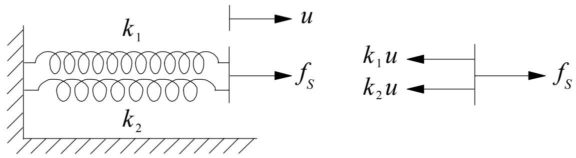
CHAPTER 1

Problem 1.1

If k_e is the effective stiffness,

$$f_S = k_e u$$



Equilibrium of forces: $f_S = (k_1 + k_2)\, u$

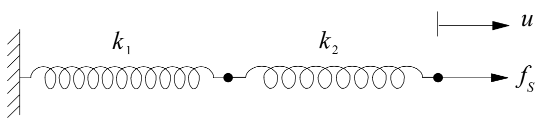
Effective stiffness: $k_e = f_S / u = k_1 + k_2$

Equation of motion: $m\ddot{u} + k_e u = p(t)$

Problem 1.2

If k_e is the effective stiffness,

$$f_S = k_e u \qquad \text{(a)}$$



If the elongations of the two springs are u_1 and u_2,

$$u = u_1 + u_2 \qquad \text{(b)}$$

Because the force in each spring is f_S,

$$f_S = k_1 u_1 \qquad f_S = k_2 u_2 \qquad \text{(c)}$$

Solving for u_1 and u_2 and substituting in Eq. (b) gives

$$\frac{f_S}{k_e} = \frac{f_S}{k_1} + \frac{f_S}{k_2} \Rightarrow \frac{1}{k_e} = \frac{1}{k_1} + \frac{1}{k_2} \Rightarrow$$

$$k_e = \frac{k_1 k_2}{k_1 + k_2}$$

Equation of motion: $m\ddot{u} + k_e u = p(t)$

Problem 1.3

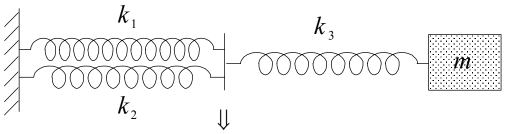


Fig. 1.3(a)

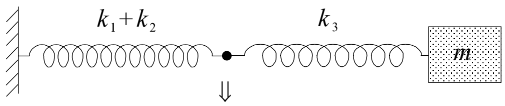


Fig. 1.3(b)

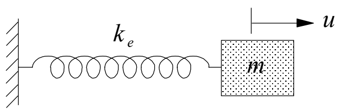


Fig. 1.3(c)

This problem can be solved either by starting from the definition of stiffness or by using the results of Problems P1.1 and P1.2. We adopt the latter approach to illustrate the procedure of reducing a system with several springs to a single equivalent spring.

First, using Problem 1.1, the parallel arrangement of k_1 and k_2 is replaced by a single spring, as shown in Fig. 1.3(b). Second, using the result of Problem 1.2, the series arrangement of springs in Fig. 1.3(b) is replaced by a single spring, as shown in Fig. 1.3(c):

$$\frac{1}{k_e} = \frac{1}{k_1 + k_2} + \frac{1}{k_3}$$

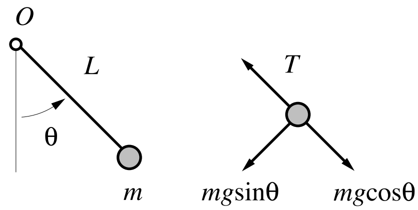
Therefore the effective stiffness is

$$k_e = \frac{(k_1 + k_2)\, k_3}{k_1 + k_2 + k_3}$$

The equation of motion is $m\ddot{u} + k_e u = p(t)$.

Problem 1.4

1. *Draw a free body diagram of the mass.*



2. *Write equation of motion in tangential direction.*

Method 1: By Newton's law.

$$-mg\sin\theta = ma$$

$$-mg\sin\theta = mL\ddot{\theta} \qquad \text{(a)}$$

$$mL\ddot{\theta} + mg\sin\theta = 0$$

This nonlinear differential equation governs the motion for any rotation θ.

Method 2: Equilibrium of moments about O yields

$$mL^2\ddot{\theta} = -mgL\sin\theta$$

or

$$mL\ddot{\theta} + mg\sin\theta = 0$$

3. *Linearize for small* θ.

