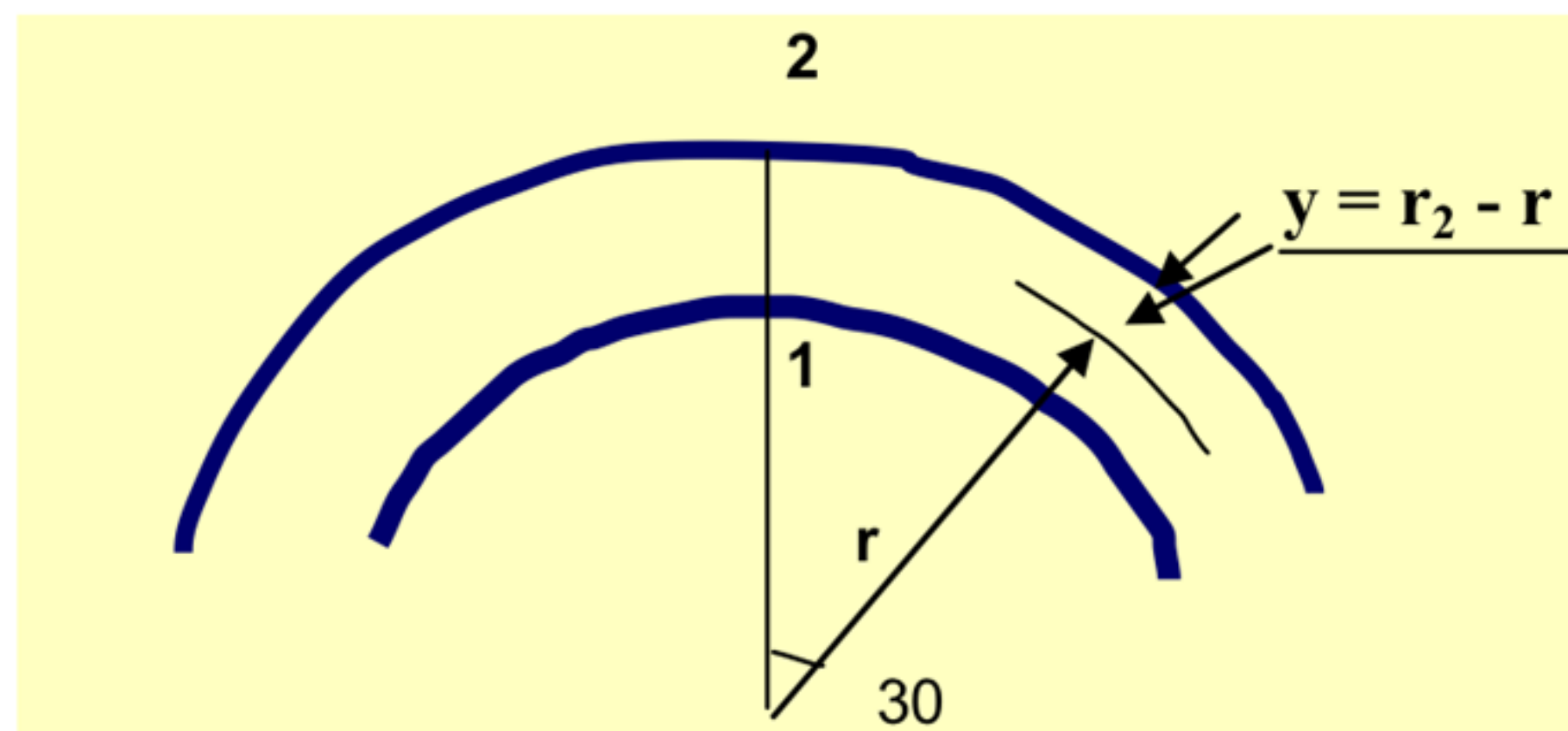


Fig. P 1.7(a)

