CHAPTER 1

P. E. 1.1

(a) $\boldsymbol{A}+\boldsymbol{B}=(1,0,3)+(5,2,-6)=(6,2,-3)$

$$|\boldsymbol{A}+\boldsymbol{B}|=\sqrt{36+4+9}=\underline{\underline{7}}$$

(b) $5\boldsymbol{A}-\boldsymbol{B}=(5,0,15)-(5,2,-6)=\underline{\underline{(0,-2,21)}}$

(c) The component of $\boldsymbol{A}$ along $\mathbf{a}_y$ is $A_y = \underline{\underline{0}}$

(d) $3\boldsymbol{A}+\boldsymbol{B}=(3,0,9)+(5,2,-6)=(8,2,3)$

A unit vector parallel to this vector is

$$\boldsymbol{a}_{11}=\frac{(8,2,3)}{\sqrt{64+4+9}}$$
$$=\underline{\underline{\pm(0.9117\boldsymbol{a}_x+0.2279\boldsymbol{a}_y+0.3419\boldsymbol{a}_z)}}$$

P. E. 1.2 (a) $\boldsymbol{r}_p=\underline{\underline{\boldsymbol{a}_x-3\boldsymbol{a}_y+5\boldsymbol{a}_z}}$

$$\boldsymbol{r}_R=\underline{\underline{3\boldsymbol{a}_y+8\boldsymbol{a}_z}}$$

(b) The distance vector is

$$\boldsymbol{r}_{QR}=\boldsymbol{r}_R-\boldsymbol{r}_Q=(0,3,8)-(2,4,6)=\underline{\underline{-2\boldsymbol{a}_x-\boldsymbol{a}_y+2\boldsymbol{a}_z}}$$

(c) The distance between Q and R is

$$|r_{QR}|=\sqrt{4+1+4}=\underline{\underline{3}}$$

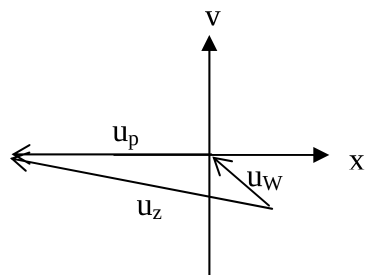
P. E. 1.3 Consider the figure shown on the next page:

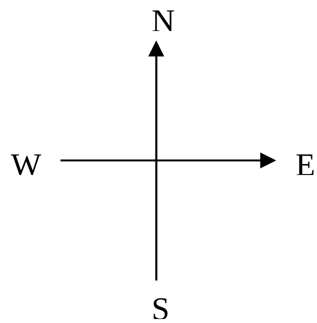
$$\boldsymbol{u}_Z=\boldsymbol{u}_P+\boldsymbol{u}_W=-350\boldsymbol{a}_x+\frac{40}{\sqrt{2}}(-\boldsymbol{a}_x+\boldsymbol{a}_y)$$
$$=-378.28\boldsymbol{a}_x+28.28a_y \text{ km/hr}$$

or

$$\boldsymbol{u}_z=379.3\angle 175.72^\circ \text{ km/hr}$$

Where $\mathbf{u}_p$ = velocity of the airplane in the absence of wind
$\mathbf{u}_w$ = wind velocity
$\mathbf{u}_z$ = observed velocity





P. E. 1.4

Using the dot product,

$$\cos\theta_{AB} = \frac{\mathbf{A}\bullet\mathbf{B}}{AB} = \frac{-13}{\sqrt{10}\sqrt{65}} = -\sqrt{\frac{13}{50}}$$

$$\underline{\underline{\theta_{AB} = 120.66^\circ}}$$

P. E. 1.5

(a) $$\boldsymbol{E}_F = (\boldsymbol{E}\cdot\boldsymbol{a}_F)\boldsymbol{a}_F = \frac{(\boldsymbol{E}\cdot\boldsymbol{F})\boldsymbol{F}}{|\boldsymbol{F}|^2} = \frac{-10(4,-10,5)}{141}$$

$$= \underline{\underline{-0.2837\boldsymbol{a}_x + 0.7092\boldsymbol{a}_y - 0.3546\boldsymbol{a}_z}}$$

(b) $$\boldsymbol{E}\times\boldsymbol{F} = \begin{vmatrix} \boldsymbol{a}_x & \boldsymbol{a}_y & \boldsymbol{a}_z \\ 0 & 3 & 4 \\ 4 & -10 & 5 \end{vmatrix} = (55,16,-12)$$

$$\boldsymbol{a}_{E\times F} = \underline{\underline{\pm(0.9398,0.2734,-0.205)}}$$

P. E. 1.6 $\boldsymbol{a} + \boldsymbol{b} + \boldsymbol{c} = 0$ showing that $\boldsymbol{a}$, $\boldsymbol{b}$, and $\boldsymbol{c}$ form the sides of a triangle.

