

CHAPTER 1

SOLUTION (1.1)

We have

$$A = 50 \times 75 = 3.75(10^{-3}) \text{ m}^2, \theta = 50^\circ, \text{ and } \sigma_x = P/A.$$

Equations (1.11), with $\theta = 50^\circ$:

$$\sigma_{x'} = 700(10^3) = \sigma_x \cos^2 50^\circ = 0.413 \sigma_x = 110.18 P$$

or $P = 6.35 \text{ kN}$

and

$$|\tau_{x'y'}| = 560(10^3) = \sigma_x \sin 50^\circ \cos 50^\circ = 0.492 \sigma_x = 131.2 P$$

Solving

$$P = 4.27 \text{ kN} = P_{all}$$



SOLUTION (1.2)

Normal stress is

$$\sigma_x = \frac{P}{A} = \frac{125(10^3)}{0.05 \times 0.05} = 50 \text{ MPa}$$

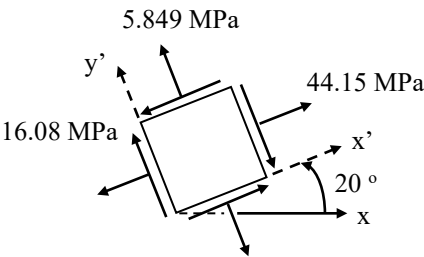
(a) Equations (1.11), with $\theta = 20^\circ$:

$$\sigma_{x'} = 50 \cos^2 20^\circ = 44.15 \text{ MPa}$$

$$\tau_{x'y'} = -50 \sin 20^\circ \cos 20^\circ = -16.08 \text{ MPa}$$



$$\sigma_{y'} = 50 \cos^2 (20^\circ + 90^\circ) = 5.849 \text{ MPa}$$



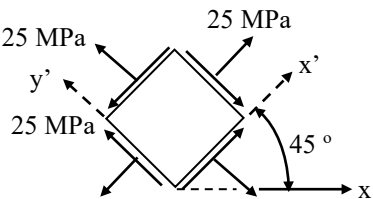
(b) Equations (1.11), with $\theta = 45^\circ$:

$$\sigma_{x'} = 50 \cos^2 45^\circ = 25 \text{ MPa}$$

$$\tau_{x'y'} = -50 \sin 45^\circ \cos 45^\circ = -25 \text{ MPa}$$



$$\sigma_{y'} = 50 \cos^2 (45^\circ + 90^\circ) = 25 \text{ MPa}$$



SOLUTION (1.3)

From Eq. (1.11a),

$$\sigma_x = \frac{\sigma_{x'}}{\cos^2 \theta} = \frac{-75}{\cos^2 30^\circ} = -100 \text{ MPa}$$

For $\theta = 50^\circ$, Eqs. (1.11) give then

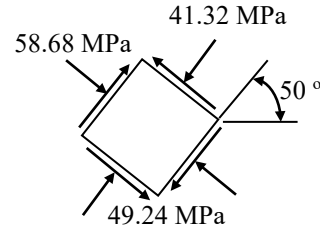
$$\sigma_{x'} = -100 \cos^2 50^\circ = -41.32 \text{ MPa}$$

$$\begin{aligned} \tau_{x'y'} &= -(-100) \sin 50^\circ \cos 50^\circ \\ &= 49.24 \text{ MPa} \end{aligned}$$

Similarly, for $\theta = 140^\circ$:

$$\sigma_{x'} = -100 \cos^2 140^\circ = -58.68 \text{ MPa}$$

$$\tau_{x'y'} = -49.24 \text{ MPa}$$



SOLUTION (1.4)

Refer to Fig. 1.6c. Equations (1.11) by substituting the double angle-trigonometric relations, or Eqs. (1.18) with $\sigma_y = 0$ and $\tau_{xy} = 0$, become

$$\sigma_{x'} = \frac{1}{2} \sigma_x + \frac{1}{2} \sigma_x \cos 2\theta \quad \text{and} \quad |\tau_{x'y'}| = \frac{1}{2} \sigma_x \sin 2\theta$$

or

$$20 = \frac{P}{2A} (1 + \cos 2\theta) \quad \text{and} \quad 10 = \frac{P}{2A} \sin 2\theta$$

The foregoing lead to

$$2 \sin 2\theta - \cos 2\theta = 1 \quad (a)$$

By introducing trigonometric identities, Eq. (a) becomes

$$4 \sin \theta \cos \theta - 2 \cos^2 \theta = 0 \quad \text{or} \quad \tan \theta = 1/2. \text{ Hence}$$

$$\theta = 26.56^\circ$$

Thus,

$$20 = \frac{P}{2(1300)} (1 + 0.6)$$

gives

$$P = 32.5 \text{ kN}$$

It can be shown that use of Mohr's circle yields readily the same result.