For small θ, $\sin\theta \approx \theta$, and Eq. (a) becomes

$$mL\ddot{\theta} + mg\theta = 0$$

$$\ddot{\theta} + \left(\frac{g}{L}\right)\theta = 0 \qquad \text{(b)}$$

4. *Determine natural frequency.*

$$\omega_n = \sqrt{\frac{g}{L}}$$

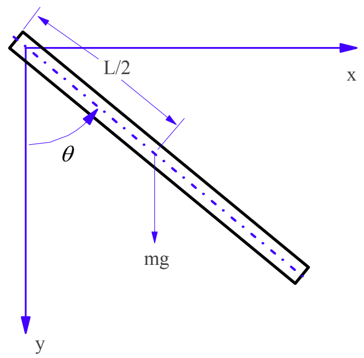
Problem 1.5

1. *Find the moment of inertia about O.*

From Appendix 8,

$$I_0 = \frac{1}{12} mL^2 + m\left(\frac{L}{2}\right)^2 = \frac{1}{3} mL^2$$

2. *Draw a free body diagram of the body in an arbitrary displaced position.*



3. *Write the equation of motion using Newton's second law of motion.*

$$\sum M_0 = I_0 \ddot{\theta}$$

$$-mg\frac{L}{2}\sin\theta = \frac{1}{3} mL^2 \ddot{\theta}$$

$$\frac{mL^2}{3}\ddot{\theta} + \frac{mgL}{2}\sin\theta = 0 \qquad \text{(a)}$$

4. *Specialize for small θ.*

For small θ, $\sin\theta \cong \theta$ and Eq. (a) becomes

$$\frac{mL^2}{3}\ddot{\theta} + \frac{mgL}{2}\theta = 0$$

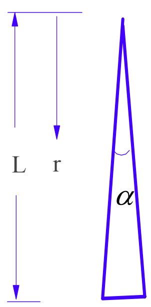
$$\ddot{\theta} + \frac{3}{2}\frac{g}{L} = 0 \qquad \text{(b)}$$

5. *Determine natural frequency.*

$$\omega_n = \sqrt{\frac{3}{2}\frac{g}{L}}$$

Problem 1.6

1. *Find the moment of inertia about about O.*



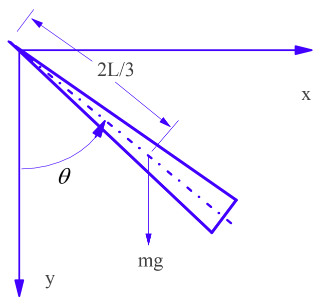
$$I_0 = \rho \int_0^L r^2 dA$$

$$= \rho \int_0^L r^2 (r \alpha dr)$$

$$= \frac{\rho}{4} L^4 \alpha$$

$$= \frac{1}{2} m L^2$$

2. *Draw a free body diagram of the body in an arbitrary displaced position.*



3. *Write the equation of motion using Newton's second law of motion.*

$$\sum M_0 = I_0 \ddot{\theta}$$

$$-mg \frac{2L}{3} \sin\theta = \frac{1}{2} m L^2 \ddot{\theta}$$

$$\frac{mL^2}{2} \ddot{\theta} + \frac{2mgL}{3} \sin\theta = 0 \qquad \text{(a)}$$

4. *Specialize for small θ.*

For small θ, $sin\theta \cong \theta$, and Eq. (a) becomes

$$\frac{mL^2}{2} \ddot{\theta} + \frac{2mgL}{3} \theta = 0$$

or

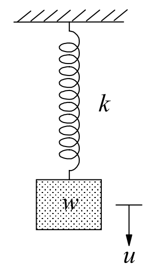
$$\ddot{\theta} + \frac{4}{3} \frac{g}{L} = 0 \qquad \text{(b)}$$

5. *Determine natural frequency.*

$$\omega_n = \sqrt{\frac{4}{3} \frac{g}{L}}$$

In each case the system is equivalent to the spring-mass system shown for which the equation of motion is

$$\left(\frac{w}{g}\right) \ddot{u} + ku = 0$$



The spring stiffness is determined from the deflection u under a vertical force f_S applied at the location of the lumped weight:

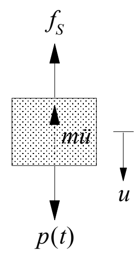
Simply-supported beam: $u = \frac{f_S L^3}{48EI} \Rightarrow k = \frac{48EI}{L^3}$

Cantilever beam: $u = \frac{f_S L^3}{3EI} \Rightarrow k = \frac{3EI}{L^3}$

Clamped beam: $u = \frac{f_S L^3}{192EI} \Rightarrow k = \frac{192EI}{L^3}$

Problem 1.7

Draw a free body diagram of the mass:



Write equation of dynamic equilibrium:

$$m\ddot{u} + f_S = p(t) \qquad \text{(a)}$$

Write the force-displacement relation:

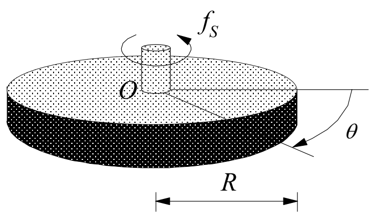
$$f_S = \left(\frac{AE}{L}\right)u \qquad \text{(b)}$$

Substitute Eq. (b) into Eq. (a) to obtain the equation of motion:

$$m\ddot{u} + \left(\frac{AE}{L}\right)u = p(t)$$

Problem 1.8

Show forces on the disk:



Write the equation of motion using Newton's second law of motion:

$$-f_S = I_O \ddot{\theta} \qquad \text{where} \qquad I_O = \frac{mR^2}{2} \qquad \text{(a)}$$

Write the torque-twist relation:

$$f_S = \left(\frac{GJ}{L}\right)\theta \qquad \text{where} \qquad J = \frac{\pi d^4}{32} \qquad \text{(b)}$$

Substitute Eq. (b) into Eq. (a):

$$I_O \ddot{\theta} + \left(\frac{GJ}{L}\right)\theta = 0$$

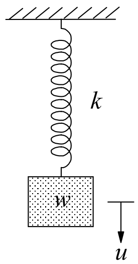
or,

$$\left(\frac{mR^2}{2}\right)\ddot{\theta} + \left(\frac{\pi d^4 G}{32L}\right)\theta = 0$$

Problems 1.9 through 1.11

In each case the system is equivalent to the spring-mass system shown for which the equation of motion is

$$\left(\frac{w}{g}\right)\ddot{u} + ku = 0$$



The spring stiffness is determined from the deflection u under a vertical force f_S applied at the location of the lumped weight:

Simply-supported beam: $u = \frac{f_S L^3}{48EI} \Rightarrow k = \frac{48EI}{L^3}$

Cantilever beam: $u = \frac{f_S L^3}{3EI} \Rightarrow k = \frac{3EI}{L^3}$

Clamped beam: $u = \frac{f_S L^3}{192EI} \Rightarrow k = \frac{192EI}{L^3}$

Problem 1.12

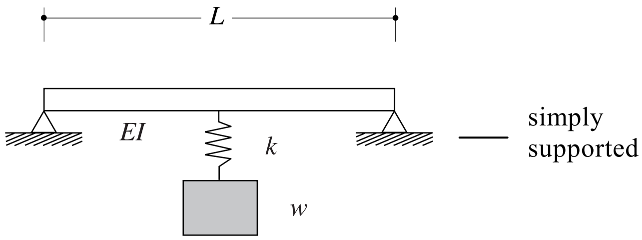


Fig. 1.12a

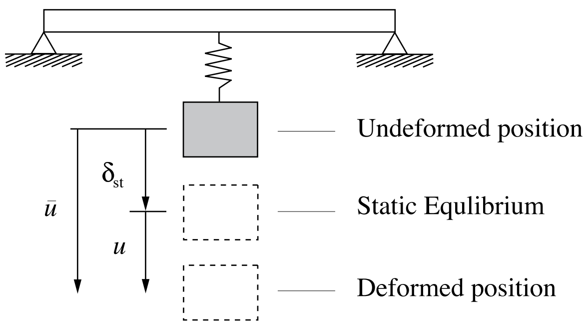


Fig. 1.12b

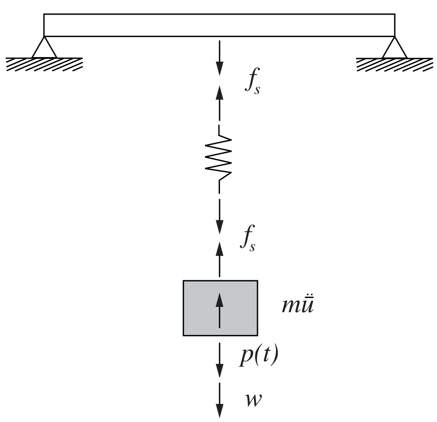


Fig. 1.12c

1. *Write the equation of motion.*

Equilibrium of forces in Fig. 1.12c gives

$$m\ddot{\bar{u}} + f_s = w + p(t) \qquad \text{(a)}$$

where

$$f_s = k_e \bar{u} \qquad \text{(b)}$$

The equation of motion is:

$$m\ddot{\bar{u}} + k_e \bar{u} = w + p(t) \qquad \text{(c)}$$

2. *Determine the effective stiffness.*

$$f_S = k_e \bar{u} \qquad \text{(d)}$$

where

$$\bar{u} = \delta_{spring} + \delta_{beam} \qquad \text{(e)}$$

$$f_s = k\delta_{spring} = k_{beam}\,\delta_{beam} \qquad \text{(f)}$$

Substitute for the δ's from Eq. (f) and for $\bar{u}$ from Eq. (d):

$$\frac{f_s}{k_e} = \frac{f_s}{k} + \frac{f_s}{k_{beam}}$$

$$k_e = \frac{k k_{beam}}{k + k_{beam}}$$

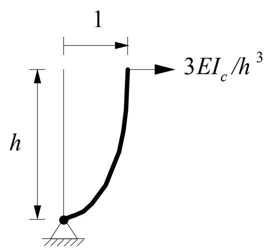
$$k_e = \frac{k\left(48EI/L^3\right)}{k + \dfrac{48EI}{L^3}}$$

3. *Determine the natural frequency.*

$$\omega_n = \sqrt{\frac{k_e}{m}}$$

Problem 1.13

Compute lateral stiffness:



$$k = 2 \times k_{column} = 2 \times \frac{3EI_c}{h^3} = \frac{6EI_c}{h^3}$$

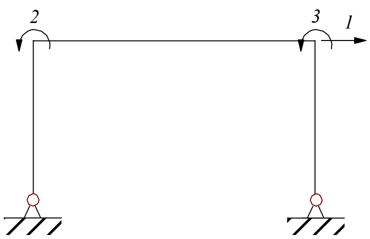
Equation of motion:

$$m\ddot{u} + ku = p(t)$$

Base fixity increases k by a factor of 4.

Problem 1.14

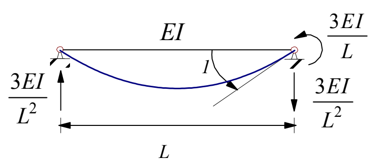
1. *Define degrees of freedom (DOF).*



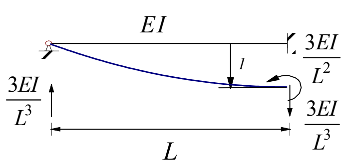
2. *Reduced stiffness coefficients.*

Since there are no external moments applied at the pinned supports, the following *reduced stiffness coefficients* are used for the columns.

Joint rotation:

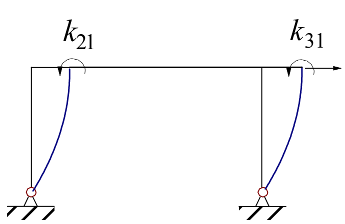


Joint translation:



3. *Form structural stiffness matrix.*

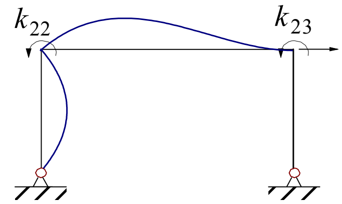
$u_1 = 1, \;\; u_2 = u_3 = 0$



$$k_{11} = 2\frac{3EI_c}{h^3} = \frac{6EI_c}{h^3}$$

$$k_{21} = k_{31} = \frac{3EI_c}{h^2}$$

$u_2 = 1, \;\; u_1 = u_3 = 0$

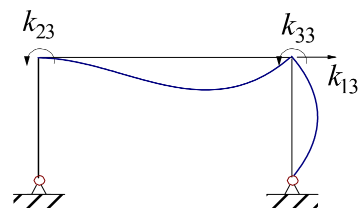


$$k_{22} = \frac{3EI_c}{h} + \frac{4EI_c}{(2h)} = \frac{5EI_c}{h}$$

$$k_{32} = \frac{2EI_c}{(2h)} = \frac{EI_c}{h}$$

$$k_{12} = \frac{3EI_c}{h^2}$$

$u_3 = 1, \;\; u_1 = u_2 = 0$



$$k_{33} = \frac{3EI_c}{h} + \frac{4EI_c}{(2h)} = \frac{5EI_c}{h}$$

$$k_{23} = \frac{2EI_c}{(2h)} = \frac{EI_c}{h}$$

$$k_{13} = \frac{3EI_c}{h^2}$$

Hence

$$\mathbf{k} = \frac{EI_c}{h^3}\begin{bmatrix} 6 & 3h & 3h \\ 3h & 5h^2 & h^2 \\ 3h & h^2 & 5h^2 \end{bmatrix}$$

