

Chapter One

Problem 1. The first law of thermodynamics for the control volume shown in Fig. 1.5 can be written as (ref. Table 1.1):

$$\frac{\partial}{\partial t} \int_{c.v} (u + \frac{V^2}{2} + \phi) \rho dV + \int_{c.s} (i + \frac{1}{2} V^2 + \phi) \rho \vec{V} \cdot \vec{n} dA = \dot{Q} - \dot{W} \quad \text{--- (1)}$$

since the flow is steady, $\dot{m} = \rho_1 V_1 A_1 = \rho_2 V_2 A_2$ --- (2)

and

$$\begin{aligned} \int_{c.s} (i + \frac{1}{2} V^2 + \phi) \rho \vec{V} \cdot \vec{n} dA &= (i_2 + \frac{1}{2} V_2^2 + g z_2) \rho_2 V_2 A_2 \\ &\quad - (i_1 + \frac{1}{2} V_1^2 + g z_1) \rho_1 V_1 A_1 \\ &= \dot{m} \left[(i_2 + \frac{1}{2} V_2^2 + g z_2) - (i_1 + \frac{1}{2} V_1^2 + g z_1) \right] \quad \text{--- (3)} \end{aligned}$$

Therefore (1) becomes,

$$\begin{aligned} \dot{Q} + \dot{m} (i_1 + \frac{1}{2} V_1^2 + g z_1) &= \dot{m} (i_2 + \frac{1}{2} V_2^2 + g z_2) + \frac{d}{dt} \left\{ \dot{m} (u + \frac{V^2}{2} + g z) \right\}_{c.v} \\ &\quad + \dot{W} \quad \text{--- (4)} \end{aligned}$$

where M , u and V are the mass, internal energy and the velocity of the control volume respectively.

Problem 2. Taking a control volume just outside the pipe-bend, we can write the momentum equation as:

$$\vec{F} = \frac{\partial}{\partial t} \int_{c.v.} \rho \vec{V} dV + \int_{c.s.} \vec{V} \rho \vec{V} \cdot \vec{n} dA \quad \text{--- (1)}$$

The first integral on the right-hand side is zero because of the steady-state conditions, and the second integral reduces to integration over section 1 and 2.

Assuming that the fluid is incompressible,

$$\int_{c.s.} \vec{V} \rho \vec{V} \cdot \vec{n} dA = -\rho V_1^2 A_1 \vec{i} + \rho V_2^2 A_2 (\cos \alpha \vec{i} - \sin \alpha \vec{j}) \quad \text{--- (2)}$$

where $\vec{V} = u \vec{i} + v \vec{j}$

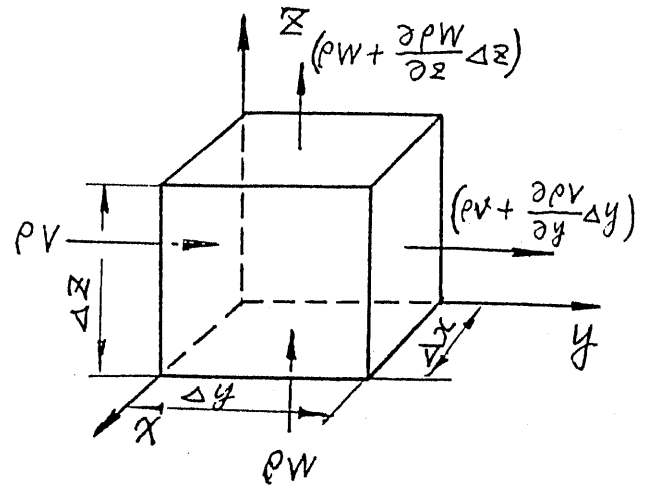
The net force acting on the c.v. is

$$\vec{F} = -R_x \vec{i} - R_y \vec{j} + P_1 A_1 \vec{i} - P_2 A_2 (\cos \alpha \vec{i} - \sin \alpha \vec{j}) \quad \text{--- (3)}$$

sub. (2) and (3) into (1) and equating the terms in x - and y -direction respectively, we get:

$$\begin{cases} R_x = (P_1 + \rho V_1^2) A_1 - (P_2 + \rho V_2^2) A_2 \cos \alpha \\ R_y = (P_2 + \rho V_2^2) A_2 \sin \alpha \end{cases}$$

Problem 3. Take the volume element $\Delta x \Delta y \Delta z$ as the control volume and assume the velocity to be uniform on each of the six surfaces.