SOLUTION (1.5)

Equations (1.12):

$$\sigma_1 = \frac{P}{A} = \frac{-150(10^3)}{\frac{\pi}{4}(50)^2} = -76.4 \text{ MPa}$$

$$\tau_{\max} = \frac{P}{2A} = 38.2 \text{ MPa}$$

SOLUTION (1.6)

Shaded transverse area:

$$A = 2at = 2(10)(75) = 1.5(10^3) \text{ mm}^2$$

Metal is capable of supporting the load

$$P = \sigma A = 90(10^6)(1.5 \times 10^{-3}) = 135 \text{ kN}$$

Apply Eqs. (1.11):

$$\sigma_{x'} = 25(10^6) = \frac{P}{1.5(10^{-3})}(\cos^2 55^\circ), \quad P = 114 \text{ kN}$$

$$\tau_{x'y'} = 12(10^6) = -\frac{P}{1.5(10^{-3})}\sin 55^\circ \cos 55^\circ, \quad P = 38.3 \text{ kN}$$

Thus,

$$P_{all} = 38.3 \text{ kN}$$


SOLUTION (1.7)

Use Eqs. (1.11):

$$\sigma_{x'} = 20(10^6) = \frac{P}{1.5(10^{-3})}(\cos^2 40^\circ), \quad P = 51.1 \text{ kN}$$

$$\tau_{x'y'} = 8(10^6) = -\frac{P}{1.5(10^{-3})}\sin 40^\circ \cos 40^\circ, \quad P = 24.4 \text{ kN}$$

Thus,

$$P_{all} = 24.4 \text{ kN}$$


SOLUTION (1.8)

$$A = 15 \times 30 = 450 \text{ mm}^2$$

Apply Eqs. (1.11):

$$\sigma_{x'} = \frac{120(10^3)}{450 \times 10^{-6}}(\cos^2 40^\circ) = 156 \text{ MPa}$$


$$\tau_{x'y'} = -\frac{120(10^3)}{450 \times 10^{-6}}\sin 40^\circ \cos 40^\circ = -131 \text{ MPa}$$

SOLUTION (1.9)

We have $A = 450(10^{-6}) \text{ m}^2$. Use Eqs. (1.11):

$$\sigma_{x'} = \frac{-100(10^3)}{450 \times 10^{-6}}(\cos^2 60^\circ) = -55.6 \text{ MPa}$$

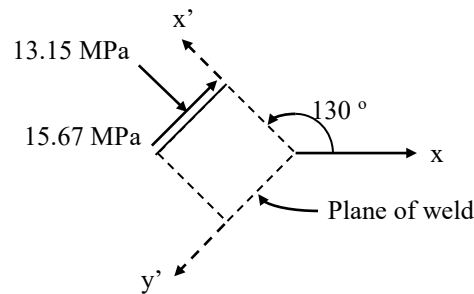

$$\tau_{x'y'} = -\frac{-100(10^3)}{450 \times 10^{-6}}\sin 60^\circ \cos 60^\circ = 96.2 \text{ MPa}$$

SOLUTION (1.10)

$$\theta = 40^\circ + 90^\circ = 130^\circ$$
$$\sigma_x = \frac{P}{A} = -\frac{150(10^3)}{\pi(0.08^2 - 0.07^2)} = -31.83 \text{ MPa}$$

Equations (1.11):

$$\sigma_{x'} = -31.83 \cos^2 130^\circ = -13.15 \text{ MPa}$$
$$\tau_{x'y'} = 31.83 \sin 130^\circ \cos 130^\circ = -15.67 \text{ MPa}$$



SOLUTION (1.11)

Use Eqs. (1.14),

$$(2x) + (-2xy) + (x) + F_x = 0$$
$$(-y^2) + (-2yz + x) + (0) + F_y = 0$$
$$(z - 4xy) + (0) + (-2z) + F_z = 0$$