$\boldsymbol{a}\cdot\boldsymbol{b} = 0$,
hence it is a right angle triangle.

$$\text{Area} = \frac{1}{2}|\boldsymbol{a}\times\boldsymbol{b}| = \frac{1}{2}|\boldsymbol{b}\times\boldsymbol{c}| = \frac{1}{2}|\boldsymbol{c}\times\boldsymbol{a}|$$

$$\frac{1}{2}|\boldsymbol{a}\times\boldsymbol{b}| = \frac{1}{2}\begin{vmatrix} 4 & 0 & -1 \\ 1 & 3 & 4 \end{vmatrix} = \frac{1}{2}|(3,-17,12)|$$

$$\text{Area} = \frac{1}{2}\sqrt{9+289+144} = \underline{\underline{10.51}}$$

P. E. 1.7

(a) $P_1P_2 = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$

$= \sqrt{25 + 4 + 64} = \underline{\underline{9.644}}$

(b) $\mathbf{r}_P = \mathbf{r}_{P_1} + \lambda(\mathbf{r}_{P_2} - \mathbf{r}_{P_1})$

$= (1,2,-3) + \lambda(-5,-2,8)$

$= \underline{\underline{(1-5\lambda, 2-2\lambda, -3+8\lambda)}}.$

(c) The shortest distance is

$$\mathbf{d} = \mathbf{P}_1\mathbf{P}_3 \sin\theta = \left|\mathbf{P}_1\mathbf{P}_3 \times \mathbf{a}_{P_1P_2}\right|$$

$$= \frac{1}{\sqrt{93}} \begin{vmatrix} 6 & -3 & 5 \\ -5 & -2 & 8 \end{vmatrix}$$

$$= \frac{1}{\sqrt{93}} \left|(-14,-73,-27)\right| = \underline{\underline{8.2}}$$

Prob.1.1

$\mathbf{r}_{OP} = 4\mathbf{a}_x - 5\mathbf{a}_y + \mathbf{a}_z$

$$\mathbf{a}_{r_{OP}} = \frac{\mathbf{r}_{OP}}{|\mathbf{r}_{OP}|} = \frac{(4,-5,1)}{\sqrt{(16+25+1)}} = \underline{\underline{0.6172\mathbf{a}_x - 0.7715\mathbf{a}_y + 0.1543\mathbf{a}_z}}$$

Prob. 1.2

Method 1:

$r_{AB} = r_B - r_A, \quad r_{BC} = r_C - r_B, \quad r_{CA} = r_A - r_C$

$r_{AB} + r_{BC} + r_{CA} = r_B - r_A + r_C - r_B + r_A - r_C = \mathbf{0}$

Method 2

$r_{AB} = r_B - r_A = (-2,0,3) - (4,-6,2) = (-6,6,1)$

$r_{BC} = r_C - r_B = (10,1,-7) - (-2,0,3) = (12,1,-10)$

$r_{CA} = r_A - r_C = (4,-6,2) - (10,1,-7) = (-6,-7,9)$

$r_{AB} + r_{BC} + r_{CA} = (0,0,0) = \mathbf{0}$

Prob. 1.3

(a)

$$\mathbf{A}-3\mathbf{B}=(4,-2,6)-3(12,18,-8)=(4,-2,6)-(36,54,-24)$$
$$=\underline{\underline{(-32,-56,-30)}}$$

(b)

$$2\mathbf{A}+5\mathbf{B}=2(4,-2,6)+5(12,18,-8)=(68,86,-28)$$
$$|\mathbf{B}|=\sqrt{12^2+18^2+8^2}=\sqrt{532}=23.065$$
$$(2\mathbf{A}+5\mathbf{B})/|\mathbf{B}|=(68,86,-28)/23.065=2.948\mathbf{a}_x+3.728\mathbf{a}_y-1.214\mathbf{a}_z$$

(c)

$$\mathbf{a}_x\times\mathbf{A}=\begin{vmatrix}1 & 0 & 0\\ 4 & -2 & 6\end{vmatrix}=\underline{\underline{-6\mathbf{a}_y-2\mathbf{a}_z}}$$

(d)

$$\mathbf{B}\times\mathbf{a}_x=\begin{vmatrix}12 & 18 & -8\\ 1 & 0 & 0\end{vmatrix}=-8\mathbf{a}_y-18\mathbf{a}_z$$
$$(\mathbf{B}\times\mathbf{a}_x)\bullet\mathbf{a}_y=-8$$

Prob. 1.4

(a) $\mathbf{A}\cdot\mathbf{B}=(10,-6,8)\cdot(1,0,2)=10+16=\underline{\underline{26}}$

(b)

$$\mathbf{A}\times\mathbf{B}=\begin{vmatrix}10 & -6 & 8\\ 1 & 0 & 2\end{vmatrix}=(-12-0)\mathbf{a}_x+(8-20)\mathbf{a}_y+(0+6)\mathbf{a}_z$$
$$=\underline{\underline{-12\mathbf{a}_x}}\,\underline{\underline{-12\mathbf{a}_y+6\mathbf{a}_z}}$$

(c) $2\mathbf{A}-3\mathbf{B}=(20,-12,16)-(3,0,6)=\underline{\underline{17\mathbf{a}_x-12\mathbf{a}_y+10\mathbf{a}_z}}$

Prob. 1.5

(a) $\mathbf{A}-\mathbf{B}+\mathbf{C}=(-2,5,1)+(-1,0,-3)+(4,-6,10)=\underline{\underline{(1,-1,8)}}$

(b)

$$\mathbf{B}\times\mathbf{C}=\begin{vmatrix}1 & 0 & 3\\ 4 & -6 & 10\end{vmatrix}=(18,2,-6)$$
$$\mathbf{A}\bullet(\mathbf{B}\times\mathbf{C})=(-2,5,1)\bullet(18,2,-6)=-36+10-6=\underline{\underline{-32}}$$

