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Answers to the exercises of chapter 1

Exercises

1.1 (a) $|\vec{e}_i| = 1$

(b) $\vec{e}_i \cdot \vec{e}_j = 0$ if $i \neq j$

$\vec{e}_i \cdot \vec{e}_j = 1$ if $i = j$

(c) $\vec{e}_x \cdot (\vec{e}_y \times \vec{e}_z) = 1$

(d) Definition of a right-handed orthonormal basis.

1.2 $\vec{F}_z = \vec{e}_x - 3\vec{e}_y - 5\vec{e}_z$

1.3 (a)

$$\underset{\sim}{a} = \begin{bmatrix} 0 \\ 0 \\ 4 \end{bmatrix} \qquad \underset{\sim}{b} = \begin{bmatrix} 0 \\ -3 \\ 4 \end{bmatrix} \qquad \underset{\sim}{c} = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}$$

(b)

$$\begin{aligned} \vec{a} + \vec{b} &= -3\vec{e}_y + 8\vec{e}_z \\ 3(\vec{a} + \vec{b} + \vec{c}) &= 3\vec{e}_x - 9\vec{e}_y + 30\vec{e}_z \end{aligned}$$

(c)

$$\begin{aligned} \vec{a} \cdot \vec{b} &= \vec{b} \cdot \vec{a} = 16 \\ \vec{a} \times \vec{b} &= 12\vec{e}_x \\ \vec{b} \times \vec{a} &= -12\vec{e}_x \end{aligned}$$

(d)

$$\begin{aligned} |\vec{a}| &= 4 \\ |\vec{b}| &= 5 \end{aligned}$$

$$|\vec{a} \times \vec{b}| = 12$$
$$|\vec{b} \times \vec{a}| = 12$$

(e) $\phi = \arccos(\frac{4}{5})$
(f) $\vec{e}_x$ or $-\vec{e}_x$
(g)

$$\vec{a} \times \vec{b} \cdot \vec{c} = 12$$
$$\vec{a} \times \vec{c} \cdot \vec{b} = -12$$

(h)

$$\vec{a}\vec{b} \cdot \vec{c} = 32\vec{e}_z$$
$$(\vec{a}\vec{b})^T \cdot \vec{c} = -24\vec{e}_y + 32\vec{e}_z$$
$$\vec{b}\vec{a} \cdot \vec{c} = -24\vec{e}_y + 32\vec{e}_z$$

(i) The vectors are independent, but not perpendicular. So the vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ form a suitable but non-orthogonal basis.

1.4

$$\vec{d} + \vec{e} = 3\vec{a} + 2\vec{b} - 3\vec{c}$$
$$\vec{d} \cdot \vec{e} = 24$$

1.5 (a) $\vec{a}_z = -25\vec{e}_z$
(b) $|\vec{a}_x| = |\vec{a}_y| = 5$; $|\vec{a}_z| = 25$
(c) $\vec{a}_x \times \vec{a}_y \cdot \vec{a}_z = 625$
(d) $\phi = \frac{\pi}{2}$
(e)

$$\vec{\alpha}_x = 4/5\ \vec{e}_x + 3/5\ \vec{e}_y$$
$$\vec{\alpha}_y = 3/5\ \vec{e}_x - 4/5\ \vec{e}_y$$
$$\vec{\alpha}_z = -\vec{e}_z$$

So the basis $\{\vec{\alpha}_x, \vec{\alpha}_y, \vec{\alpha}_z\}$ is right-handed and orthogonal.

(f)

$$\vec{b} = 2\vec{e}_x + 3\vec{e}_y + \vec{e}_z$$
so with respect to $\{\vec{e}_x, \vec{e}_y, \vec{e}_z\}$, $\underset{\sim}{b} = [\ 2\ 3\ 1\ ]^T$

$$\vec{b} = \frac{17}{25}\,\vec{a}_x - \frac{6}{25}\,\vec{a}_y - \frac{1}{25}\,\vec{a}_z$$

so with respect to $\{\vec{a}_x, \vec{a}_y, \vec{a}_z\}$, $\underset{\sim}{b} = \frac{1}{25}[\ 17\ \ -6\ \ -1\ ]^T$

$$\vec{b} = \frac{17}{5}\,\vec{\alpha}_x - \frac{6}{5}\,\vec{\alpha}_y - \vec{\alpha}_z$$

so with respect to $\{\vec{\alpha}_x, \vec{\alpha}_y, \vec{\alpha}_z\}$, $\underset{\sim}{b} = \frac{1}{5}[\ 17\ \ -6\ \ -1\ ]^T$

1.6 The triple product is zero. This means that the vectors are not independent. The vector $\vec{a}$ is lying in the plane, that is defined by the vectors $\vec{b}$ and $\vec{c}$. Relation: $2\vec{a} - \vec{b} - \vec{c} = \vec{0}$.

1.7 Both operators are associated with a rotation.

1.8 (a) $a_x\vec{e}_x$
(b) $a_x\vec{e}_x + a_y\vec{e}_y$
(c) no effect
(d) $a_y\vec{e}_x - a_x\vec{e}_y + a_z\vec{e}_z$
(e) $a_x\vec{e}_x - a_y\vec{e}_y + a_z\vec{e}_z$

2
Answers to the exercises of chapter 2

Exercises

2.1 (a) $\vec{F} = \frac{5}{2}\sqrt{2}\ \vec{\epsilon}_1 + \frac{1}{2}\sqrt{2}\ \vec{\epsilon}_2 - 4\ \vec{\epsilon}_3$
(b) $|\vec{F}| = \sqrt{29}$

2.2 (a) $\vec{M}_P = \vec{0}$; $\vec{M}_R = -2\ \vec{e}_z$
(b) $\underset{\sim}{M}_P = [\ 0\ 0\ 0\ ]^T$; $\underset{\sim}{M}_R = [\ 0\ 0\ \ -2\ ]^T$

2.3 (a) 0
(b) $2F\ell$ positive if counterclockwise
(c) $F\ell$ positive if counterclockwise
(d) $2F\ell$ positive if counterclockwise
(e) $2F\ell$ positive if counterclockwise

2.4 $f = \frac{R}{r}\ F$

2.5 $\vec{M}_P = 6\vec{e}_x - 9\vec{e}_y$

2.6 $\vec{M}_S = 12\vec{e}_z$; $\vec{M}_E = 3\vec{e}_z$

2.7 (a) $\vec{M}_S = 3\vec{e}_y + 3\vec{e}_z$
(b) $\vec{M}_O = -2\vec{e}_x + 22\vec{e}_y + 13\vec{e}_z$