$$\frac{p}{\gamma} = (r_2 - r) \cos \theta - \frac{a_n}{g} (r_2 - r).$$

$$= \text{depth below free surface} = (r_2 - r).$$

$$\text{Hence } \frac{p}{\gamma} = y \cos \theta - \frac{a_n}{g} y$$

$$\text{When } \theta = 30^\circ \text{ and } a_n = 0.4 g$$

$$\frac{p}{\gamma} = 0.866y - 0.4y = 0.466y$$

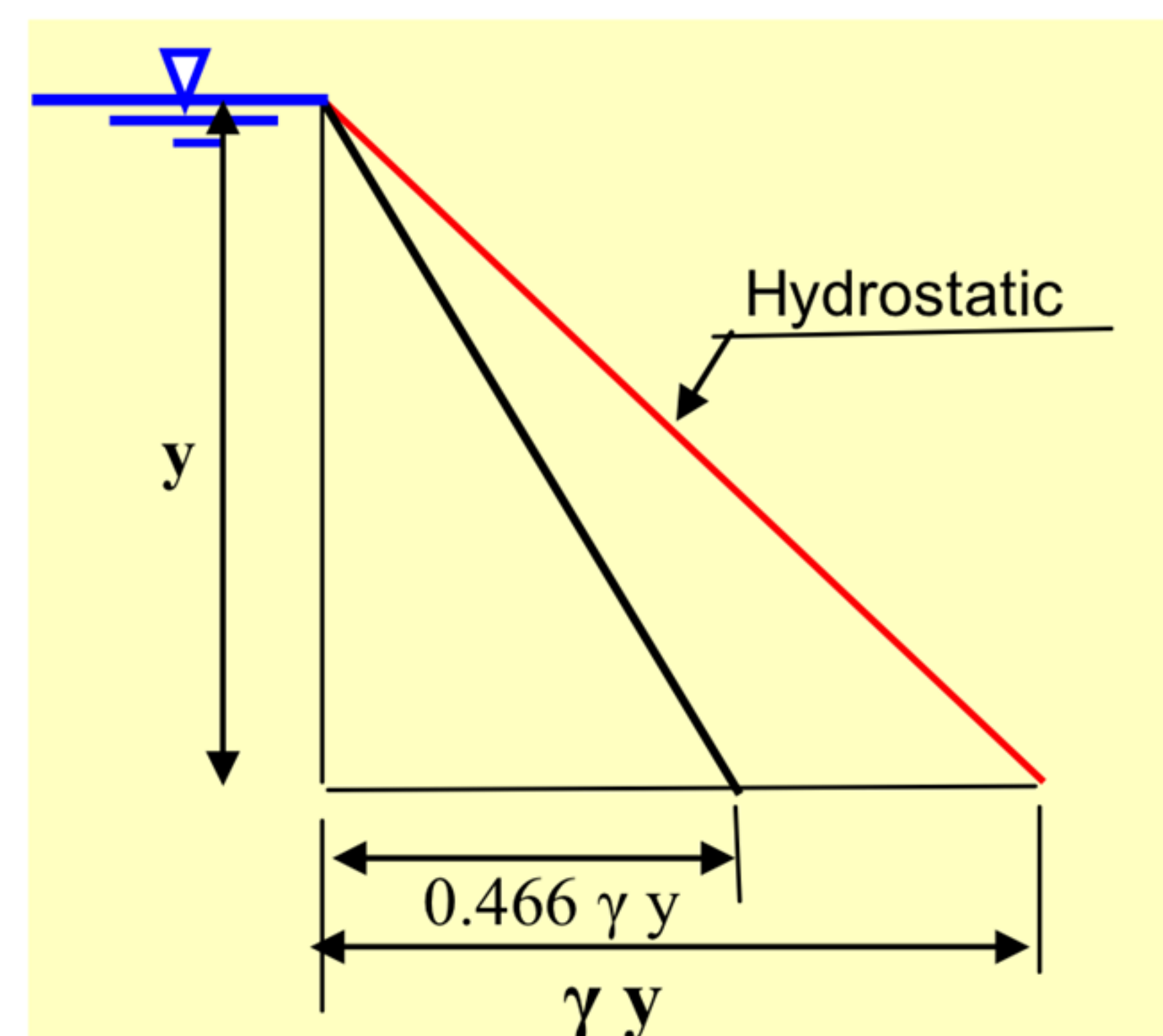


Fig. P 1.7(b)

$$\text{At the free surface } y = 0 \text{ and hence } \frac{p}{\gamma} = 0$$

$$\text{At the bottom of the channel, } y = 0.75 \text{ m, and } \frac{p}{\gamma} = 0.35 \text{ m.}$$

$$\text{Hydrostatic pressure distribution gives } \frac{p}{\gamma} = y.$$

1.8 Refer to Fig. P-1.8.

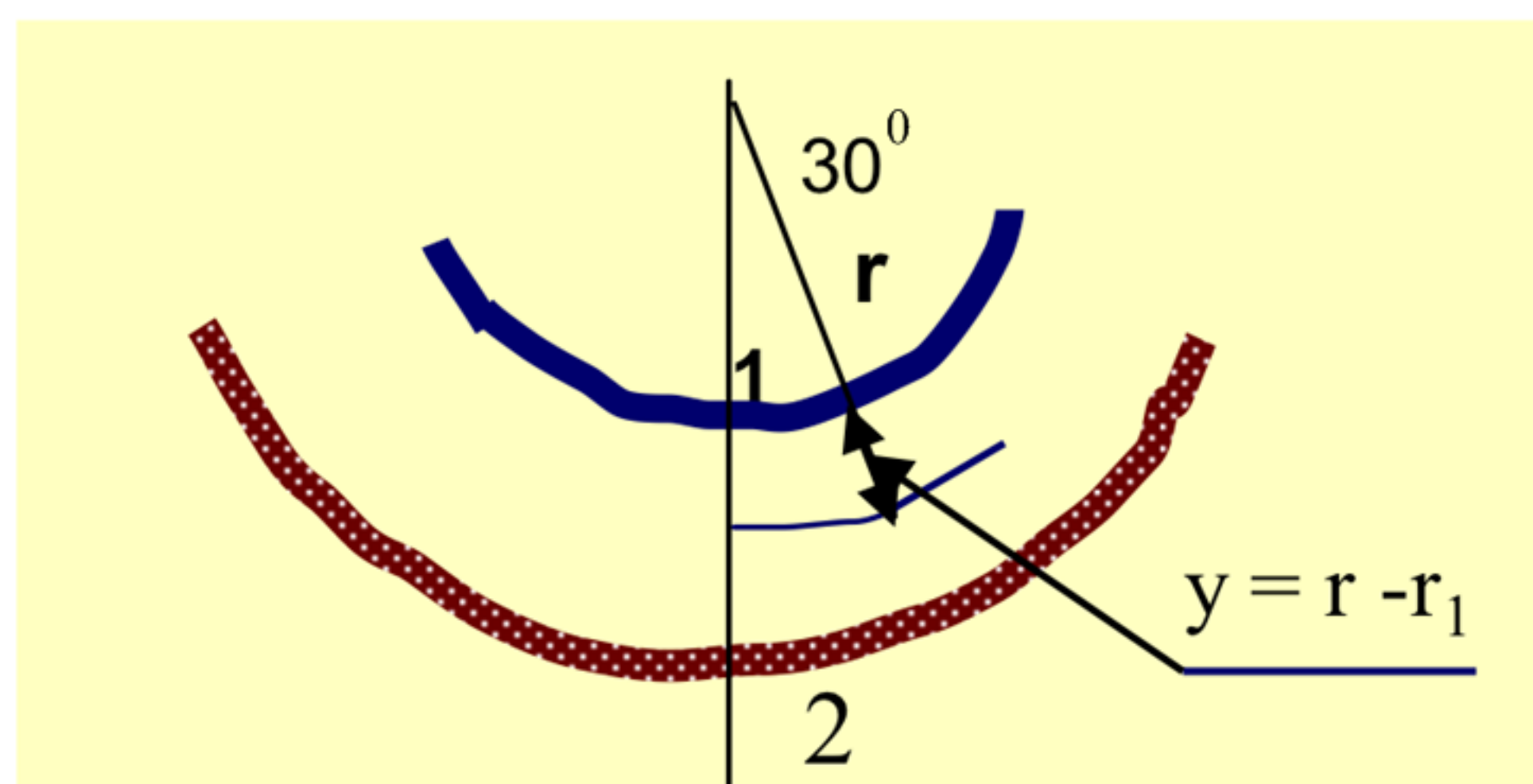


Fig. P-1.8.