(c) $\cos\theta_{AB}=\dfrac{\mathbf{A}\bullet\mathbf{B}}{AB}=\dfrac{-2+0+3}{\sqrt{4+25+1}\sqrt{1+9}}=0.05773 \quad\rightarrow\quad \theta_{AB}=\underline{\underline{86.69^o}}$

Prob. 1.6

(a)

$$\mathbf{B}\times\mathbf{C}=\begin{vmatrix}1 & 1 & 1\\ 0 & 1 & 2\end{vmatrix}=\mathbf{a}_x-2\mathbf{a}_y+\mathbf{a}_z$$
$$\mathbf{A}\cdot(\mathbf{B}\times\mathbf{C})=(1,0,-1)\cdot(1,-2,1)=1+0-1=\underline{\underline{0}}$$

(b) $\boldsymbol{A}\times\boldsymbol{B} = \begin{vmatrix} 1 & 0 & -1 \\ 1 & 1 & 1 \end{vmatrix} = \boldsymbol{a}_x - 2\boldsymbol{a}_y + \boldsymbol{a}_z$

$(\boldsymbol{A}\times\boldsymbol{B})\cdot\boldsymbol{C} = (1,-2,1)\cdot(0,1,2) = 0-2+2 = \underline{\underline{0}}$

(c) $\boldsymbol{A}\times(\boldsymbol{B}\times\boldsymbol{C}) = \begin{vmatrix} 1 & 0 & -1 \\ 1 & -2 & 1 \end{vmatrix} = \underline{\underline{-2\boldsymbol{a}_x - 2\boldsymbol{a}_y - 2\boldsymbol{a}_z}}$

(d) $(\boldsymbol{A}\times\boldsymbol{B})\times\boldsymbol{C} = \begin{vmatrix} 1 & -2 & 1 \\ 0 & 1 & 2 \end{vmatrix} = \underline{\underline{-5\boldsymbol{a}_x - 2\boldsymbol{a}_y + \boldsymbol{a}_z}}$

Prob.1.7

(a) T = $\underline{(4, 6, -1)}$ and S = $\underline{(10, 12, 8)}$

(b) $r_{TS} = r_S - r_T = (10,12,8)-(4,6,-1) = \underline{\underline{6\boldsymbol{a}_x + 6\boldsymbol{a}_y + 9\boldsymbol{a}_z}}$

(c) $TS = |r_{TS}| = \sqrt{36+36+81} = \underline{\underline{12.37}}$

Prob. 1.8

(a) If **A** and **B** are parallel, **B**=k**A**, where k is a constant.

$B_x = kA_x, \quad B_y = kA_y, \quad B_z = kA_z$

For B_z, $\quad 3 = k(-1) \quad \rightarrow \quad k=-3$

$B_x = \alpha = kA_x = (-3)(4) = -12$

$B_y = \beta = kA_y = (-3)(2) = -6$

Hence, $\quad \underline{\underline{\alpha = -12, \beta = -6}}$

(b) If A and B are perpendicular to each other,

$$\boldsymbol{A}\bullet\boldsymbol{B} = 0 \quad \longrightarrow \quad \underline{\underline{4\alpha + 2\beta - 3 = 0}}$$

Prob. 1.9

(a) $\boldsymbol{A}\times\boldsymbol{a}_y = \begin{vmatrix} 10 & 5 & -2 \\ 0 & 1 & 0 \end{vmatrix} = \underline{\underline{2\boldsymbol{a}_x + 10\boldsymbol{a}_z}}$

(b) $\boldsymbol{A}\bullet\boldsymbol{a}_z = \underline{\underline{-2}}$

(c) $\cos\theta_z = \dfrac{\boldsymbol{A}\bullet\boldsymbol{a}_z}{\sqrt{100+25+4}} = \dfrac{-2}{11.358} \quad \rightarrow \quad \theta_z = \underline{\underline{100.14^o}}$

Prob. 1.10

(a) $A \cdot B = \text{AB}\cos\theta_{AB}$

$A \times B = \text{AB}\sin\theta_{AB} a_n$

$(A \cdot B)^2 + |A \times B|^2 = (AB)^2(\cos^2\theta_{AB} + \sin^2\theta_{AB}) = (\text{AB})^2$

(b) $\boldsymbol{a}_x \cdot (\boldsymbol{a}_y \times \boldsymbol{a}_z) = \boldsymbol{a}_x \cdot \boldsymbol{a}_x = 1.$ Hence,

$$\frac{\boldsymbol{a}_y \times \boldsymbol{a}_z}{\boldsymbol{a}_x \cdot \boldsymbol{a}_y \times \boldsymbol{a}_z} = \frac{\boldsymbol{a}_x}{1} = \boldsymbol{a}_x$$

$$\frac{\boldsymbol{a}_z \times \boldsymbol{a}_x}{\boldsymbol{a}_x \cdot \boldsymbol{a}_y \times \boldsymbol{a}_z} = \frac{\boldsymbol{a}_y}{1} = \boldsymbol{a}_y$$

$$\frac{\boldsymbol{a}_x \times \boldsymbol{a}_y}{\boldsymbol{a}_x \cdot \boldsymbol{a}_y \times \boldsymbol{a}_z} = \frac{\boldsymbol{a}_z}{1} = \boldsymbol{a}_z$$

