FACILITIES DESIGN

5th Edition Solutions

CHAPTER-1

1.4 Solutions

1. This could be given as a project for students. The article by Suskind (1989) in IE Magazine is very helpful in conducting an operations review – a crucial step that must be undertaken before a facility decision is made.

2. See Section 1.1.

CHAPTER – 2

2.6 Solutions

CHAPTER-3

3.10 Solutions

1. As indicated in Section 3.1 as well as Chapter 1, facility location must be addressed in the very early stages of facility design – as soon as the products to be produced and markets to be served have been identified.
2. Facility location is a sub-problem in logistics management. As defined in Section 3.1, logistics management covers not only location, allocation, but also the distribution methods, costs, and a number of other logistics related factors.
3. It should serve the needs of the company, minimize costs, must mesh well with the existing network of supply chain of suppliers, plants and customers, etc. This is a good question for in-class discussion.
4. See Section 3.1.
5. See Section 3.1.
6. See Table 3.1.
7. This is an interesting project for undergraduate students.
8. Based on the calculations done in the table below, we find that site #6 is the best location based on the subjective factors.

	Location Factors				
	0.15	0.2	0.4	0.25	
Site Number	1	2	3	4	Score
1	90	40	80	70	71.00
2	100	30	80	50	65.50
3	50	80	85	60	72.50
4	60	90	60	75	69.75
5	90	30	65	80	65.50
6	90	30	85	85	74.75

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12. It is a trivial exercise for the student to verify the results in Example 3.2 using a transportation problem.

13. It is a trivial exercise for the student to verify the results in Example 3.3 using a transportation problem.

14. It is a trivial exercise for the student to verify the results in Example 3.3 using a transportation problem.

15. Setting up the transportation problem for the two cases, we get the solutions shown below. Because the site 1 solution has a cost of $56,000 and site 2 a cost of $62,000, we choose site 1.

Distribution pattern with new facility at site 1

	Buffalo	Syracuse	Rochester	Supply Capacity
Ex. Plant 1			1000	1000
Ex. Plant 2	1000		1000	2000
New Site 1	2000	3000		5000
Demand	3000	3000	2000	

Distribution pattern with new facility at site 2

	Buffalo	Syracuse	Rochester	Supply Capacity
Ex. Plant 1			1000	1000
Ex. Plant 2		1000	1000	2000
New Site 2	3000	2000		5000
Demand	3000	3000	2000	

16. It is a trivial exercise for the student to verify the results in Exercise 15 using a transportation problem.

17. Once again, setting up the transportation problem for the three cases, we get the solutions shown below. The student must add the handling cost per pallet to the unit transportation costs. Because the distribution pattern is the same for all three cases, we show only one table assuming the plant is at Los Angeles. Phoenix has a cost of $2017.60, Los Angeles a cost of $1713.60, and Las Vegas a cost of $1717.6, we therefore choose Los Angeles. Of course, because the Los Angeles and Las Vegas locations have such a close total distribution cost, we must include other subjective factors before making a final decision.

Distribution pattern with new hub at Los Angeles

	A	R	P	U	SF	Supply Capacity
D	33		15			50
H	2			18		20
B		20	5			25
LA					40	40
Demand	35	20	20	18	40	

18. It is a trivial exercise for the student to verify the results in Exercise 17 using a transportation problem.
19. Scanning each row for the maximum element and then choosing the minimum of these, we find that the optimal location of the library (to minimize the maximum distance traveled by people in any population zone) is location 3. Then, the maximum distance traveled by people in any population zone is 14 units.
20. See Section 3.3.3. Another question for in-class discussion.
21. Based on the calculations done in the table below, we find that site #3 is the best location based on subjective and objective factors. Because we considered the latter and critical factors, it turned out that site #6 was not the best location. Once again, we have site #4 that is a close second-best location.

	Critical Factors		Objective Factors			Subjective Factors				
						0.15	0.2	0.4	0.25	
Site No	1	2	1	2		1	2	3	4	Score
1	1	0	-100	85	-15	90	40	80	70	0.00
2	1	1	-175	185	10	100	30	80	50	26.22
3	1	1	-110	20	-90	50	80	85	60	29.23
4	1	1	-200	60	-140	60	90	60	75	28.23
5	1	1	-350	80	-270	90	30	65	80	26.80
6	0	1	-80	100	20	90	30	85	85	0.00

22. When the subjective factors in Exercise 14 are worth twice that of the objective factors we still get the same result as in Exercise 14, but not Exercise 8 (see the table below).