4. *Determine lateral stiffness.*

The lateral stiffness k of the frame can be obtained by static condensation since there is no force acting on DOF 2 and 3:

$$\frac{EI_c}{h^3}\begin{bmatrix} 6 & 3h & 3h \\ 3h & 5h^2 & h^2 \\ 3h & h^2 & 5h^2 \end{bmatrix}\begin{Bmatrix} u_1 \\ u_2 \\ u_3 \end{Bmatrix} = \begin{Bmatrix} f_S \\ 0 \\ 0 \end{Bmatrix}$$

First partition $\mathbf{k}$ as

$$\mathbf{k} = \frac{EI_c}{h^3}\left[\begin{array}{c|cc} 6 & 3h & 3h \\ \hline 3h & 5h^2 & h^2 \\ 3h & h^2 & 5h^2 \end{array}\right] = \begin{bmatrix} \mathbf{k}_{tt} & \mathbf{k}_{t0} \\ \mathbf{k}_{t0} & \mathbf{k}_{00} \end{bmatrix}$$

where

$$\mathbf{k}_{tt} = \frac{EI_c}{h^3}\begin{bmatrix}6\end{bmatrix}$$

$$\mathbf{k}_{t0} = \frac{EI_c}{h^3}\begin{bmatrix}3h & 3h\end{bmatrix}$$

$$\mathbf{k}_{00} = \frac{EI_c}{h^3}\begin{bmatrix}5h^2 & h^2 \\ h^2 & 5h^2\end{bmatrix}$$

Then compute the lateral stiffness k from

$$k = \mathbf{k}_{tt} - \mathbf{k}_{t0}\,\mathbf{k}_{00}^{-1}\,\mathbf{k}_{t0}^{T}$$

Since

$$\mathbf{k}_{00}^{-1} = \frac{h}{24EI_c}\begin{bmatrix}5 & -1 \\ -1 & 5\end{bmatrix}$$

we get

$$k = \frac{6EI_c}{h^3} - \frac{EI_c}{h^3}\begin{bmatrix}3h & 3h\end{bmatrix}\cdot\frac{h}{24EI_c}\begin{bmatrix}5 & -1 \\ -1 & 5\end{bmatrix}\cdot\frac{EI_c}{h^3}\begin{bmatrix}3h \\ 3h\end{bmatrix}$$

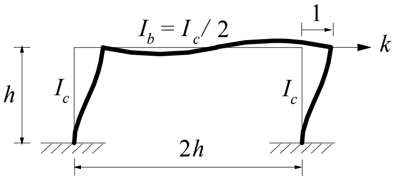
$$k = \frac{EI_c}{h^3}\begin{bmatrix}6-3\end{bmatrix}$$

$$k = \frac{3EI_c}{h^3}$$

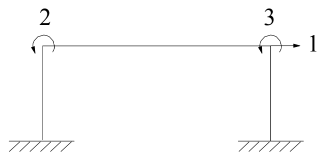
5. *Equation of motion.*

$$m\ddot{u} + \frac{3EI_c}{h^3}u = p(t)$$

Problem 1.15

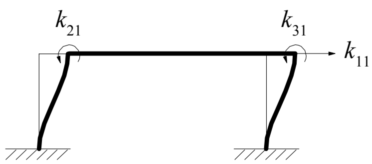


Define degrees of freedom (DOF):



Form structural stiffness matrix:

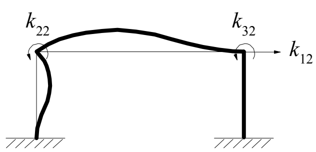
$u_1 = 1, \quad u_2 = u_3 = 0$



$$k_{11} = 2\,\frac{12EI_c}{h^3} = \frac{24EI_c}{h^3}$$

$$k_{21} = k_{31} = \frac{6EI_c}{h^2}$$

$u_2 = 1, \quad u_1 = u_3 = 0$

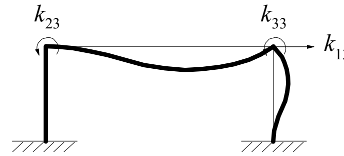


$$k_{22} = \frac{4EI_c}{h} + \frac{4EI_b}{(2h)} = \frac{4EI_c}{h} + \frac{EI_c}{h} = \frac{5EI_c}{h}$$

$$k_{32} = \frac{2EI_b}{(2h)} = \frac{EI_c}{2h}$$

$$k_{12} = \frac{6EI_c}{h^2}$$

$u_3 = 1, \quad u_1 = u_2 = 0$



$$k_{33} = \frac{4EI_c}{h} + \frac{4EI_b}{(2h)} = \frac{4EI_c}{h} + \frac{EI_c}{h} = \frac{5EI_c}{h}$$

$$k_{23} = \frac{2EI_b}{(2h)} = \frac{EI_c}{2h}$$

$$k_{13} = \frac{6EI_c}{h^2}$$

Hence

$$\mathbf{k} = \frac{EI_c}{h^3}\begin{bmatrix} 24 & 6h & 6h \\ 6h & 5h^2 & \frac{1}{2}h^2 \\ 6h & \frac{1}{2}h^2 & 5h^2 \end{bmatrix}$$