Prob. 1.11

(a) $\boldsymbol{P} + \boldsymbol{Q} = (6, 2, 0), \boldsymbol{P} + \boldsymbol{Q} - \boldsymbol{R} = (7, 1, -2)$

$|\boldsymbol{P} + \boldsymbol{Q} - \boldsymbol{R}| = \sqrt{49+1+4} = \sqrt{54} = \underline{\underline{7.3485}}$

(b) $$\boldsymbol{P.Q} \times \boldsymbol{R} = \begin{vmatrix} 2 & -1 & -2 \\ 4 & 3 & 2 \\ -1 & 1 & 2 \end{vmatrix} = 2(6-2) + (8+2) - 2(4+3) = 8 + 10 - 14 = \underline{\underline{4}}$$

$$\boldsymbol{Q} \times \boldsymbol{R} = \begin{vmatrix} 4 & 3 & 2 \\ -1 & 1 & 2 \end{vmatrix} = (4, -10, 7)$$

$$\boldsymbol{P.Q} \times \boldsymbol{R} = (2, -1, -2) \cdot (4, -10, 7) = 8 + 10 - 14 = \underline{\underline{4}}$$

(c) $$\boldsymbol{Q} \times \boldsymbol{P} = \begin{vmatrix} 4 & 3 & 2 \\ 2 & -1 & -2 \end{vmatrix} = (-4, 12, -10)$$

$$\boldsymbol{Q} \times \boldsymbol{P} \cdot \boldsymbol{R} = (-4, 12, -10) \cdot (-1, 1, 2) = 4 + 12 - 20 = \underline{\underline{-4}}$$

or $$\boldsymbol{Q} \times \boldsymbol{P} \cdot \boldsymbol{R} = \boldsymbol{R} \cdot \boldsymbol{Q} \times \boldsymbol{P} = \begin{vmatrix} -1 & 1 & 2 \\ 4 & 3 & 2 \\ 2 & -1 & -2 \end{vmatrix} = -(-6+2) - (-8-4) + 2(-4-6) = \underline{\underline{-4}}$$

(d) $$(\boldsymbol{P} \times \boldsymbol{Q}) \cdot (\boldsymbol{Q} \times \boldsymbol{R}) = (4, -12, 10) \cdot (4, -10, 7) = 16 + 120 + 70 = \underline{\underline{206}}$$

(e) $$(\boldsymbol{P} \times \boldsymbol{Q}) \times (\boldsymbol{Q} \times \boldsymbol{R}) = \begin{vmatrix} 4 & -12 & 10 \\ 4 & -10 & 7 \end{vmatrix} = \underline{\underline{16a_x + 12a_y + 8a_z}}$$

(f) $\cos\theta_{PR} = \frac{\mathbf{P}\cdot\mathbf{R}}{|\boldsymbol{P}||\boldsymbol{R}|} = \frac{(-2-1-4)}{\sqrt{4+1+4}\sqrt{1+1+4}} = \frac{-7}{3\sqrt{6}} = -0.9526$

$\theta_{PR} = 162.3^\circ$

(g) $\sin\theta_{PQ} = \frac{|P\times Q|}{|P||Q|} = \frac{\sqrt{16+144+100}}{3\sqrt{16+9+4}} = \frac{\sqrt{260}}{3\sqrt{29}} = 0.998$

$\theta_{PQ} = 86.45^\circ$

Prob. 1.12

$\boldsymbol{A}\cdot\boldsymbol{B} = (4,-6,1)\cdot(2,0,5) = 8-0+5 = 13$

(a) $|\boldsymbol{B}|^2 = 2^2 + 5^2 = 29$

$\boldsymbol{A}\cdot\boldsymbol{B} + 2|\boldsymbol{B}|^2 = 13 + 2\times 29 = 71$

(b)

$\boldsymbol{a}_\perp = \pm\frac{\boldsymbol{A}\times\boldsymbol{B}}{|\boldsymbol{A}\times\boldsymbol{B}|}$

Let $\boldsymbol{C} = \boldsymbol{A}\times\boldsymbol{B} = \begin{vmatrix} 4 & -6 & 1 \\ 2 & 0 & 5 \end{vmatrix} = (-30,-18,12)$

$\boldsymbol{a}_\perp = \pm\frac{\boldsymbol{C}}{|\boldsymbol{C}|} = \pm\frac{(-30,-18,12)}{\sqrt{30^2+18^2+12^2}} = \pm(-0.8111\boldsymbol{a}_x - 0.4867\boldsymbol{a}_y + 0.3244\boldsymbol{a}_z)$

Prob. 1.13

$\boldsymbol{P}\cdot\boldsymbol{Q} = (2,-6,5)\cdot(0,3,1) = 0-18+5 = -13$

$\boldsymbol{P}\times\boldsymbol{Q} = \begin{vmatrix} 2 & -6 & 5 \\ 0 & 3 & 1 \end{vmatrix} = -21\boldsymbol{a}_x - 2\boldsymbol{a}_y + 6\boldsymbol{a}_z$

$\cos\theta_{PQ} = \frac{\boldsymbol{P}\cdot\boldsymbol{Q}}{PQ} = \frac{-13}{\sqrt{10}\sqrt{65}} = -0.51 \longrightarrow \theta_{PQ} = 120.66^o$

Prob. 1.14

P and **Q** are orthogonal if the angle between them is 90°. Hence

$\boldsymbol{P}\bullet\boldsymbol{Q} = PQ\cos\theta = 0$

$\boldsymbol{P}\bullet\boldsymbol{Q} = (2,4,-6)\bullet(5,2,3) = 10+8-18 = 0$

showing that they are perpendicular or orthogonal.