The figure shows the mass fluxes in the y - and z -directions. The net mass flow rate into the control volume is,

$$-\left\{\frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\partial z}\right\} \Delta x \Delta y \Delta z$$

The rate of increase of mass within the control volume is,

$$\frac{\partial \rho}{\partial t} \Delta x \Delta y \Delta z$$

From the law of conservation of mass,

$$\frac{\partial \rho}{\partial t} \Delta x \Delta y \Delta z = -\left\{\frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\partial z}\right\} \Delta x \Delta y \Delta z$$

$$\therefore \frac{\partial \rho}{\partial t} + \frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\partial z} = 0$$

$$\text{or. } \frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot (\rho \vec{V}) = 0$$

$$\text{where } \vec{V} = u \vec{i} + v \vec{j} + w \vec{k}$$

Problem 4. The heat transfer coefficient is defined as,

$$h = \frac{-k_f \left(\frac{\partial T_f}{\partial n} \right)_s}{T_w - T_\infty} \quad \text{--- (1)}$$

$$T_f(y) = 20 + 80 e^{-800y}$$

$$\left(\frac{\partial T_f}{\partial n} \right)_s = \left(\frac{dT_f}{dy} \right)_{y=0} = -800 \times 80 e^{-800y} \Big|_{y=0} = -64 \times 10^3 \text{ } ^\circ\text{C/m} \quad \text{--- (2)}$$

$$T_w = (T_f)_{y=0} = 20 + 80 e^{-800y} \Big|_{y=0} = 100 \text{ } ^\circ\text{C} \quad \text{--- (3)}$$

$$T_\infty = \lim_{y \rightarrow \infty} (20 + 80 e^{-800y}) = 20 \text{ } ^\circ\text{C} \quad \text{--- (4)}$$

Sub. (2), (3) and (4) into (1), we obtain,

$$h = \frac{(-0.62 \text{ W/mK}) (-64 \times 10^3 \text{ } ^\circ\text{C/m})}{100 \text{ } ^\circ\text{C} - 20 \text{ } ^\circ\text{C}} = 496 \text{ W/m}^2\text{K}$$

Problem 5. Assume that the surface area of the cylinder is A , then the solar energy absorbed by the cylinder is,

$$Q_{abs} = \alpha_s \cdot 1500 A \quad (\text{W}) \quad \text{--- (1)}$$

and the energy emitted by the cylinder is

$$Q_{em} = \epsilon A \sigma T_{cy}^4 \quad (\text{W}) \quad \text{--- (2)}$$

since $\alpha = \epsilon$, at equilibrium we have,

$$\sigma T_{cy}^4 = 1500$$

$$\therefore T_{cy} = \left(\frac{1500 \text{ W/m}^2}{5.6697 \times 10^{-8} \text{ W/m}^2\text{K}^4} \right)^{1/4} = 403.3 \text{ K}$$