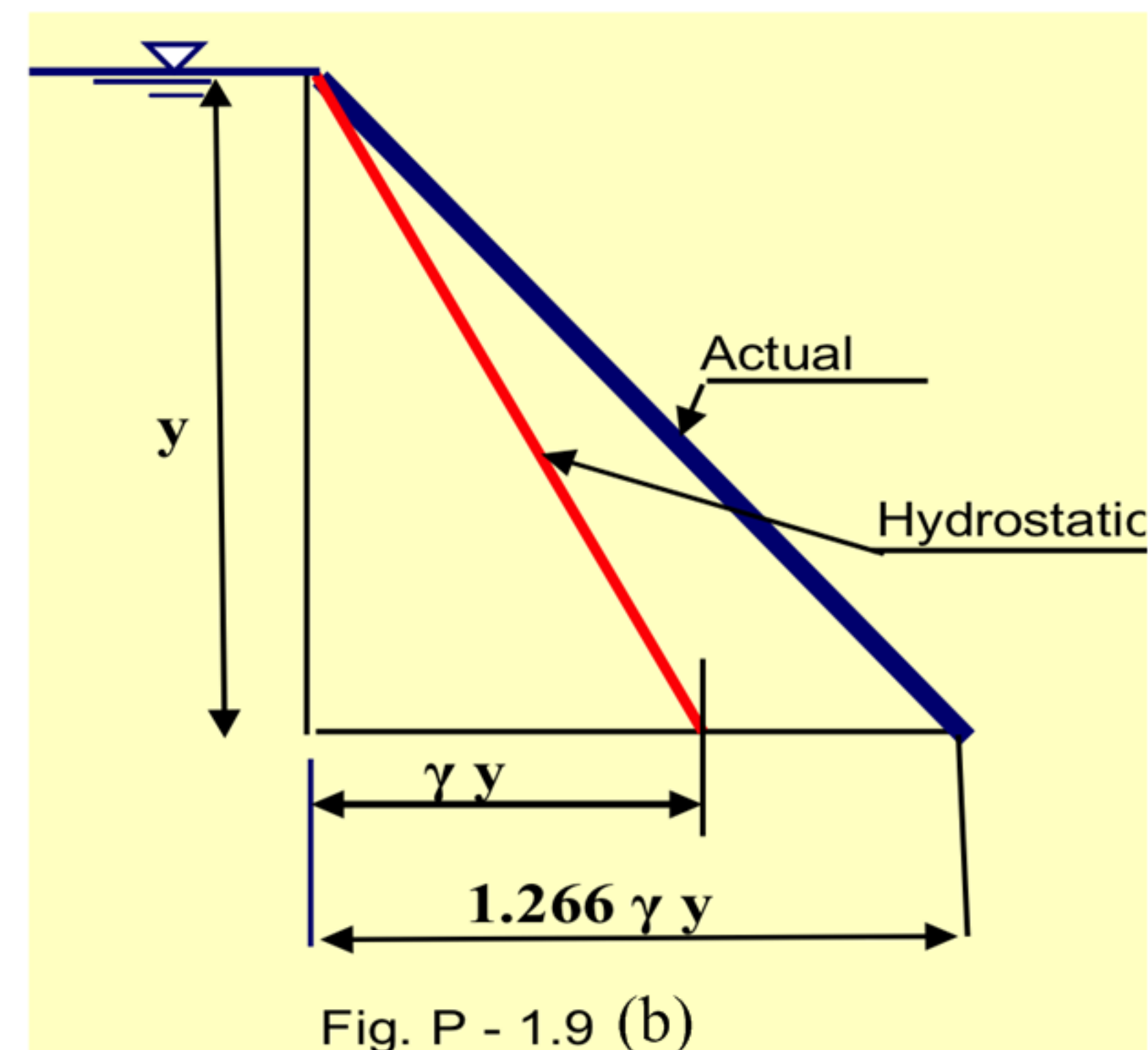
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$$\frac{p}{\gamma} = y \cos \theta + \frac{a_n}{g} y = 0.866y + 0.4y = 1.266y$$