Prob. 1.15

(a) Using the fact that

$$(\boldsymbol{A}\times\boldsymbol{B})\times\boldsymbol{C} = (\boldsymbol{A}\cdot\boldsymbol{C})\boldsymbol{B} - (\boldsymbol{B}\cdot\boldsymbol{C})\boldsymbol{A},$$

we get

$$\boldsymbol{A}\times(\boldsymbol{A}\times\boldsymbol{B}) = -(\boldsymbol{A}\times\boldsymbol{B})\times\boldsymbol{A} = \underline{\underline{(\boldsymbol{B}\cdot\boldsymbol{A})\boldsymbol{A} - (\boldsymbol{A}\cdot\boldsymbol{A})\boldsymbol{B}}}$$

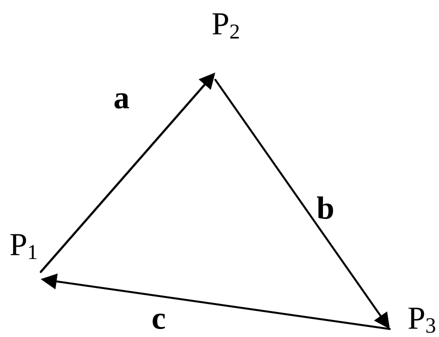
(b) $$\boldsymbol{A}\times(\boldsymbol{A}\times(\boldsymbol{A}\times\boldsymbol{B})) = \boldsymbol{A}\times\left[(\boldsymbol{A}\bullet\boldsymbol{B})\boldsymbol{A} - (\boldsymbol{A}\bullet\boldsymbol{A})\boldsymbol{B}\right]$$

$$= (\boldsymbol{A}\bullet\boldsymbol{B})\ (\boldsymbol{A}\times\boldsymbol{A}) - (\boldsymbol{A}\bullet\boldsymbol{A})\ (\boldsymbol{A}\times\boldsymbol{B})$$

$$= \underline{\underline{-A^2\ (\boldsymbol{A}\times\boldsymbol{B})}}$$

since **A**x**A** = **0**

Prob. 1.16



$$\boldsymbol{a} = \boldsymbol{r}_{p2} - \boldsymbol{r}_{p1} = (1,-2,4) - (5,-3,1) = (-4,1,3)$$

(a) $$\boldsymbol{b} = \boldsymbol{r}_{p3} - \boldsymbol{r}_{p2} = (3,3,5) - (1,-2,4) = (2,5,1)$$

$$\boldsymbol{c} = \boldsymbol{r}_{p1} - \boldsymbol{r}_{p3} = (5,-3,1) - (3,3,5) = (2,-6,-4)$$

Note that **a** + **b** + **c** = **0**

$\boldsymbol{a}\cdot\boldsymbol{b} = -8+5+3 = 0 \quad \longrightarrow \quad$ perpendicular

$\boldsymbol{b}\cdot\boldsymbol{c} = 4-30-4 \neq 0$

$\boldsymbol{c}\cdot\boldsymbol{a} = -8-6-12 \neq 0$

Hence $\underline{\underline{P_2}}$ is a right angle.

(b) $$\text{Area} = \frac{1}{2}|\boldsymbol{a}\times\boldsymbol{b}| = \frac{1}{2}\begin{vmatrix} -4 & 1 & 3 \\ 2 & 5 & 1 \end{vmatrix} = \frac{1}{2}|(1-15)\boldsymbol{a}_x + (6+4)\boldsymbol{a}_y + (-20-2)\boldsymbol{a}_z|$$

$$= \frac{1}{2}|(-14,10,-22)| = \frac{1}{2}\sqrt{196+100+484} = \underline{\underline{13.96}}$$

Prob. 1.17

Given $\boldsymbol{r}_P = (-1,4,8), \quad \boldsymbol{r}_Q = (2,-1,3), \quad \boldsymbol{r}_R = (-1,2,3)$

(a) $|PQ| = \sqrt{9+25+25} = \underline{\underline{7.6811}}$

(b) $\boldsymbol{PR} = \underline{\underline{-2\boldsymbol{a}_y - 5\boldsymbol{a}_z}}$

(c)

$$\boldsymbol{QP} = (-1,4,8)-(2,-1,3) = (-3,5,5)$$

$$\boldsymbol{QR} = (-1,2,3)-(2,-1,3) = (-3,3,0)$$

$$\frac{\boldsymbol{QP}\cdot\boldsymbol{QR}}{|\boldsymbol{QP}||\boldsymbol{QR}|} = \frac{9+15+0}{\sqrt{59}\sqrt{18}} = 0.7365$$

$$\angle PQR = \cos^{-1}\left(0.7365\right) = \underline{\underline{42.64^o}}$$

(d) $\text{Area} = \frac{1}{2}\left|QP \times QR\right| = \frac{1}{2}\begin{vmatrix} -3 & 5 & 5 \\ -3 & 3 & 0 \end{vmatrix} = 0.5\left|(-15,-15,8)\right| = 0.5\sqrt{225+225+36} = \underline{\underline{10.677}}$

(e) Perimeter $= PQ + QR + RP = \sqrt{59} + \sqrt{18} + \sqrt{29} = \underline{\underline{17.31}}$