The lateral stiffness k of the frame can be obtained by static condensation since there is no force acting on DOF 2 and 3:

$$\frac{EI_c}{h^3}\begin{bmatrix} 24 & 6h & 6h \\ 6h & 5h^2 & \frac{1}{2}h^2 \\ 6h & \frac{1}{2}h^2 & 5h^2 \end{bmatrix}\begin{Bmatrix} u_1 \\ u_2 \\ u_3 \end{Bmatrix} = \begin{Bmatrix} f_S \\ 0 \\ 0 \end{Bmatrix}$$

First partition $\mathbf{k}$ as

$$\mathbf{k} = \frac{EI_c}{h^3}\left[\begin{array}{c|cc} 24 & 6h & 6h \\ \hline 6h & 5h^2 & \frac{1}{2}h^2 \\ 6h & \frac{1}{2}h^2 & 5h^2 \end{array}\right] = \begin{bmatrix} \mathbf{k}_{tt} & \mathbf{k}_{t0} \\ \mathbf{k}_{t0}^T & \mathbf{k}_{00} \end{bmatrix}$$

where

$$\mathbf{k}_{tt} = \frac{EI_c}{h^3}[24]$$

$$\mathbf{k}_{t0} = \frac{EI_c}{h^3}\begin{bmatrix} 6h & 6h \end{bmatrix}$$

$$\mathbf{k}_{00} = \frac{EI_c}{h^3}\begin{bmatrix} 5h^2 & \frac{1}{2}h^2 \\ \frac{1}{2}h^2 & 5h^2 \end{bmatrix}$$

Then compute the lateral stiffness k from

$$k = \mathbf{k}_{tt} - \mathbf{k}_{t0}\,\mathbf{k}_{00}^{-1}\,\mathbf{k}_{t0}^T$$

Since

$$\mathbf{k}_{00}^{-1} = \frac{4h}{99EI_c}\begin{bmatrix} 5 & -\frac{1}{2} \\ -\frac{1}{2} & 5 \end{bmatrix}$$

we get

$$\begin{aligned} k &= \frac{24EI_c}{h^3} - \frac{EI_c}{h^3}\begin{bmatrix}6h & 6h\end{bmatrix} \cdot \frac{4h}{99EI_c}\begin{bmatrix}5 & -\frac{1}{2} \\ -\frac{1}{2} & 5\end{bmatrix} \cdot \frac{EI_c}{h^3}\begin{bmatrix}6h \\ 6h\end{bmatrix} \\ &= \frac{EI_c}{h^3}\left(24 - \frac{144}{11}\right) \\ &= \frac{120}{11}\frac{EI_c}{h^3} \end{aligned}$$

This result can be checked against Eq. 1.3.5:

$$k = \frac{24EI_c}{h^3}\left(\frac{12\rho+1}{12\rho+4}\right)$$

Substituting $\rho = I_b/4I_c = 1/8$ gives

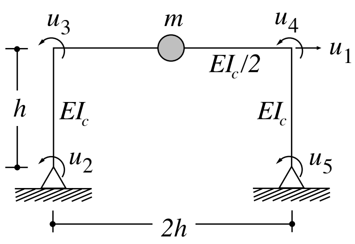
$$k = \frac{24EI_c}{h^3}\left(\frac{12\frac{1}{8}+1}{12\frac{1}{8}+4}\right) = \frac{24EI_c}{h^3}\left(\frac{5}{11}\right) = \frac{120}{11}\frac{EI_c}{h^3}$$