Hydrostatic pressure distribution gives $\frac{p}{\gamma} = y$

Pressure at the bottom of the channel

$$\frac{p}{\gamma} = 1.266 \times 0.75 = \mathbf{0.95 \text{ m.}}$$



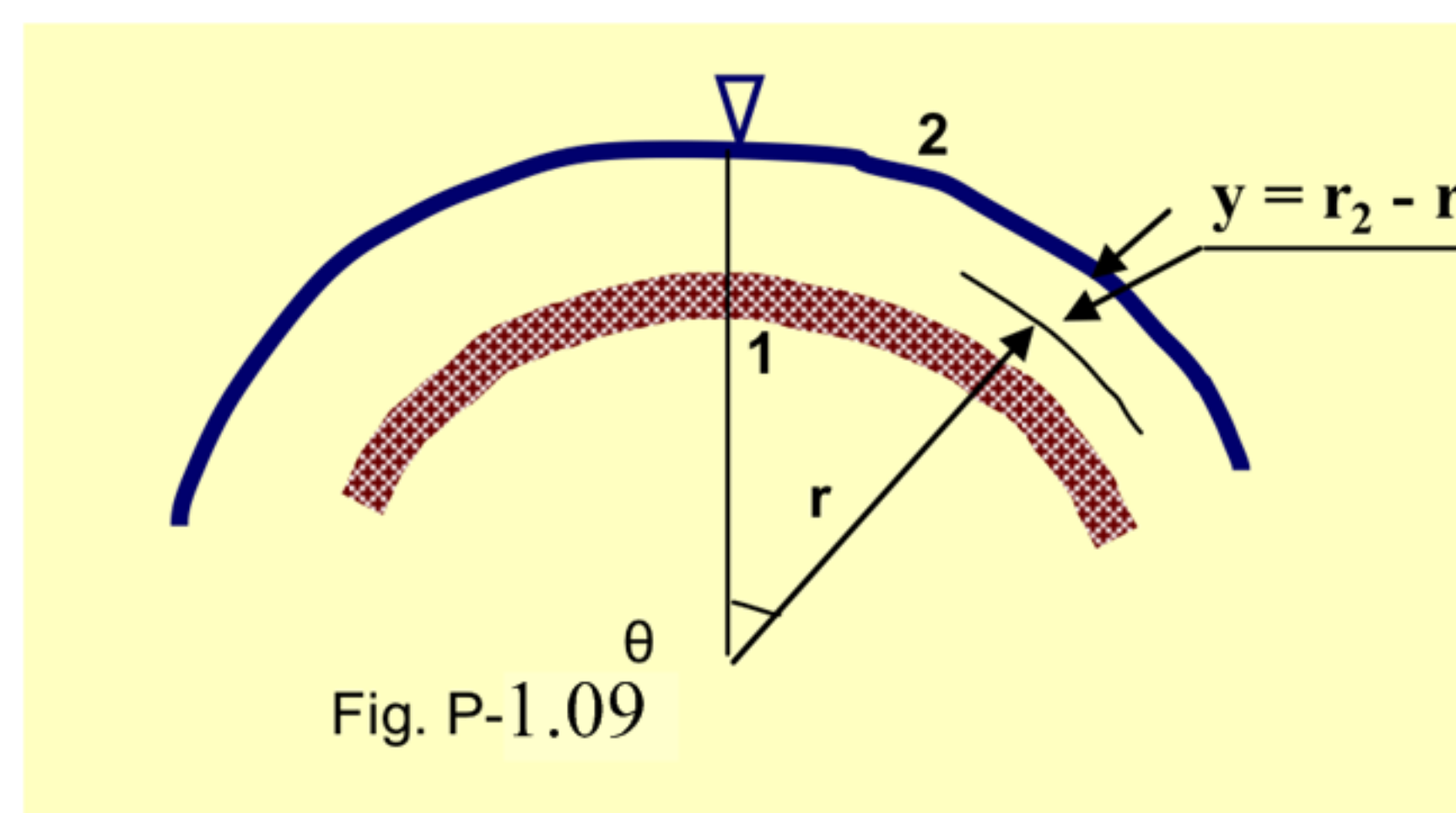
1.09 Refer to Fig. P- 1.09

$$\frac{p}{\gamma} = y \cos \theta - \frac{a_n}{g} y.$$

At point 1 on the crest,

$$\frac{p}{\gamma} = 0 = \text{atmospheric.}$$

$$\text{Then } \frac{p_1}{\gamma} = y_1 \cos \theta - \frac{a_n}{g} y_1 = 0 \quad \text{or} \quad a_n = g \cos \theta.$$



$$\begin{aligned} \left(\frac{p}{\gamma} + Z \right) &= \int \frac{v^2}{gr} + \text{const} = \int \frac{C^2}{gr^3} + \text{const} \\ &= \frac{C^2}{g} \left[-\frac{1}{2r^2} \right] + \text{const} \end{aligned}$$

At point 1 on the water surface $p_1 = 0$.

$$Z_1 = -\frac{C^2}{2gr_1^2} + \text{const.} \quad \text{or} \quad \text{const} = Z_1 + \frac{C^2}{2gr_1^2}$$

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$$\frac{p}{\gamma} = (Z_1 - Z) + \frac{C^2}{g} \left(\frac{1}{r_1^2} - \frac{1}{r^2} \right).$$

At a point h below the free surface, $(Z_1 - Z_2) = h \cos \theta$ and

$$\frac{p}{\gamma} = h \cos \theta + \frac{(v_1^2 - v_2^2)}{2g}$$

1.11

$$\begin{aligned} v = Cr \quad \text{and hence} \quad \left(\frac{p}{\gamma} + Z \right) &= \int \frac{C^2 r}{g} + \text{const} \\ &= \frac{C^2 r^2}{2g} + \text{const} \end{aligned}$$

At point 1 on the free surface $r = r_1$, $Z = Z_1$ and $\frac{p}{\gamma} = 0$.

$$\therefore \frac{p}{\gamma} = (Z_1 - Z) + \frac{C^2}{2g} (r^2 - r_1^2).$$

At a depth h below the free surface, $(Z_1 - Z_2) = h \cos \theta$ and hence

$$\frac{p}{\gamma} = h \cos \theta + \frac{(v_2^2 - v_1^2)}{2g}$$

1.12

$$\begin{aligned} q &= (0.10 \times 1.20 \cos 4^0) + (0.10 \times 1.00 \cos 8^0) + (0.10 \times 0.90 \cos 12^0) \\ &\quad + (0.10 \times 0.85 \cos 15^0) \\ &= \mathbf{0.3889 \text{ m}^3/\text{s}} \end{aligned}$$

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- 1.13 $B_1 = 3.0$ m, $y_1 =$ depth in 3.0 m channel = 2.0 m
 $B_2 = 2.5$ m, $y_2 =$ depth in 2.5 m channel, $\Delta Z =$ drop = 0.2 m
 Since the drop in water surface = 0.1 m
 $y_1 + 0.2 = y_2 + 0.1$
 $2.0 + 0.2 = y_2 + 0.1$ Hence $y_2 = 2.1$ m Considering the energy above the bed level of the downstream channel

$$\Delta Z + y_1 + \frac{V_1^2}{2g} = y_2 + \frac{V_2^2}{2g}$$

$$0.2 + 2.0 + \frac{V_1^2}{2g} = 2.1 + \frac{V_2^2}{2g}$$

$$\frac{(V_2^2 - V_1^2)}{2g} = 2.2 - 2.1 = 0.1$$

$$\text{Also, } B_1 y_1 V_1 = B_2 y_2 V_2$$

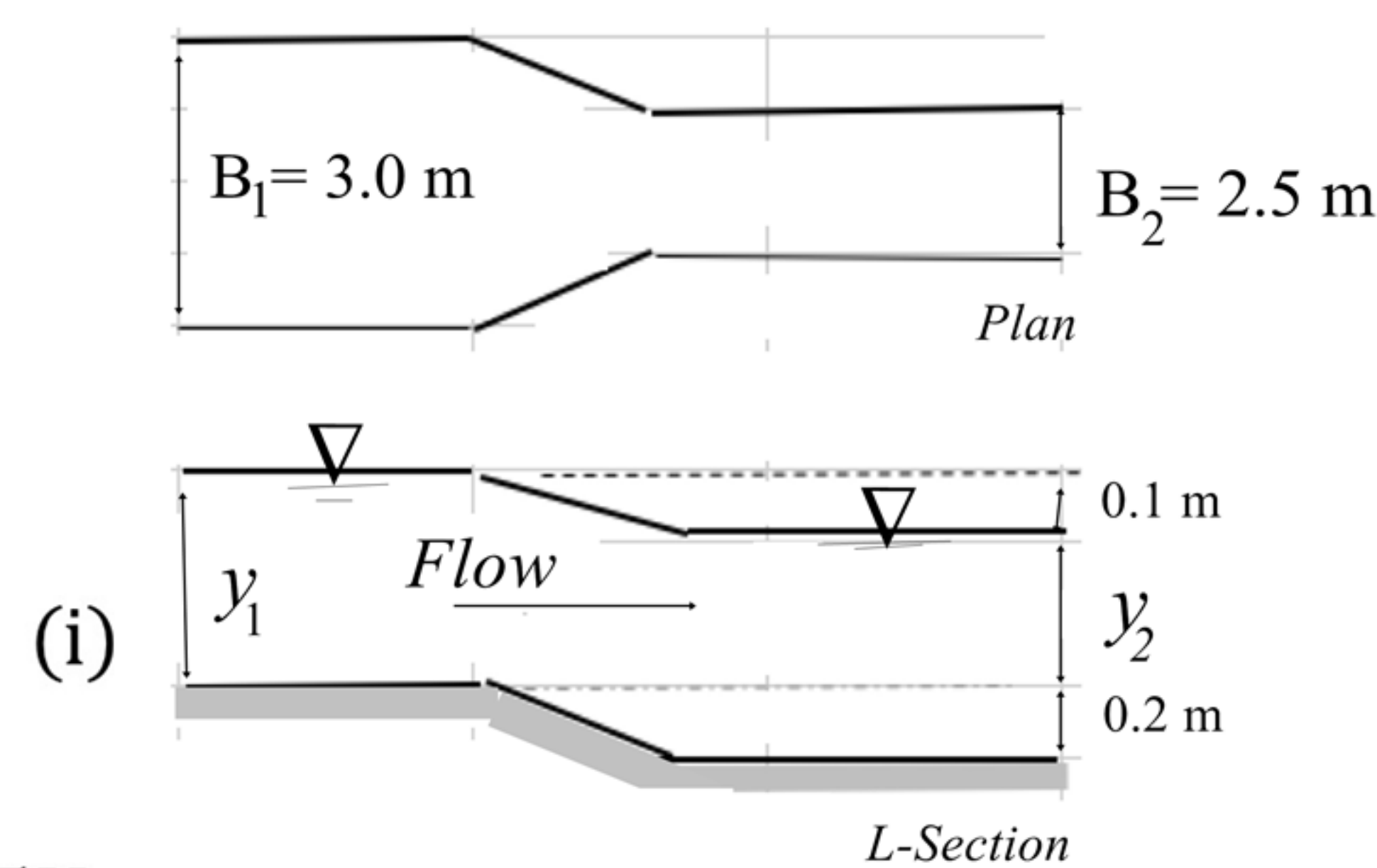
$$3.0 \times 2.0 \times V_1 = 2.5 \times 2.1 \times V_2 \quad \therefore V_1 = 0.875 V_2$$