Prob.1.18

Let R be the midpoint of PQ.

$$\boldsymbol{r}_R = \frac{1}{2}\{(2,4,-1)+(12,16,9)\} = (7,10,4)$$

$$OR = \sqrt{49+100+16} = \sqrt{165} = 12.845$$

$$t = \frac{OR}{v} = \frac{12.845}{300} = \underline{\underline{42.82 \text{ ms}}}$$

Prob. 1.19

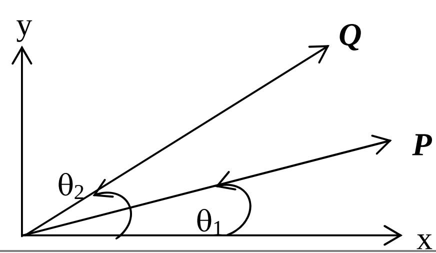
Area = Twice the area of a triangue

$$= |\boldsymbol{D}\times\boldsymbol{E}| = \begin{vmatrix} 4 & 1 & 5 \\ -1 & 2 & 3 \end{vmatrix} = |(3-10)\boldsymbol{a}_x + (-5-12)\boldsymbol{a}_y + (8+1)\boldsymbol{a}_z|$$

$$= |(-7,-19,9)| = \sqrt{49+361+81} = \underline{\underline{22.16}}$$

Prob. 1.20

(a) Let **P** and **Q** be as shown below:



$|\boldsymbol{P}| = \cos^2\theta_1 + \sin^2\theta_1 = 1, |\boldsymbol{Q}| = \cos^2\theta_2 + \sin^2\theta_2 = 1,$

Hence **P** and **Q** are unit vectors.

(b) $\boldsymbol{P}\cdot\boldsymbol{Q} = (1)(1)\cos(\theta_2 - \theta_1)$

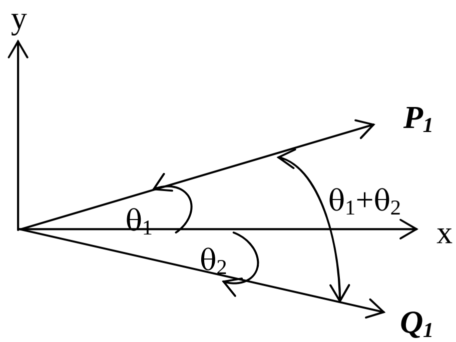
But $\boldsymbol{P}\cdot\boldsymbol{Q} = \cos\theta_1\cos\theta_2 + \sin\theta_1\sin\theta_2$. Thus,

$$\underline{\underline{\cos(\theta_2 - \theta_1) = \cos\theta_1\cos\theta_2 + \sin\theta_1\sin\theta_2}}$$

Let $\boldsymbol{P}_1 = \boldsymbol{P} = \cos\theta_1\boldsymbol{a}_x + \sin\theta_1\boldsymbol{a}_y$ and

$\boldsymbol{Q}_1 = \cos\theta_2\boldsymbol{a}_x - \sin\theta_2\boldsymbol{a}_y$.

$\boldsymbol{P_1}$ and $\boldsymbol{Q_1}$ are unit vectors as shown below:



$\boldsymbol{P}_1\cdot\boldsymbol{Q}_1 = (1)(1)\cos(\theta_1 + \theta_2)$

But $\boldsymbol{P}_1\cdot\boldsymbol{Q}_1 = \cos\theta_1\cos\theta_2 - \sin\theta_1\sin\theta_2$,

$$\underline{\underline{\cos(\theta_2 + \theta_1) = \cos\theta_1\cos\theta_2 - \sin\theta_1\sin\theta_2}}$$

Alternatively, we can obtain this formula from the previous one by replacing θ_2 by $-\theta_2$ in $\boldsymbol{Q}$.

(c)

$$\frac{1}{2}|\boldsymbol{P}-\boldsymbol{Q}| = \frac{1}{2}|(\cos\theta_1 - \cos\theta_2)a_x + (\sin\theta_1 - \sin\theta_2)a_y|$$

$$= \frac{1}{2}\sqrt{\cos^2\theta_1 + \sin^2\theta_1 + \cos^2\theta_2 + \sin^2\theta_2 - 2\cos\theta_1\cos\theta_2 - 2\sin\theta_1\sin\theta_2}$$

$$= \frac{1}{2}\sqrt{2 - 2(\cos\theta_1\cos\theta_2 + \sin\theta_1\sin\theta_2)} = \frac{1}{2}\sqrt{2 - 2\cos(\theta_2 - \theta_1)}$$

Let $\theta_2 - \theta_1 = \theta$, the angle between **P** and **Q.**

$$\frac{1}{2}|\boldsymbol{P}-\boldsymbol{Q}| = \frac{1}{2}\sqrt{2 - 2\cos\theta}$$

But cos 2A = 1 – 2 sin ^{2}A.

$$\frac{1}{2}|\boldsymbol{P}-\boldsymbol{Q}| = \frac{1}{2}\sqrt{2-2+4\sin^2\theta/2} = \sin\theta/2$$

Thus,

$$\frac{1}{2}|\boldsymbol{P}-\boldsymbol{Q}| = |\sin\frac{\theta_2-\theta_1}{2}|$$