Problem 6. From vector identity:

$$\int_S \vec{Y} (\vec{x} \cdot d\vec{s}) = \int_V \{ \vec{Y} \nabla \cdot \vec{x} + (\vec{x} \cdot \nabla) \vec{Y} \} dV$$

we have,

$$\int_{c.s} \vec{v} \rho \vec{v} \cdot \vec{n} dA = \int_{c.v} \{ \vec{v} \nabla \cdot (\rho \vec{v}) + (\rho \vec{v} \cdot \nabla) \vec{v} \} dV$$

$$\begin{aligned} \text{so, } \vec{F} &= \frac{\partial}{\partial t} \int_{c.v} \rho \vec{v} dV + \int_{c.s} \vec{v} \rho \vec{v} \cdot \vec{n} dA \\ &= \int_{c.v} \left\{ \vec{v} \frac{\partial \rho}{\partial t} + \rho \frac{\partial \vec{v}}{\partial t} \right\} dV + \int_{c.v} \{ \vec{v} \nabla \cdot (\rho \vec{v}) + (\rho \vec{v} \cdot \nabla) \vec{v} \} dV \\ &= \int_{c.v} \left\{ \vec{v} \left(\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{v}) \right) + \rho \left(\frac{\partial \vec{v}}{\partial t} + (\vec{v} \cdot \nabla) \vec{v} \right) \right\} dV \\ &= \int_{c.v} \rho \left(\frac{\partial \vec{v}}{\partial t} + (\vec{v} \cdot \nabla) \vec{v} \right) dV \\ &= \int_{c.v} \rho \frac{D\vec{v}}{Dt} dV \end{aligned}$$

Problem 7. By the use of Gauss' Theorem the L.H.S of Eq. (1.32) can be written as,

$$\begin{aligned} &\frac{\partial}{\partial t} \int_{c.v} e \rho dV + \int_{c.v} \nabla \cdot (e \rho \vec{v}) dV \\ &= \int_{c.v} \left\{ e \frac{\partial \rho}{\partial t} + \rho \frac{\partial e}{\partial t} + e \nabla \cdot (\rho \vec{v}) + \rho (\vec{v} \cdot \nabla) e \right\} dV \\ &= \int_{c.v} \left\{ e \left(\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{v}) \right) + \rho \left(\frac{\partial e}{\partial t} + (\vec{v} \cdot \nabla) e \right) \right\} dV \\ \text{so, } &\int_{c.v} \rho \frac{De}{Dt} dV = \dot{q}_{c.s} - \dot{W} \end{aligned}$$

Chapter Two

Problem 2.1 Under the conditions given in the problem, the Navier-Stokes equations reduce to:

$$\frac{1}{r} \frac{d}{dr} \left(r \frac{dw}{dr} \right) = \frac{1}{\mu} \frac{dP}{dz} \quad \text{--- (1)}$$

where $\frac{dP}{dz}$ is a constant, since the R.H.S. is a function of z only and the L.H.S. is a function of r only.

The boundary conditions are:

$$\begin{cases} w=0 & \text{at } r=r_0 \end{cases} \quad \text{--- (2)}$$

$$\begin{cases} \frac{dw}{dr}=0 & \text{at } r=0 \end{cases} \quad \text{--- (3)}$$

Integrating (1) twice:

$$w = \frac{1}{4\mu} \frac{dP}{dz} r^2 + C_1 \ln r + C_2 \quad \text{--- (4)}$$

From B.C.s (2) and (3),

$$C_1 = 0, \quad C_2 = -\frac{1}{4\mu} \frac{dP}{dz} r_0^2$$

$$\text{so, } w = \frac{r_0^2}{4\mu} \frac{dP}{dz} \left(\left(\frac{r}{r_0} \right)^2 - 1 \right) \quad \text{--- (5)}$$

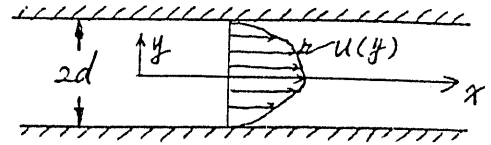
The average velocity:

$$W_m = \frac{1}{\pi r_0^2} \int_0^{r_0} 2\pi r w dr = \frac{r_0^2}{8\mu} \frac{dP}{dz} \quad \text{--- (6)}$$

$$\text{so, } \frac{w}{W_m} = 2 \left(1 - \left(\frac{r}{r_0} \right)^2 \right) \quad \text{--- (7)}$$

Problem 2.2 From the statements of the problem,

$$V=0, W=0, \frac{\partial u}{\partial x}=0, \frac{\partial u}{\partial z}=0.$$



The N-S equations in y- and

z-direction reduce to: $\frac{\partial P}{\partial y}=0$ and $\frac{\partial P}{\partial z}=0$ respectively.