$$\text{Substituting in Eq. (i), } \frac{V_2^2}{2g} (1 - (0.875)^2) = 0.1$$

$$V_2^2 = 2 \times 9.81 \times \frac{0.1}{0.2345} = 8.366$$

$$V_2 = 2.892 \text{ m/s and } V_1 = 2.531 \text{ m/s}$$

$$\text{Discharge } Q = A_1 V_1 = 3 \times 2 \times 2.531 = \mathbf{15.186 \text{ m}^3/\text{s}}$$



Schematic sketch of Problem 1.13

- 1.14 $Q_1 = 20 \text{ m}^3/\text{s}$, $\frac{\Delta y}{\Delta t} = 0.45 \text{ m/h} = \frac{0.45}{(60 \times 60)} \text{ m/s}$
 $T =$ top width = 25 m, $\Delta x = 1.5 \text{ km} = 1500 \text{ m}$
 $\frac{\partial Q}{\partial x} + T \frac{\partial y}{\partial t} = 0$
 $\frac{(Q_2 - Q_1)}{\Delta x} + T \frac{\Delta y}{\Delta t} = 0$
 $\frac{(Q_2 - 20)}{1500} + 25 \frac{0.45}{(60 \times 60)} = 0$
 $Q_2 = 20 - 25 \times \frac{0.45}{(60 \times 60)} \times 1500 = \mathbf{15.3125 \text{ m}^3/\text{s}}$

- 1.15 Hydraulic jump: $q = \frac{Q}{B} = \frac{44.30}{5.0} = 8.86 \text{ m}^3/\text{s/m}$
 $E_1 = y_1 + \frac{V_1^2}{2g} = 0.5 + \frac{(8.86)^2}{2 \times 9.81 \times (0.5)^2} = 16.50 \text{ m}$
 $E_2 = y_2 + \frac{V_2^2}{2g} = 0.5 + \frac{(8.86)^2}{2 \times 9.81 \times (5.4)^2} = 5.547 \text{ m}$
 Energy loss $E_L = E_1 - E_2 = 16.50 - 5.547 = \mathbf{10.95 \text{ m}}$

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1.16

Rectangular channel: $A_1 = 2.0 \times 1.5 = 3.0 \text{ m}^2$

$$V_1 = 10.0/3.0 = 3.33 \text{ m/s}$$

$$y_1 = 1.50 \text{ m, and } \frac{V_1^2}{2g} = 0.566 \text{ m}$$

$$E_1 = y_1 + \frac{V_1^2}{2g} = 2.066 \text{ m}$$

Trapezoidal channel: $A_2 = [3.0 + (1.5 \times 1.0)] = 4.5 \text{ m}^2$

$$V_2 = 10.0/4.5 = 2.222 \text{ m/s}$$

$$y_2 = 1.0 \text{ m, and } \frac{V_2^2}{2g} = 0.252 \text{ m}$$

Energy loss $E_L = 0.2 \left(\frac{V_1^2}{2g} - \frac{V_2^2}{2g} \right) = 0.063 \text{ m}$