	Critical Factors		Objective Factors			Subjective Factors				
						0.15	0.2	0.4	0.25	
Site No	1	2	1	2		1	2	3	4	Score
1	1	0	-100	85	-15	90	40	80	70	0.00
2	1	1	-175	185	10	100	30	80	50	39.31
3	1	1	-110	20	-90	50	80	85	60	43.65
4	1	1	-200	60	-140	60	90	60	75	42.07
5	1	1	-350	80	-270	90	30	65	80	39.70
6	0	1	-80	100	20	90	30	85	85	0.00

23. Based on the calculations done in the table below, we find that site C is the best location based on subjective and objective factors.

	Objective Factor	Subjective Factors				
		0.2	0.3	0.3	0.2	
Site		1	2	3	4	Score
A	3000000	0.5	1	0.25	0.75	0.77
B	4500000	0.25	0.75	0.25	1	0.27
C	2500000	0.75	0.5	0.5	0.25	0.90
D	2800000	0.25	0.75	0.5	1	0.83
E	5000000	0.25	1	0.5	0.5	0.12
F	3500000	1	0.5	1	0.5	

24. See Section 3.4. An interesting modeling related question for in-class discussion.
25. Given the explanation in Section 3.4.1, it is a trivial exercise to provide the required "proof".
26. Once again, given the explanation in Section 3.4.1, it is a trivial exercise to provide the required "proof".
27. Arranging the x coordinates in increasing order and calculating the cumulative trips, we find the optimal x coordinate is 5. Similarly, rearranging the y coordinates in increasing order and calculating the cumulative trips, we find the optimal y coordinate is 2 (see the tables below).

Machine #	x coordinate	y coordinate	Cumulative trips to machines
2	2	10	10
4	5	1	16
1	10	10	22
3	10	2	32

Machine #	x coordinate	y coordinate	Cumulative trips to machines
4	5	1	6
3	10	2	16
1	10	10	22
2	2	10	32

28. If the number of trips between each existing machine and the new one is 10 units each, using the median method, we find that the optimal x and y coordinates are 5, 2 - the same answer as in Exercise 20 (see the tables below).

Machine #	x coordinate	y coordinate	Cumulative trips to machines
1	2	10	10
2	5	1	20
3	10	10	30
4	10	2	40

Machine #	x coordinate	y coordinate	Cumulative trips to machines
1	5	1	10
2	10	2	20
3	10	10	30
4	2	10	40

29. Arranging the x coordinates in increasing order and calculating the cumulative trips, we find the optimal x coordinate is 32. Similarly, rearranging the y coordinates in increasing order and calculating the cumulative trips, we find the optimal y coordinate is 58 (see the tables below). The training facility should therefore be located in Center 1.

Center	x coordinate	y coordinate	Number of Operators
3	11	69	6000
1	32	58	12800
4	42	31	16200
2	49	42	20500

Center	x coordinate	y coordinate	Cumulative Number of Operators
4	42	31	3400
2	49	42	7700
1	32	58	14500
3	11	69	20500

30a. Arranging the x coordinates in increasing order and calculating the cumulative trips, we find the optimal x coordinate is 8. Similarly, rearranging the x coordinates in increasing order and calculating the cumulative trips, we find the optimal y coordinate is 12 (see the tables below). The service center must be located near customer 1.

Customer	x coordinate	y coordinate	Cumulative Number of Visits
2	2	15	350
1	8	12	850
4	8	10	1600
3	9	18	2150

Customer	x coordinate	y coordinate	Cumulative Number of Visits
4	8	10	750
1	8	12	1250
2	2	15	1600
3	9	18	2150

30b. The LINDO input and output are shown below. The answer is the same as in Exercise 23a.