The one in the x-direction reduces to,

$$\frac{dP}{dx} = \mu \frac{d^2 u}{dy^2} \quad \text{--- (1)}$$

We can deduce that $\frac{dP}{dx} = \text{const.}$, since the R.H.S is a function of y only and the L.H.S. is a function of x only.

The boundary conditions are:

$$\begin{cases} u=0 & \text{at } y=d \end{cases} \quad \text{--- (2)}$$

$$\begin{cases} \frac{du}{dy}=0 & \text{at } y=0 \end{cases} \quad \text{--- (3)}$$

Integrating (1) twice,

$$u = \frac{1}{2\mu} \frac{dP}{dx} y^2 + Ay + B \quad \text{--- (4)}$$

From B.C.s (2) and (3),

$$A=0, \text{ and } B = -\frac{1}{2\mu} \frac{dP}{dx} d^2$$

$$\text{So, } u = \frac{1}{2\mu} \frac{dP}{dx} (y^2 - d^2) \quad \text{--- (5)}$$

Problem 2.2 (Continued)

At $y=0$, $u=u_{max}$,

$$u_{max} = -\frac{1}{2\mu} \frac{dP}{dx} d^2, \quad \text{--- (6)}$$

so, the velocity distribution in terms of u_{max} is,

$$\frac{u}{u_{max}} = 1 - \left(\frac{y}{d}\right)^2 \quad \text{--- (7)}$$

The average velocity is,

$$u_m = \frac{1}{d} \int_0^d u dy = -\frac{1}{3\mu} \frac{dP}{dx} d^2 \quad \text{--- (8)}$$

so, the velocity distribution in terms of u_m is,

$$\frac{u}{u_m} = \frac{3}{2} \left[1 - \left(\frac{y}{d}\right)^2 \right] \quad \text{--- (9)}$$

Also,

$$u_{max} = \frac{3}{2} u_m \quad \text{--- (10)}$$

Problem 2.3 Assume the downward direction to be the positive y -direction. The N-S equation is,

$$\rho \frac{DV}{Dt} = \rho g - \frac{\partial P}{\partial y} + \frac{\partial}{\partial x} \left(\mu \left(\frac{\partial V}{\partial x} + \frac{\partial u}{\partial y} \right) \right) + \frac{\partial}{\partial y} \left(\mu \left(2 \frac{\partial V}{\partial y} - \frac{2}{3} \nabla \cdot \vec{V} \right) \right) + \frac{\partial}{\partial z} \left(\mu \left(\frac{\partial w}{\partial y} + \frac{\partial v}{\partial z} \right) \right) -$$

Introducing the dimensionless variables:

$$\bar{x} = \frac{x}{L}, \quad \bar{y} = \frac{y}{L}, \quad \bar{z} = \frac{z}{L}, \quad \bar{u} = \frac{u}{U_0}, \quad \bar{w} = \frac{w}{U_0}$$

$$\bar{P} = \frac{P - P_0}{\rho U_0^2}, \quad \bar{t} = \frac{t U_0}{L}$$

where L is the characteristic length of the problem, U_0 is a reference velocity, and P_0 is the reference pressure.