Equation of motion:

$$m\ddot{u} + \left(\frac{120}{11}\frac{EI_c}{h^3}\right)u = p(t)$$

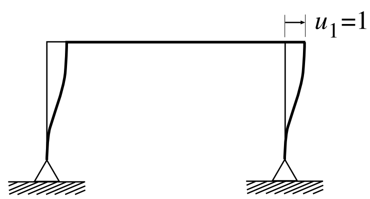
Problem 1.16

1. *Define degrees of freedom (DOF).*



2. *Form the structural stiffness matrix.*

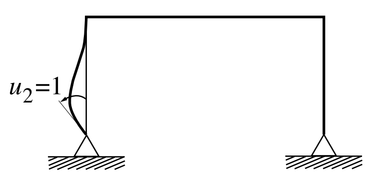
$u_1 = 1$, $u_2 = u_3 = u_4 = u_5 = 0$



$$k_{11} = 2\frac{12EI_c}{h^3} = \frac{24EI_c}{h^3}$$

$$k_{21} = k_{31} = k_{41} = k_{51} = \frac{6EI_c}{h^2}$$

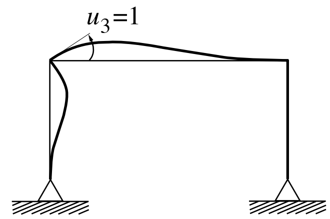
$u_2 = 1$, $u_1 = u_3 = u_4 = u_5 = 0$



$$k_{22} = \frac{4EI_c}{h} \qquad k_{12} = \frac{6EI_c}{h^2}$$

$$k_{32} = \frac{2EI_c}{h} \qquad k_{42} = k_{52} = 0$$

$u_3 = 1$, $u_1 = u_2 = u_4 = u_5 = 0$

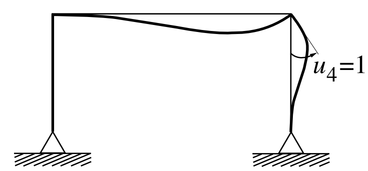


$$k_{33} = \frac{4EI_c}{h} + \frac{4EI_c}{2(2h)} = \frac{5EI_c}{h}$$

$$k_{13} = \frac{6EI_c}{h^2} \qquad k_{23} = \frac{2EI_c}{h}$$

$$k_{43} = \frac{2EI_c}{2(2h)} = \frac{EI_c}{2h} \qquad k_{53} = 0$$

$u_4 = 1$, $u_1 = u_2 = u_3 = u_5 = 0$

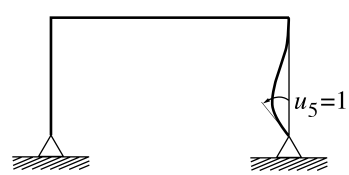


$$k_{44} = \frac{4EI_c}{h} + \frac{4EI_c}{2(2h)} = \frac{5EI_c}{h}$$

$$k_{14} = \frac{6EI_c}{h^2} \qquad k_{24} = 0$$

$$k_{34} = \frac{2EI_c}{2(2h)} = \frac{EI_c}{2h} \qquad k_{54} = \frac{2EI_c}{h}$$

$u_5 = 1$, $u_1 = u_2 = u_3 = u_4 = 0$



$$k_{55} = \frac{4EI_c}{h} \qquad k_{15} = \frac{6EI_c}{h^2}$$

$$k_{45} = \frac{2EI_c}{h} \qquad k_{25} = k_{35} = 0$$

Assemble the stiffness coefficients:

$$k = \frac{EI_c}{h^3}\begin{bmatrix} 24 & 6h & 6h & 6h & 6h \\ 6h & 4h^2 & 2h^2 & 0 & 0 \\ 6h & 2h^2 & 5h^2 & \frac{1}{2}h^2 & 0 \\ 6h & 0 & \frac{1}{2}h^2 & 5h^2 & 2h^2 \\ 6h & 0 & 0 & 2h^2 & 4h^2 \end{bmatrix}$$

3. *Determine the lateral stiffness of the frame.*

First partition **k**.

$$k = \frac{EI_c}{h^3}\left[\begin{array}{c|cccc} 24 & 6h & 6h & 6h & 6h \\ \hline 6h & 4h^2 & 2h^2 & 0 & 0 \\ 6h & 2h^2 & 5h^2 & \frac{1}{2}h^2 & 0 \\ 6h & 0 & \frac{1}{2}h^2 & 5h^2 & 2h^2 \\ 6h & 0 & 0 & 2h^2 & 4h^2 \end{array}\right] = \begin{bmatrix} \mathbf{k}_{tt} & \mathbf{k}_{t0} \\ \mathbf{k}_{t0}^T & \mathbf{k}_{00} \end{bmatrix}$$

Compute the lateral stiffness.