MIN 500 XP1 + 500 XN1 + 500 YP1 + 500 YN1 + 350 XP2 + 350 XN2
+ 350 YP2 + 350 YN2 + 550 XP3 + 550 XN3 + 550 YP3 + 550 YN3 + 750 XP4
+ 750 XN4 + 750 YP4 + 750 YN4

SUBJECT TO

C2) XP1 - XN1 + XBAR = 8

C3) XP2 - XN2 + XBAR = 2

C4) XP3 - XN3 + XBAR = 9

C5) XP4 - XN4 + XBAR = 8

C6) YP1 - YN1 + YBAR = 12

C7) YP2 - YN2 + YBAR = 15

C9) YP4 - YN4 + YBAR = 10

END

LP OPTIMUM FOUND AT STEP 9

OBJECTIVE FUNCTION VALUE

1) 8500.000

VARIABLE	VALUE	REDUCED COST
XP1	.000000	.000000
XN1	.000000	1000.000000
YP1	.000000	650.000000
YN1	.000000	350.000000
XP2	.000000	700.000000
XN2	6.000000	.000000
YP2	3.000000	.000000
YN2	.000000	700.000000
XP3	1.000000	.000000
XN3	.000000	1100.000000
YP3	6.000000	.000000
YN3	.000000	1100.000000
XP4	.000000	1450.000000
XN4	.000000	50.000000

YP4	.000000	1500.000000
YN4	2.000000	.000000
XBAR	8.000000	.000000
YBAR	12.000000	.000000

NO. ITERATIONS= 9

31. The LINDO input and output are shown below. The answer is the same as in Exercise 20.

MIN 6 XP1 + 6 XN1 + 6 YP1 + 6 YN1 + 10 XP2 + 10 XN2 + 10 YP2+ 10 YN2 + 10 XP3 + 10 XN3 + 10 YP3 + 10 YN3 + 6 XP4 + 6 XN4 + 6 YP4 + 6 YN4

SUBJECT TO

C2) XP1 - XN1 + XBAR = 10

C3) XP2 - XN2 + XBAR = 2

C4) XP3 - XN3 + XBAR = 10

C5) XP4 - XN4 + XBAR = 5

C6) YP1 - YN1 + YBAR = 10

C7) YP2 - YN2 + YBAR = 10

C8) YP3 - YN3 + YBAR = 2

C9) YP4 - YN4 + YBAR = 1

END

LP OPTIMUM FOUND AT STEP 9

OBJECTIVE FUNCTION VALUE

1) 244.0000

VARIABLE	VALUE	REDUCED COST
XP1	5.000000	.000000
XN1	.000000	12.000000

YP1	8.000000	.000000
YN1	.000000	12.000000
XP2	.000000	20.000000
XN2	3.000000	.000000
YP2	8.000000	.000000
YN2	.000000	20.000000
XP3	5.000000	.000000
XN3	.000000	20.000000
YP3	.000000	20.000000
YN3	.000000	.000000
XP4	.000000	12.000000
XN4	.000000	.000000
YP4	.000000	12.000000
YN4	1.000000	.000000
XBAR	5.000000	.000000
YBAR	2.000000	.000000