Sub. all the dimensionless variables into (1), we get,

$$\frac{D\bar{V}}{D\bar{t}} = \bar{g} - \frac{\partial \bar{P}}{\partial \bar{y}} + \frac{\partial}{\partial \bar{x}} \left(\frac{1}{Re} \left(\frac{\partial \bar{V}}{\partial \bar{x}} + \frac{\partial \bar{u}}{\partial \bar{y}} \right) \right) + \frac{\partial}{\partial \bar{y}} \left(\frac{1}{Re} \left(2 \frac{\partial \bar{V}}{\partial \bar{y}} - \frac{2}{3} \nabla \cdot \vec{\bar{V}} \right) \right) + \frac{\partial}{\partial \bar{z}} \left(\frac{1}{Re} \left(\frac{\partial \bar{w}}{\partial \bar{y}} + \frac{\partial \bar{v}}{\partial \bar{z}} \right) \right)$$

where $\bar{g} = \frac{U_0^2}{gL} = Fr$, called Froude Number,

and $Re = \frac{\rho U_0 L}{\mu}$, called Reynolds Number.

So, the conditions for dynamical similarity are

$$Fr_1 = Fr_2 \quad \text{and} \quad Re_1 = Re_2$$

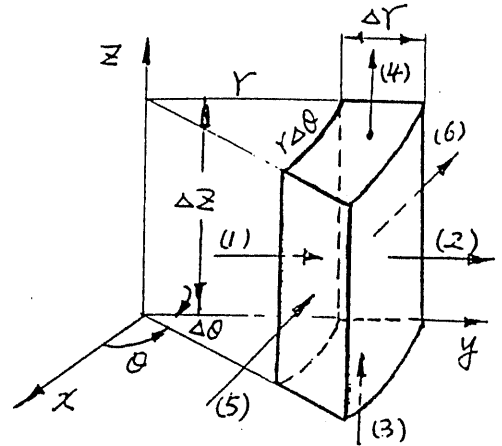
Problem 2.4 The control volume is as shown in the Fig.

The mass flow rate into the C.V. through surface (1) is $(\rho V_r) \cdot r \Delta \theta \cdot \Delta z$.

Taking linear approximation, the mass out-flow through surface (2)

is, $(\rho V_r) r \Delta \theta \Delta z + \frac{\partial}{\partial r}(\rho V_r r \Delta \theta \Delta z) \Delta r$.

So, the net mass in-flow is, $-\frac{\partial}{\partial r}(\rho r V_r) \Delta r \Delta \theta \Delta z$



The mass in-flow through surface (3) is $\rho V_z \cdot \Delta r \cdot r \Delta \theta$, and the mass out-flow through surface (4) is,

$\rho V_z r \Delta r \Delta \theta + \frac{\partial}{\partial z}(\rho V_z r \Delta r \Delta \theta) \Delta z$, so the net mass in-flow is $-\frac{\partial}{\partial z}(\rho V_z) r \Delta r \Delta \theta \Delta z$.

Similarly, the net mass in-flow rate in the tangential direction is

$$-\rho V_\theta \Delta r \Delta z - \frac{\partial}{\partial \theta}(\rho V_\theta \Delta r \Delta z) \Delta \theta + \rho V_\theta \Delta r \Delta z = -\frac{\partial}{\partial \theta}(\rho V_\theta) \Delta r \Delta \theta \Delta z$$

The increase rate of mass in the C.V. is

$$\frac{\partial}{\partial t}(\rho \Delta r \Delta z r \Delta \theta)$$

From the law of mass conservation:

$$\frac{\partial \rho}{\partial t} \Delta r \Delta z \Delta \theta = -\frac{\partial}{\partial r}(\rho r V_r) \Delta r \Delta \theta \Delta z - \frac{\partial}{\partial z}(\rho V_z) r \Delta r \Delta \theta \Delta z - \frac{\partial}{\partial \theta}(\rho V_\theta) \Delta r \Delta \theta \Delta z$$

$$\text{or. } \frac{\partial \rho}{\partial t} + \frac{1}{r} \frac{\partial}{\partial r}(\rho r V_r) + \frac{1}{r} \frac{\partial}{\partial \theta}(\rho V_\theta) + \frac{\partial}{\partial z}(\rho V_z) = 0$$