Solving, we have (in MN/m^3):

$$F_x = -3x + 2xy \quad F_y = -x + y^2 + 2yz \quad F_z = 4xy + z \quad (a)$$

Substituting $x=-0.01 \text{ m}$, $y=0.03 \text{ m}$, and $z=0.06 \text{ m}$, Eqs. (a) yield the following values

$$F_x = 29.4 \text{ kN/m}^3 \quad F_y = 14.5 \text{ kN/m}^3 \quad F_z = 58.8 \text{ kN/m}^3$$

Resultant body force is thus

$$F = \sqrt{F_x^2 + F_y^2 + F_z^2} = 67.32 \text{ kN/m}^3$$

SOLUTION (1.12)

Equations (1.14):

$$-2c_1y - 2c_1y + 0 + 0 = 0, \quad 4c_1y \neq 0$$
$$0 + c_3z + 0 + 0 = 0, \quad c_3z \neq 0$$
$$0 + 0 + 0 + 0 = 0$$

No. Eqs. (1.14) are not satisfied.

SOLUTION (1.13)

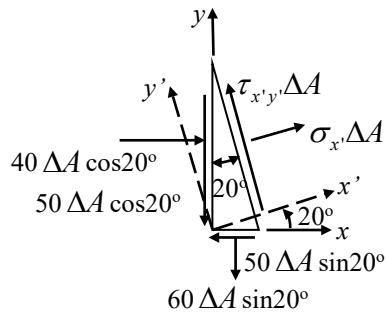
- (a) No. Eqs. (1.14) are not satisfied. ▶
 (b) Yes. Eqs. (1.14) are satisfied. ▶
-

SOLUTION (1.14)

Eqs. (1.14) for the given stress field yield: ▶

$$F_x = F_y = F_z = 0$$

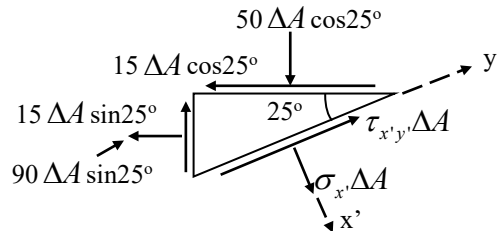
SOLUTION (1.15)



$$\begin{aligned} \sum F_{x'} = 0: \quad & \sigma_{x'}\Delta A + 40\Delta A \cos^2 20^\circ - 60\Delta A \sin^2 20^\circ \\ & - 2(50\Delta A \sin 20^\circ \cos 20^\circ) = 0 \\ \sigma_{x'} = & -35.32 + 7.02 + 32.14 = 3.8 \text{ MPa} \end{aligned} \quad \blacktriangleleft$$

$$\begin{aligned} \sum F_{y'} = 0: \quad & \tau_{x'y'}\Delta A - 40\Delta A \sin 20^\circ \cos 20^\circ \\ & - 60\Delta A \sin 20^\circ \cos 20^\circ - 50\Delta A \cos^2 20^\circ \\ & + 50\Delta A \sin^2 20^\circ = 0 \\ \tau_{x'y'} = & 12.86 + 19.28 + 44.15 - 5.85 = 70.4 \text{ MPa} \end{aligned} \quad \blacktriangleleft$$

SOLUTION (1.16)



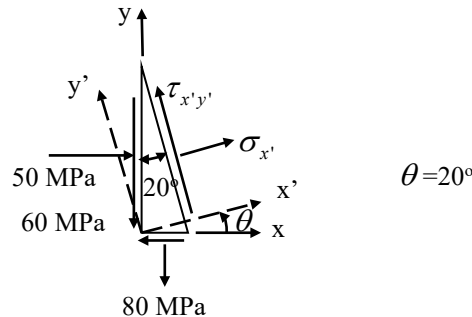
$$\begin{aligned} \sum F_{x'} = 0: \quad & \sigma_{x'}\Delta A + 50\Delta A \cos^2 25^\circ \\ & - 90\Delta A \sin^2 25^\circ - 2(15\Delta A \sin 25^\circ \cos 25^\circ) = 0 \\ \sigma_{x'} = & -41.07 + 16.07 + 11.49 = -13.5 \text{ MPa} \end{aligned} \quad \blacktriangleleft$$

(CONT.)

1.16 (CONT.)