Prob. 1.21

$$\boldsymbol{\omega} = \frac{\omega(1,-2,2)}{3} = (1,-2,2), \quad r = r_p - r_o = (1,3,4)-(2,-3,1) = (-1,6,3)$$

$$u = \boldsymbol{\omega}\times r = \begin{vmatrix} 1 & -2 & 2 \\ -1 & 6 & 3 \end{vmatrix} = (-18,-5,4)$$

$$u = -18a_x - 5a_y + 4a_z$$

Prob. 1.22

$$\boldsymbol{r}_1 = (1,1,1), \qquad \boldsymbol{r}_2 = (1,0,1)-(0,1,0) = (1,-1,1)$$

$$\cos\theta = \frac{\boldsymbol{r}_1\cdot\boldsymbol{r}_2}{r_1 r_2} = \frac{(1-1+1)}{\sqrt{3}\sqrt{3}} = \frac{1}{3} \quad \longrightarrow \quad \theta = 70.53^o$$

Prob. 1.23

(a) $T_s = T\cdot a_s = \dfrac{T\cdot S}{|S|} = \dfrac{(2,-6,3)\cdot(1,2,1)}{\sqrt{6}} = \dfrac{-7}{\sqrt{6}} = -2.8577$

(b) $\boldsymbol{S}_{\boldsymbol{T}} = (\boldsymbol{S}\cdot\boldsymbol{a}_{\boldsymbol{T}})\boldsymbol{a}_{\boldsymbol{T}} = \dfrac{(\boldsymbol{S}\cdot\boldsymbol{T})\boldsymbol{T}}{\boldsymbol{T}^2} = \dfrac{-7(2,-6,3)}{7^2}$

$$= -0.2857\boldsymbol{a}_x + 0.8571\boldsymbol{a}_y - 0.4286\boldsymbol{a}_z$$

(c) $\sin\theta_{TS} = \dfrac{|\boldsymbol{T}\times\boldsymbol{S}|}{|\boldsymbol{T}||\boldsymbol{S}|} = \begin{vmatrix} 2 & -6 & 3 \\ 1 & 2 & 1 \end{vmatrix} = \dfrac{|(-12,1,10)|}{7\sqrt{6}} = \dfrac{\sqrt{245}}{7\sqrt{6}} = 0.9129$

$$\Rightarrow \theta_{TS} = 65.91^\circ$$

Prob. 1.24

Let $\boldsymbol{A} = \boldsymbol{A}_{B\parallel} + \boldsymbol{A}_{B\perp}$

$$\boldsymbol{A}_{B\parallel} = (\boldsymbol{A}\cdot\boldsymbol{a}_B)\boldsymbol{a}_B = \frac{\boldsymbol{A}\cdot\boldsymbol{B}}{\boldsymbol{B}\cdot\boldsymbol{B}}\boldsymbol{B}$$

Hence,

$$\boldsymbol{A}_{B\perp} = \boldsymbol{A} - \boldsymbol{A}_{B\parallel} = \boldsymbol{A} - \frac{\boldsymbol{A}\cdot\boldsymbol{B}}{\boldsymbol{B}\cdot\boldsymbol{B}}\boldsymbol{B}$$

Prob. 1.25

(a) $\boldsymbol{A}\bullet\boldsymbol{B} = 20 + 0 - 10 = \underline{\underline{10}}$

(b) $\boldsymbol{A}\times\boldsymbol{B} = \begin{vmatrix} 20 & 15 & -10 \\ 1 & 0 & 1 \end{vmatrix} = \underline{\underline{15\boldsymbol{a}_x - 30\boldsymbol{a}_y - 15\boldsymbol{a}_z}}$

(c) $\boldsymbol{A}_B = (\boldsymbol{A}\bullet\boldsymbol{a}_B)\boldsymbol{a}_B = \dfrac{(\boldsymbol{A}\bullet\boldsymbol{B})\boldsymbol{B}}{B^2} = \dfrac{10(\boldsymbol{a}_x + \boldsymbol{a}_z)}{2} = \underline{\underline{5\boldsymbol{a}_x + 5\boldsymbol{a}_z}}$

Prob. 1.26

$$\boldsymbol{A}\bullet\boldsymbol{a}_x = A_x = A\cos\alpha \quad \rightarrow \quad \cos\alpha = \frac{A_x}{A} = \frac{2}{\sqrt{4+16+36}} = 0.2673 \quad \rightarrow \quad \underline{\underline{\alpha = 74.5^o}}$$

$$\cos\beta = \frac{A_y}{A} = \frac{-4}{\sqrt{56}} = -0.5345 \quad \rightarrow \quad \underline{\underline{\beta = 122.31^o}}$$

$$\cos\gamma = \frac{A_z}{A} = \frac{6}{\sqrt{56}} = 0.8018 \quad \rightarrow \quad \underline{\underline{\gamma = 36.7^o}}$$

Prob.1.27

(a) $\boldsymbol{H}(1,3,-2) = 6\boldsymbol{a}_x + \boldsymbol{a}_y + 4\boldsymbol{a}_z$

$$\boldsymbol{a}_H = \frac{(6,1,4)}{\sqrt{36+1+16}} = \underline{\underline{0.8242\boldsymbol{a}_x + 0.1374\boldsymbol{a}_y + 0.5494\boldsymbol{a}_z}}$$