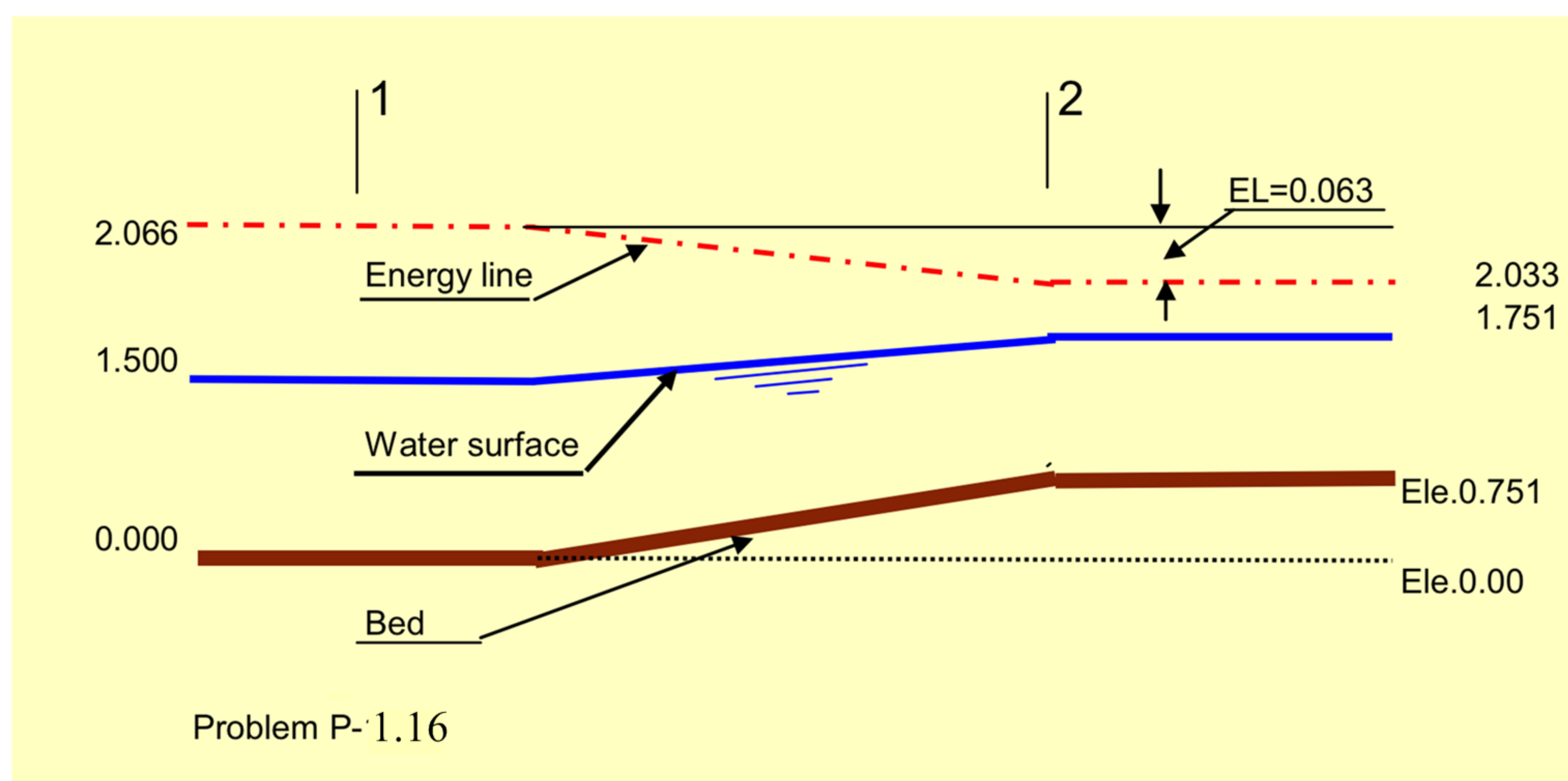


Fig. P-1.16 Schematic Features of the transition

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Refer to Fig. P-1.20. With bed level of section 1 as datum,

Elevation of energy line at 2 = $2.066 - 0.063 = 2.003$ m

Elevation of water surface at 2 = $2.003 - 0.252 = 1.751$ m

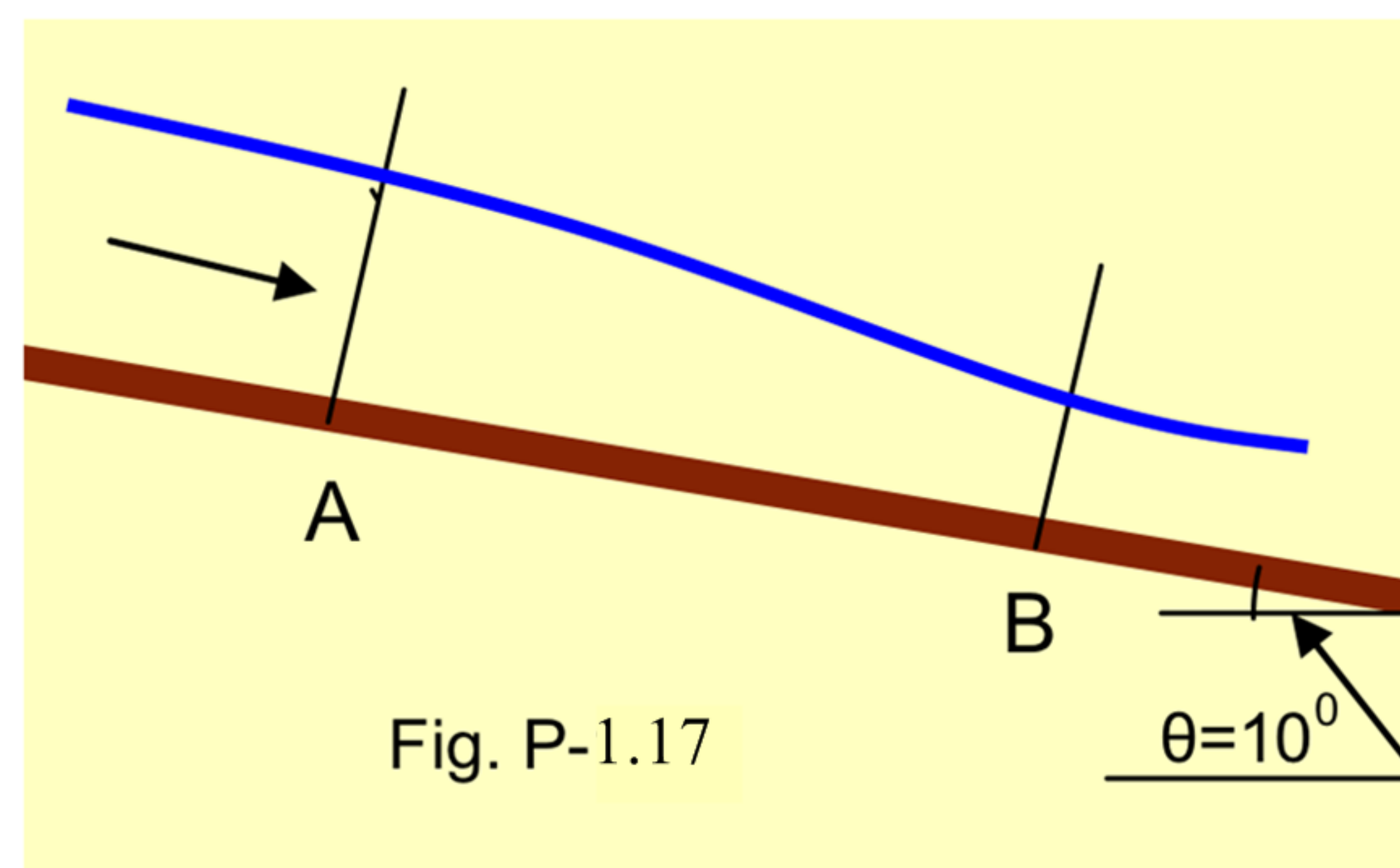
Elevation of bed at 2 = $1.751 - 1.000 = 0.751$ m

$$\Delta Z = \mathbf{0.751\ m} \text{ and } \Delta W_s = \mathbf{0.251\ m}$$

1.17

Refer to Fig. P-1.17.

$$H_A = Z_A + y_A \cos \theta + \alpha_A \frac{V_A^2}{2g}$$



$$H_A = 15.00 + 1.3 \cos 10^\circ + 1.03 \frac{(3.0)^2}{2 \times 9.81} = \mathbf{16.753\ m}$$

h_{pA} = elevation of piezometric head at A

$$Z_A + y_A \cos \theta = 15.00 + 1.3 \cos 10^\circ = \mathbf{16.280\ m}$$

$$\begin{aligned} \text{Similarly } H_B &= Z_B + y_B \cos \theta + \alpha_B \frac{V_B^2}{2g} \\ &= 14.60 + 1.2 \cos 10^\circ + 1.02 \frac{(V_B)^2}{2 \times 9.81} \end{aligned}$$