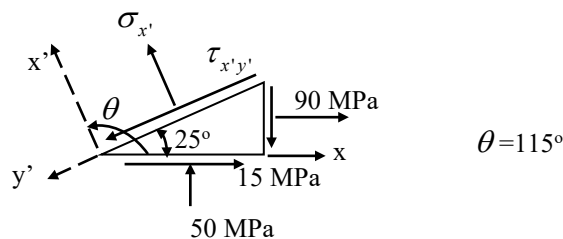
$$\begin{aligned}\sum F_{y'} &= 0: \tau_{x'y'}\Delta A - 50\Delta A \sin 25^\circ \cos 25^\circ \\ &\quad - 90\Delta A \sin 25^\circ \cos 25^\circ - 15\Delta A \cos^2 25^\circ + 15\Delta A \sin^2 25^\circ = 0 \\ \tau_{x'y'} &= 19.15 + 34.47 + 12.32 - 2.68 = 63.3 \text{ MPa}\end{aligned}$$

SOLUTION (1.17)



$$\begin{aligned}\sigma_{x'} &= \frac{1}{2}(-40 + 60) + \frac{1}{2}(-40 - 60)\cos 40^\circ + 50\sin 40^\circ \\ &= 10 - 38.3 + 32.1 = 3.8 \text{ MPa} \\ \tau_{x'y'} &= -\frac{1}{2}(-40 - 60)\sin 40^\circ + 50\cos 40^\circ \\ &= 32.14 + 38.3 = 70.4 \text{ MPa}\end{aligned}$$

SOLUTION (1.18)



$$\begin{aligned}\sigma_{x'} &= \frac{1}{2}(90 - 50) + \frac{1}{2}(90 + 50)\cos 230^\circ - 15\sin 230^\circ \\ &= 20 - 45 + 11.5 = -13.5 \text{ MPa} \\ \tau_{x'y'} &= -\frac{1}{2}(90 + 50)\sin 230^\circ - 15\cos 230^\circ \\ &= 53.62 + 9.64 = 63.3 \text{ MPa}\end{aligned}$$

SOLUTION (1.19)

Transform from $\theta = 40^\circ$ to $\theta = 0$. For convenience in computations, Let

$$\sigma_x = -160 \text{ MPa}, \quad \sigma_y = -80 \text{ MPa}, \quad \tau_{xy} = 40 \text{ MPa} \text{ and } \theta = -40^\circ$$

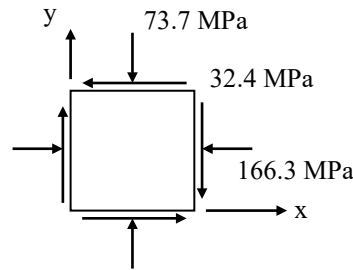
Then

$$\begin{aligned} \sigma_{x'} &= \frac{1}{2}(\sigma_x + \sigma_y) + \frac{1}{2}(\sigma_x - \sigma_y) \cos 2\theta + \tau_{xy} \sin 2\theta \\ &= \frac{1}{2}(-160 - 80) + \frac{1}{2}(-160 + 80) \cos(-80^\circ) + 40 \sin(-80^\circ) \\ &= -166.3 \text{ MPa} \end{aligned} \quad \blacktriangle$$

$$\begin{aligned} \tau_{x'y'} &= -\frac{1}{2}(\sigma_x - \sigma_y) \sin 2\theta + \tau_{xy} \cos 2\theta \\ &= -\frac{1}{2}(-160 + 80) \sin(-80^\circ) + 40 \cos(-80^\circ) \\ &= -32.4 \text{ MPa} \end{aligned} \quad \blacktriangle$$

So $\sigma_{y'} = \sigma_x + \sigma_y - \sigma_{x'} = -160 - 80 + 166.3 = -73.7 \text{ MPa}$ $\blacktriangle$

For $\theta = 0^\circ$:

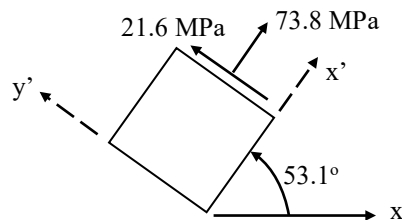


SOLUTION (1.20)

$$\theta = \tan^{-1} \frac{4}{3} = 53.1^\circ$$

$$\begin{aligned} \sigma_{x'} &= \frac{45 + 90}{2} + \frac{45 - 90}{2} \cos 106.2^\circ \\ &= 67.5 + 6.28 = 73.8 \text{ MPa} \end{aligned} \quad \blacktriangleleft$$

$$\tau_{x'y'} = -\frac{45 - 90}{2} \sin 106.2^\circ = 21.6 \text{ MPa}$$



SOLUTION (1.21)

$$\tau_{xy} = 0 \quad \theta = 70^\circ$$

$$(a) \quad \tau_{x'y'} = -30 = -\frac{\sigma - 60}{2} \sin 140^\circ \quad \sigma = 153.3 \text{ MPa} \quad \blacktriangle$$

$$(b) \quad \sigma_{x'} = 80 = \frac{\sigma + 60}{2} + \frac{\sigma - 60}{2} \cos 140^\circ \quad \sigma = 231 \text{ MPa} \quad \blacktriangle$$