NO. ITERATIONS= 9

32. The LINDO input and output are shown below. Once again, the answer is the same as in Exercise 21.

```
MIN     10 XP1 + 10 XN1 + 10 YP1 + 10 YN1 + 10 XP2 + 10 XN2 + 10 YP2
    + 10 YN2 + 10 XP3 + 10 XN3 + 10 YP3 + 10 YN3 + 10 XP4 + 10 XN4
    + 10 YP4 + 10 YN4
SUBJECT TO
     C2)   XP1 - XN1 + XBAR =    10
     C3)   XP2 - XN2 + XBAR =    2
     C4)   XP3 - XN3 + XBAR =    10
     C5)   XP4 - XN4 + XBAR =    5
     C6)   YP1 - YN1 + YBAR =    10
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C7) YP2 - YN2 + YBAR = 10

C8) YP3 - YN3 + YBAR = 2

C9) YP4 - YN4 + YBAR = 1

END

LP OPTIMUM FOUND AT STEP 10

OBJECTIVE FUNCTION VALUE

1) 300.0000

VARIABLE	VALUE	REDUCED COST
XP1	5.000000	.000000
XN1	.000000	20.000000
YP1	8.000000	.000000
YN1	.000000	20.000000
XP2	.000000	20.000000
XN2	3.000000	.000000
YP2	8.000000	.000000
YN2	.000000	20.000000
XP3	5.000000	.000000
XN3	.000000	20.000000
YP3	.000000	20.000000
YN3	.000000	.000000
XP4	.000000	20.000000
XN4	.000000	.000000
YP4	.000000	20.000000
YN4	1.000000	.000000
XBAR	5.000000	.000000
YBAR	2.000000	.000000

NO. ITERATIONS= 10

33. This is a trivial exercise for the student because we have already proved this for the x_i^+ and x_i^- variables using Equation (18).

34. The revised table is shown below.

Centroid Coordinates and Average Number of Trips to Copiers

Department #	x coordinate	y coordinate	Average number of daily trips to copiers
1	10	2	6
2	10	10	20
3	8	6	8
4	12	5	4

Using the median method, we obtain the optimal x and y coordinates as 10, 10.

Department #	x coordinates in non-decreasing order	Weights	Cumulative weights
3	8	8	8
1	10	6	14
2	10	20	34
4	12	4	38

Department #	y coordinates in non-decreasing order	Weights	Cumulative weights
1	2	6	6
4	5	4	10
3	6	8	18
2	10	20	38

35. Arranging the x coordinates in increasing order and calculating the cumulative trips, we find the optimal x coordinate is 10. Similarly, rearranging the y coordinates in increasing order and calculating the cumulative trips, we find the optimal y coordinate is 10 (see the tables below).

Machine #	x coordinate	y coordinate	Cumulative trips to machines
2	2	10	10
4	5	1	16
1	10	10	46
3	10	2	56

Machine #	x coordinate	y coordinate	Cumulative trips to machines
4	5	1	6
3	10	2	16
1	10	10	46
2	2	10	56

Because the optimal location coincides with that of an existing facility, we use the contour method to determine alternate feasible locations.

Step 1: The vertical and horizontal lines V_1, V_2, V_3 and H_1, H_2, H_3 are drawn as shown in the following figure. In addition to these lines, we also draw lines V_0, V_4 and H_0, H_4 so that the "exterior" regions can be identified.

Step 2: The weights V1, V2, V3, H1, H2, H3 are calculated by adding the weights of the points that fall on the respective lines. Note that for this example, p = 3, and q = 3.

Step 3: Since $\sum_{i=1}^{4} w_i = 56$, set $N_0 = D_0 = -56$

Step 4: Set

$N_1 = -56 + 2(10) = -36$; $D_1 = -56 + 2(6) = -44$

$N_2 = -36 + 2(6) = -24$; $D_2 = -44 + 2(10) = -24$