(b) $|\boldsymbol{H}| = 10 = \sqrt{4x^2y^2 + (x+z)^2 + z^4}$

or

$$\underline{\underline{100 = 4x^2y^2 + x^2 + 2xz + z^2 + z^4}}$$

Prob. 1.28

$\boldsymbol{R} = R\boldsymbol{a}_R, \quad R = 4$

$$\boldsymbol{a}_R = \frac{\boldsymbol{P}\times\boldsymbol{Q}}{|\boldsymbol{P}\times\boldsymbol{Q}|}$$

$$\boldsymbol{P}\times\boldsymbol{Q} = \begin{vmatrix} 2 & -4 & 1 \\ 1 & 2 & 0 \end{vmatrix} = -2\boldsymbol{a}_x + \boldsymbol{a}_y + 8a_z$$

$$\boldsymbol{a}_R = \frac{-2\boldsymbol{a}_x + \boldsymbol{a}_y + 8a_z}{\sqrt{4+1+64}} = -0.2408\boldsymbol{a}_x + 0.1204\boldsymbol{a}_y + 0.9631\boldsymbol{a}_z$$

$$\boldsymbol{R} = R\boldsymbol{a}_R = 4(-0.2408\boldsymbol{a}_x + 0.1204\boldsymbol{a}_y + 0.9631\boldsymbol{a}_z) = \underline{\underline{-0.9631\boldsymbol{a}_x + 0.4815\boldsymbol{a}_y + 3.852\boldsymbol{a}_z}}$$

An alternate choice of $\boldsymbol{R}$ is $-0.9631\boldsymbol{a}_x + 0.4815\boldsymbol{a}_y + 3.852\boldsymbol{a}_z$

Prob. 1.29

(a) At (1, -2, 3), x = 1, y = -2, z = 3.
$\boldsymbol{G} = \boldsymbol{a}_x + 2\boldsymbol{a}_y + 6\boldsymbol{a}_z, \quad \boldsymbol{H} = -6\boldsymbol{a}_x + 3\boldsymbol{a}_y - 3\boldsymbol{a}_z$

$$G = \sqrt{1+4+36} = \underline{\underline{6.403}}$$

$$H = \sqrt{36+9+9} = \underline{\underline{7.348}}$$

(b) $\boldsymbol{G}\cdot\boldsymbol{H} = -6+6-18 = \underline{\underline{-18}}$

(c)
$$\cos\theta_{GH} = \frac{\boldsymbol{G}\cdot\boldsymbol{H}}{GH} = \frac{-18}{6.403\times 7.348} = -0.3826$$

$$\underline{\underline{\theta_{GH} = 112.5^o}}$$

Prob. 1.30

(a) $\boldsymbol{H} = 10(2)(16)\boldsymbol{a}_x - 8(-8)\boldsymbol{a}_y + 12(4)\boldsymbol{a}_z = 320\boldsymbol{a}_x + 64\boldsymbol{a}_y + 48\boldsymbol{a}_z$

Let $\boldsymbol{F} = \boldsymbol{a}_x - \boldsymbol{a}_y$

(b)
$$\boldsymbol{H}_F = (\boldsymbol{H}\bullet\boldsymbol{a}_F)\boldsymbol{a}_F = \frac{(\boldsymbol{H}\bullet\boldsymbol{F})\boldsymbol{F}}{F^2} = \frac{(320-64)(1,-1,0)}{1+1} = 128\boldsymbol{a}_x - 128\boldsymbol{a}_y$$

Prob. 1.31
(a) At (1,2,3), $\boldsymbol{E}$ = (2,1,6)

$$|\boldsymbol{E}| = \sqrt{4+1+36} = \sqrt{41} = \underline{\underline{6.403}}$$

(b) At (1,2,3), $\boldsymbol{F}$ = (2,-4,6)

$$\mathbf{E}_F = (\mathbf{E}\cdot\mathbf{a}_F)\mathbf{a}_F = \frac{(\mathbf{E}\cdot\mathbf{F})\mathbf{F}}{|\mathbf{F}|^2} = \frac{36}{56}(2,-4,6)$$

$$= \underline{\underline{1.286\mathbf{a}_x - 2.571\mathbf{a}_y + 3.857\mathbf{a}_z}}$$

(c) At (0,1,-3), **E** = (0,1,-3), **F** = (0,-1,0)

$$\mathbf{E}\times\mathbf{F} = \begin{vmatrix} 0 & 1 & -3 \\ 0 & -1 & 0 \end{vmatrix} = (-3,0,0)$$

$$\mathbf{a}_{E\times F} = \pm\frac{\mathbf{E}\times\mathbf{F}}{|\mathbf{E}\times\mathbf{F}|} = \underline{\underline{\pm\mathbf{a}_x}}$$

Prob. 1.32

(a) At P, x = -1, y = 2, z = 4

$$\mathbf{D} = 8\mathbf{a}_x - 4\mathbf{a}_y - 2\mathbf{a}_z, \qquad \mathbf{E} = -10\mathbf{a}_x + 24\mathbf{a}_y + 128\mathbf{a}_z$$

$$\mathbf{C} = \mathbf{D} + \mathbf{E} = -2\mathbf{a}_x + 20\mathbf{a}_y + 126\mathbf{a}_z$$

(b)

$$\mathbf{C}\cdot\mathbf{a}_x = C\cos\theta_x \longrightarrow \cos\theta_x = \frac{\mathbf{C}\cdot\mathbf{a}_x}{C} = \frac{-2}{\sqrt{2^2+20^2+126^2}} = -0.01575$$

$$\underline{\underline{\theta_x = 90.9^o}}$$