$$k = \mathbf{k}_{tt} - \mathbf{k}_{t0}\mathbf{k}_{00}^{-1}\mathbf{k}_{t0}^T$$

$$k = \frac{24EI_c}{h^3} - \frac{22EI_c}{h^3} = \frac{2EI_c}{h^3}$$

4. *Write the equation of motion.*

$$m\ddot{u} + ku = p(t)$$

$$m\ddot{u} + \left(\frac{2EI_c}{h^3}\right)u = p(t)$$

Problem 1.17

(a) *Equation of motion in the x-direction.*

The lateral stiffness of each wire is the same as the lateral stiffness of a brace derived in Eq. (c) of Example 1.2:

$$k_w = \left(\frac{AE}{L}\right)\cos^2\theta$$

$$= \left(\frac{AE}{h\sqrt{2}}\right)\cos^2 45^\circ = \frac{1}{2\sqrt{2}}\frac{AE}{h}$$

Each of the four sides of the structure includes two wires. If they were not pretensioned, under lateral displacement, only the wire in tension will provide lateral resistance and the one in compression will go slack and will not contribute to the lateral stiffness. However, the wires are pretensioned to a high stress; therefore, under lateral displacement the tension will increase in one wire, but decrease in the other; and both wires will contribute to the lateral direction. Consequently, four wires contribute to the stiffness in the x-direction:

$$k_x = 4k_w = \sqrt{2}\,\frac{AE}{h}$$

Then the equation of motion in the x-direction is

$$m\ddot{u}_x + k_x u_x = 0$$

(b) *Equation of motion in the y-direction.*

The lateral stiffness in the y direction, $k_y = k_x$, and the same equation applies for motion in the y-direction:

$$m\ddot{u}_y + k_y u_y = 0$$

Problem 1.18

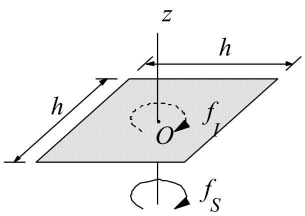


Fig. 1.18(a)

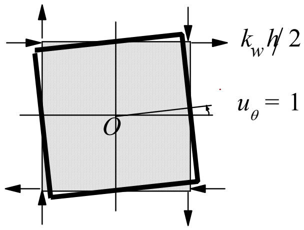


Fig. 1.18(b)

1. *Set up equation of motion.*

The elastic resisting torque f_S and inertia force f_I are shown in Fig. 1.18(a). The equation of dynamic equilibrium is

$$f_I + f_S = 0 \quad \text{or} \quad I_O \ddot{u}_\theta + f_S = 0 \qquad \text{(a)}$$

where

$$I_O = m\frac{h^2 + h^2}{12} = \frac{mh^2}{6} \qquad \text{(b)}$$

2. *Determine torsional stiffness, k_θ.*

$$f_S = k_\theta u_\theta \qquad \text{(c)}$$

Introduce $u_\theta = 1$ in Fig. 1.18(b) and identify the resisting forces due to each wire. All the eight forces are the same; each is $k_w h/2$, where, from Problem 1.17,

$$k_w = \frac{1}{2\sqrt{2}}\frac{AE}{h}$$

The torque required to equilibrate these resisting forces is

$$k_\theta = 8k_w \frac{h}{2}\frac{h}{2} = 2k_w h^2 = \frac{2}{2\sqrt{2}}\left(\frac{AE}{h}\right)h^2$$

$$= \frac{AEh}{\sqrt{2}} \qquad \text{(d)}$$

3. *Set up equation of motion*.

Substituting Eq. (d) in (c) and then Eqs. (c) and (b) in (a) gives the equation of motion:

$$\frac{mh^2}{6}\ddot{u}_\theta + \frac{AEh}{\sqrt{2}}u_\theta = 0$$

Problem 1.19

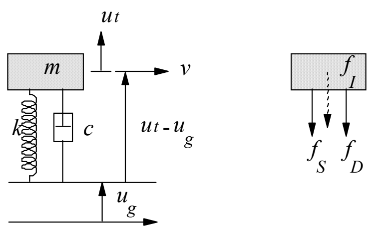


Fig. 1.19(a)

Fig. 1.19(b)

Displacement u^t is measured from the static equilibrium position under the weight mg.

From the free-body diagram in Fig. 1.19(b)

$$f_I + f_D + f_S = 0 \tag{a}$$

where

$$f_I = m\ddot{u}^t$$
$$f_D = c(\dot{u}^t - \dot{u}_g) \tag{b}$$
$$f_S = k(u^t - u_g)$$

Substituting Eqs. (b) in Eq. (a) gives

$$m\ddot{u}^t + c(\dot{u}^t - \dot{u}_g) + k(u^t - u_g) = 0$$

Noting that $x = vt$ and transferring the excitation terms to the right side gives the equation of motion:

$$m\ddot{u}^t + c\dot{u}^t + ku^t = c\dot{u}_g(vt) + ku_g(vt)$$