$N_3 = -24 + 2(40) = 56$; $D_3 = -24 + 2(40) = 56$

(These values are entered at the bottom of each column and left of each row in the following figure.)

Step 5: Compute the slope of each region.

S_{00}=-(-56/-56) =-1; S_{01}=-(-56/-44) =-1.27; S_{02}=-(-56/-24) =-2.33;

S_{03}=-(-56/56) =1; S_{10}=-(-36/-56) =-0.64; S_{11}=-(-36/-44) =-0.82;

S_{12}=-(-36/-24) =-1.5; S_{13}=-(-36/56) =0.64; S_{20}=-(-24/-56) =-0.43;

S_{21}=-(-24/-44) =-0.55; S_{22}=-(-24/-24) =-1; S_{23}=-(-24/56) =0.43;

S_{30}=-(56/-56) =1; S_{31}=-(56/-44) =1.27; S_{32}=-(56/-24) =2.33;

S_{33}=-(56/56) =-1

(The above slope values are shown inside each region.)

Step 6: Using the slopes, it is easy for the student to draw a contour line through any point, for example, (9,10) and obtain a region of points with the same or lower cost as done in Example 5.

H_4	D_4=56	1	0.64	0.43	-1	
H_3=40	D_2=-24	-2.33	-1.5	-1	2.33	
H_2=10	D_1=-44	-1.27	-0.82	-0.55	1.27	
H_1=6	D_0=-56	-1	-0.64	-0.43	1	
H_0		N_0=-56	N_1=-36	N_2=-24	N_3=56	
		0	=10	=6	=40	4

36. The optimal x and y coordinates determined using the Gravity Method are shown in the table below.

Place	xi	yi	wi	wi*xi	wi*yi
35. 36. Machine 1	37. 38. 10	39. 40. 10	41. 42. 6	43. 44. 60	45. 46. 60
Machine 2	2	10	10	20	100
Machine 3	10	2	10	100	20
Machine 4	5	1	6	30	6
			32	210	186
Optimal x		**6.56**			
Optimal y		**5.81**			

37. If squared Euclidean distance metric is more appropriate for Exercises 21, 27 and 29, here are the optimal x and y coordinates.

Exercise 21

Place	xi	yi	wi	wi*xi	wi*yi
Machine 1	10	10	10	100	100
Machine 2	2	10	10	20	100
Machine 3	10	2	10	100	20
Machine 4	5	1	10	50	10
			40	270	230
Optimal x		**6.75**			
Optimal y		**5.75**			

Exercise 27

Place	xi	yi	wi	wi*xi	wi*yi
Machine 1	10	2	6	60	12
Machine 2	10	10	20	200	200
Machine 3	8	6	8	64	48
Machine 4	12	5	4	48	20
			38	372	280
Optimal x		**9.79**			
Optimal y		**7.37**			

Exercise 29

Place	xi	yi	wi	wi*xi	wi*y
Machine 1	10	10	30	300	300
Machine 2	2	10	10	20	100
Machine 3	10	2	10	100	20
Machine 4	5	1	6	30	6
			56	450	426
Optimal x		**8.04**			
Optimal y		**7.61**			

38. The optimal x and y coordinates determined using Weizfield Method are shown in the table below.

Place	xi	yi	wi	wi*xi	wi*yi		
Mach 1	10	10	6	60	60		
Mach 2	2	10	10	20	100		
Mach 3	10	2	10	100	20		
Mach 4	5	1	6	30	6		
			32	210	186		
Initial x		6.56					
Initial y		5.81					
Iteration #	x*	y*	B+C				TC
0	6.56	5.81					
1	6.78	5.52	1.11	1.61	1.95	1.19	175.46
2	6.91	5.31	1.09	1.53	2.10	1.24	175.16
3	6.99	5.16	1.07	1.47	2.21	1.27	175.03
4	7.04	5.06	1.05	1.44	2.29	1.30	174.97
5	7.07	5.00	1.04	1.42	2.35	1.32	174.94
6	7.09	4.95	1.04	1.41	2.39	1.33	174.92
7	**7.11**	**4.92**	1.03	1.40	2.41	1.34	**174.92**

39. If squared Euclidean distance metric is more appropriate for Exercises 21, 27 and 29, here are the optimal x and y coordinates.

<u>Exercise 21</u>

Place	xi	yi	wi	wi*xi	wi*yi		
Mach 1	10	10	10	100	100		
Mach 2	2	10	10	20	100		
Mach 3	10	2	10	100	20		
Mach 4	5	1	10	50	10		
			40	270	230		
Initial x		6.75					
Initial y		5.75					

Iteration #	x*	y*	B+C				TC
0	6.75	5.75					