By continuity $y_A V_A = y_B V_B$,

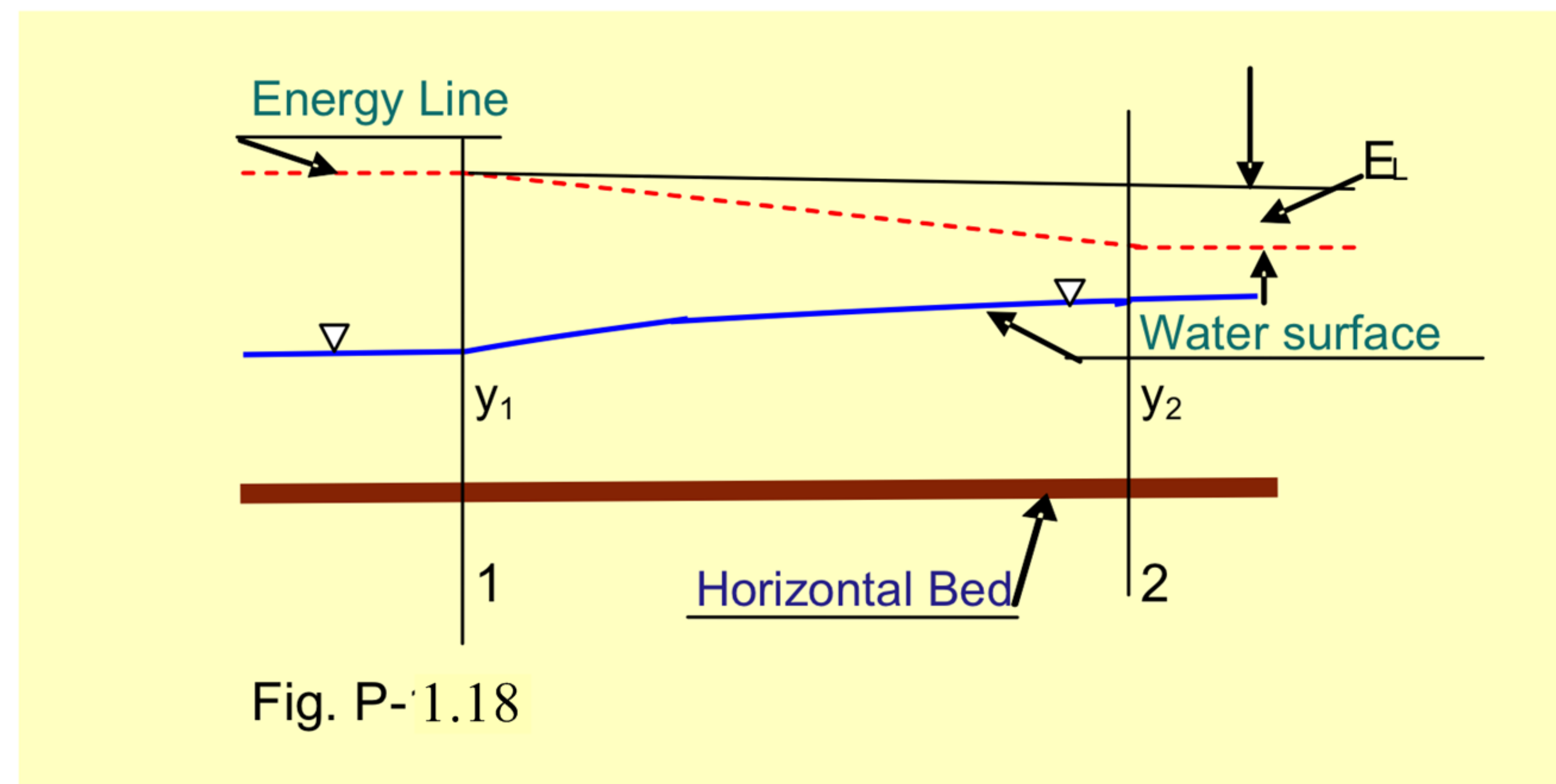
and hence $V_B = (1.3 \times 3.0)/1.2 = 3.25$ m/s

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$$H_B = 14.60 + 1.2 \cos 10^\circ + 1.03 \frac{(3.25)^2}{2 \times 9.81} = \mathbf{16.331 \text{ m}}$$

$$\begin{aligned} h_{pB} &= \text{elevation of piezometric head at } B \\ &= 14.60 + 1.2 \cos 10^\circ = \mathbf{15.782 \text{ m}} \end{aligned}$$