1	6.98	5.44	1.87	1.57	2.02	1.98	216.60
2	7.07	5.23	1.83	1.48	2.19	2.06	216.30
3	7.11	5.10	1.79	1.44	2.29	2.12	216.18
4	7.12	5.01	1.76	1.41	2.36	2.17	216.13
5	7.13	4.96	1.74	1.40	2.40	2.20	216.11
6	7.14	4.92	1.72	1.39	2.43	2.23	216.10
7	7.14	4.90	1.71	1.38	2.44	2.24	216.10
8	**7.14**	**4.88**	1.71	1.38	2.46	2.25	**216.09**

Exercise 27

Place	xi	yi	wi	wi*xi	wi*yi
Mach 1	10	2	6	60	12
Mach 2	10	10	20	200	200
Mach 3	8	6	8	64	48
Mach 4	12	5	4	48	20
			38	372	280
Initial x		9.79			
Initial y		7.37			

Iteration #	x*	y*	B+C				TC
0	9.79	7.37					
1	9.66	7.83	1.12	7.58	3.55	1.23	113.45
2	9.70	8.15	1.03	9.08	3.25	1.09	111.93
3	9.76	8.43	0.97	10.70	2.91	1.02	110.78
4	9.80	8.66	0.93	12.56	2.67	0.98	109.86
5	9.84	8.85	0.90	14.73	2.49	0.94	109.12

6	9.86	9.02	0.88	17.26	2.36	0.91	108.51
7	9.89	9.16	0.85	20.23	2.25	0.88	108.01
8	9.90	9.29	0.84	23.70	2.17	0.86	107.59
9	9.92	9.39	0.82	27.79	2.11	0.84	107.24
10	9.93	9.48	0.81	32.58	2.05	0.82	106.95
11	9.94	9.56	0.80	38.22	2.01	0.81	106.70
12	9.95	9.62	0.79	44.84	1.97	0.80	106.49
13	9.96	9.68	0.79	52.63	1.94	0.79	106.31
14	9.97	9.73	0.78	61.79	1.92	0.78	106.16
15	9.97	9.77	0.78	72.57	1.90	0.78	106.03
16	9.98	9.80	0.77	85.24	1.88	0.77	105.93
17	9.98	9.83	0.77	100.16	1.87	0.77	105.83
18	9.98	9.86	0.77	117.70	1.86	0.76	105.76
19	9.98	9.88	0.76	138.34	1.85	0.76	105.69
20	9.99	9.90	0.76	162.63	1.84	0.76	105.63
21	9.99	9.91	0.76	191.20	1.83	0.76	105.59
22	9.99	9.92	0.76	224.81	1.82	0.75	105.55
23	9.99	9.94	0.76	264.36	1.82	0.75	105.51

24	9.99	9.95	0.76	310.89	1.81	0.75	105.48
25	9.99	9.95	0.76	365.64	1.81	0.75	105.46
26	10.00	9.96	0.75	430.05	1.81	0.75	105.44
27	**10.00**	**9.97**	0.75	505.84	1.80	0.75	**105.42**

Exercise 29

Place	xi	yi	wi	wi*xi	wi*yi
Mach 1	10	10	30	300	300
Mach 2	2	10	10	20	100
Mach 3	10	2	10	100	20
Mach 4	5	1	6	30	6
			56	450	426
Initial x		8.04			
Initial y		7.61			

1	8.80	8.48	9.69	1.54	1.68	0.83	244.00
2	9.21	9.03	15.50	1.43	1.52	0.72	235.38
3	9.48	9.37	24.05	1.37	1.41	0.66	230.40
4	9.66	9.59	36.81	1.33	1.35	0.63	227.34
5	9.77	9.73	55.91	1.30	1.32	0.61	225.41
6	9.85	9.82	84.56	1.29	1.29	0.60	224.17
7	9.90	9.88	127.56	1.27	1.28	0.60	223.35
8	9.93	9.92	192.11	1.27	1.27	0.59	222.82
9	9.96	9.95	289.03	1.26	1.26	0.59	222.47
10	9.97	9.96	434.55	1.26	1.26	0.59	222.24
11	**9.98**	**9.98**	653.05	1.25	1.26	0.59	**222.08**

40. If Euclidean distance metric is more appropriate for Exercise 23, here are the optimal x and y coordinates.

Place	xi	yi	wi	wi*xi	wi*yi
Mach 1	8	12	500	4000	6000
Mach 2	2	15	350	700	5250
Mach 3	9	18	550	4950	9900
Mach 4	8	10	750	6000	7500
			2150	15650	28650
Initial x		7.28			
Initial y		13.33			

Iteration #	x*	y*	B+C				TC
0	7.28	13.33					
1	7.63	12.57	331.36	63.20	110.42	220.41	7512.08
2	7.79	12.15	738.21	57.07	98.16	289.17	7293.87
3	7.90	12.01	1941.88	54.22	92.14	346.64	7222.80
4	7.96	11.99	5197.08	52.88	90.31	372.87	7204.78
5	7.98	12.00	12342.09	52.42	90.21	376.42	7198.34
6	7.99	12.00	27858.78	52.28	90.31	375.78	7195.60

41. This is an interesting semester long project for undergraduate as well as graduate students.