1.18 Refer to Fig. P-1.18



$$A_1 = 2.0 \times 1.2 = 2.40 \text{ m}^2$$

$$V_1 = 7.20/2.40 = 3.0 \text{ m/s}$$

$$\frac{V_1^2}{2g} = 0.459, y_1 = 1.20 \text{ m},$$

$$\alpha_1 = 1.05$$

$$H_1 = y_1 + \alpha_1 \frac{V_1^2}{2g} = 1.682 \text{ m}$$

$$\text{For section 2: } A_2 = 3.0 \times 1.4 = 4.20 \text{ m}^2$$

$$V_2 = 7.20/4.20 = 1.714 \text{ m/s}$$

$$\frac{V_2^2}{2g} = 0.150, y_2 = 1.40 \text{ m},$$

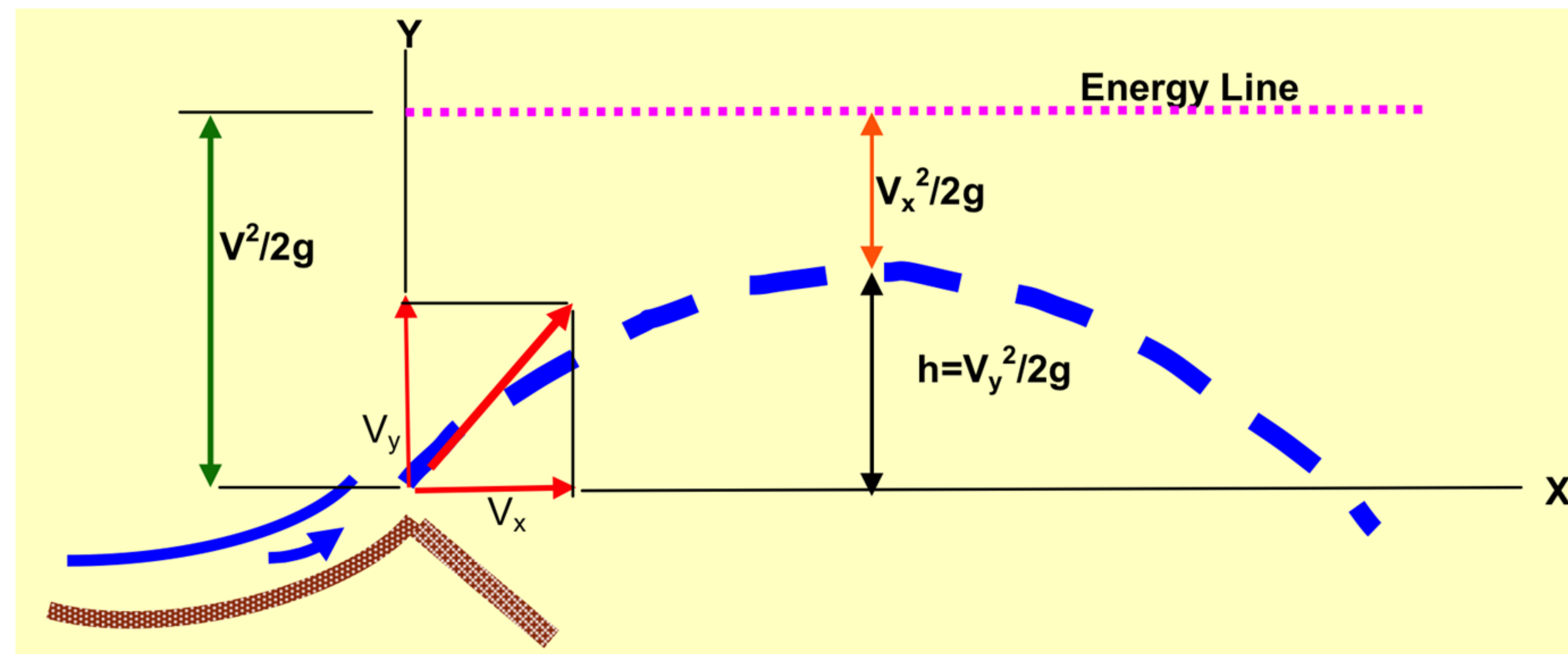
$$\alpha_2 = 1.15$$

$$H_2 = y_2 + \alpha_2 \frac{V_2^2}{2g} = 1.573 \text{ m}$$

$$\text{Energy loss} = E_L = 1.682 - 1.573 = \mathbf{0.109 \text{ m}}$$

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1.19 Refer to Fig. P-1.19. At the maximum point of the trajectory.



See Fig. P –1.19 Trajectory of liquid jet.

From this Fig.

$$h = \frac{V_y^2}{2g} = \frac{(V \sin \theta)^2}{2g} = \frac{(20 \times \sin 40^\circ)^2}{2 \times 9.81} = \mathbf{8.42 \text{ m}}$$

1.20 (a) $q = 2.5 \text{ m}^3/\text{s/m}$ and $y_2 = 0.30 \text{ m}$

$$V_2 = \frac{q}{y_2} = \frac{2.5}{0.30} = 8.333 \text{ m/s}, \quad V_1 = \frac{2.5}{y_1}$$

$$y_1 + \frac{V_1^2}{2g} = y_2 + \frac{V_2^2}{2g}$$

$$y_1 + \frac{(2.5)^2}{2 \times 9.81 \times y_1^2} = 0.30 + \frac{(8.333)^2}{2 \times 9.81} = 3.8395 \text{ m}$$

By trial and error $y_1 = \mathbf{3.818 \text{ m}}$

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1.20 (b) $q = 2.0 \text{ m}^3/\text{s}/\text{m} = V_1 y_1 = V_2 y_2$ and $y_1 = 4.0 \text{ m}$

$$y_1 + \frac{V_1^2}{2g} = y_2 + \frac{V_2^2}{2g}$$

$$4.00 + \frac{(2.0)^2}{2 \times 9.81 \times (4.0)^2} = y_2 + \frac{(2.0)^2}{2 \times 9.81 \times y_2^2}$$

$$y_2 + \frac{0.2039}{y_2^2} = 4.013 \text{ and by trial and error } y_2 = \mathbf{0.232 \text{ m}}$$

1.20 (c) $q = 2.0 \text{ m}^3/\text{s}/\text{m} = V_1 y_1 = V_2 y_2$ and $y_1 = 4.0 \text{ m}$

$$y_1 + \frac{V_1^2}{2g} = y_2 + \frac{V_2^2}{2g} + 0.1 \frac{V_2^2}{2g} = y_2 + 1.1 \frac{V_2^2}{2g}$$

$$4.00 + \frac{(2.0)^2}{2 \times 9.81 \times (4.0)^2} = y_2 + \frac{1.1 \times (2.0)^2}{2 \times 9.81 \times y_2^2}$$

$$y_2 + \frac{0.22427}{y_2^2} = 4.013 \text{ and by trial and error } y_2 = \mathbf{0.244 \text{ m}}$$

1.20 (d) $y_1 + \frac{q^2}{2gy_1^2} = y_2 + \frac{q^2}{2gy_2^2}$

$$(y_1 - y_2) = \frac{q^2}{2g} \left(\frac{1}{y_2^2} - \frac{1}{y_1^2} \right) = \frac{q^2}{2g} \cdot \frac{(y_1 + y_2)}{y_1^2 y_2^2}$$

$$q = y_1 y_2 \sqrt{\left(\frac{2g}{(y_1 + y_2)} \right)} = (3.0 \times 0.25) \sqrt{\frac{(2 \times 9.81)}{(3.0 + 0.25)}} = \mathbf{1.843 \text{ m}^3/\text{s}/\text{m}}$$

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Referring to Example 1.21, Force on a sluice gate during flow is evaluated by use of momentum equation as:

$$\begin{aligned} \frac{1}{2} \gamma y_1^2 - \frac{1}{2} \gamma y_2^2 - R &= \rho q^2 \left(\frac{y_1 - y_2}{y_1 y_2} \right) \\ R &= \frac{1}{2} \rho g (y_1^2 - y_2^2) - \rho q^2 \left(\frac{y_1 - y_2}{y_1 y_2} \right) \\ &= \rho (y_1 - y_2) \left\{ \frac{g}{2} (y_1 + y_2) - \frac{q^2}{(y_1 y_2)} \right\} \quad (1) \end{aligned}$$

The force on the gate is equal and opposite to R . Values of R (and hence F) are calculated by using Eq.1 for all the four cases of problem 1.20 and are listed below:

y_1 (m)	y_2 (m)	q	F (Force per metre width of gate (kN/m))
3.818	0.300	2.500	51.76
4.000	0.232	2.000	61.85
4.000	0.244	2.000	62.67
3.000	0.250	1.843	31.32