SOLUTION (1.22)

Equations(1.18) with $\theta = 60^\circ$, $\sigma_x = 110 \text{ MPa}$, $\sigma_y = 0$, $\tau_{xy} = 50 \text{ MPa}$ give

$$\sigma_{x'} = \frac{1}{2}(110) + \frac{1}{2}(110) \cos 120^\circ + 50 \sin 120^\circ = 70.8 \text{ MPa}$$

$$\tau_{x'y'} = -\frac{1}{2}(110) \sin 120^\circ + 50 \cos 120^\circ = -72.6 \text{ MPa}$$

$$\sigma_{y'} = \frac{1}{2}(110) - \frac{1}{2}(110) \cos 120^\circ - 50 \sin 120^\circ = 39.2 \text{ MPa}$$

SOLUTION (1.23)

Equations(1.18) with $\theta = 30^\circ$, $\sigma_x = 110 \text{ MPa}$, $\sigma_y = 0$, $\tau_{xy} = 50 \text{ MPa}$ result in

$$\sigma_{x'} = \frac{1}{2}(110) + 55 \cos 60^\circ + 50 \sin 60^\circ = 125.8 \text{ MPa}$$

$$\tau_{x'y'} = -\frac{1}{2}(110) \sin 60^\circ + 50 \cos 60^\circ = -22.6 \text{ MPa}$$

$$\sigma_{y'} = \frac{1}{2}(110) - 55 \cos 60^\circ - 50 \sin 60^\circ = -15.8 \text{ MPa}$$

SOLUTION (1.24)

We have

$$\theta = 25 + 90 = 115^\circ$$

$$\sigma_x = -10 \text{ MPa}$$

$$\sigma_y = 30 \text{ MPa}$$

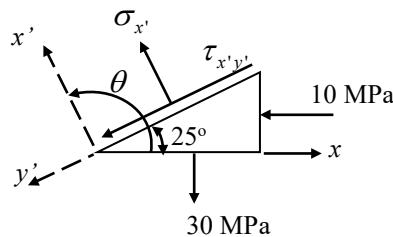
$$\tau_{xy} = 0$$

$$(a) \quad \sigma_{x'} = \frac{1}{2}(\sigma_x + \sigma_y) + \frac{1}{2}(\sigma_x - \sigma_y) \cos 2\theta$$

$$= \frac{1}{2}(-10 + 30) + \frac{1}{2}(-10 - 30) \cos 230^\circ = 22.86 \text{ MPa} \quad \blacktriangleleft$$

Thus,

$$\sigma_w = \sigma_{x'} = 22.86 \text{ MPa}$$



(CONT.)

1.24 (CONT.)

$$\begin{aligned} \text{(b)} \quad \tau_{x'y'} &= -\frac{1}{2}(\sigma_x - \sigma_y) \sin 2\theta \\ &= -\frac{1}{2}(-10 - 30) \sin 230^\circ = -15.32 \text{ MPa} \end{aligned}$$

So

$$\tau_w = \tau_{x'y'} = -15.32 \text{ MPa}$$



SOLUTION (1.25)

$$\begin{aligned} \text{(a)} \quad \sigma_1 &= 80 = \frac{0 + 50}{2} + \sqrt{\left(\frac{0 - 50}{2}\right)^2 + \tau^2} \\ \tau &= 49 \text{ MPa} \end{aligned}$$



$$\text{(b)} \quad \tau_{\max} = \sqrt{\left(\frac{-50}{2}\right)^2 + 49^2} = 55 \text{ MPa}$$

$$\sigma' = \frac{50}{2} = 25 \text{ MPa}$$

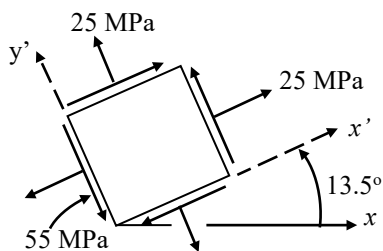


$$2\theta_s = \tan^{-1}\left[-\frac{0 - 50}{2(49)}\right] = 27^\circ$$

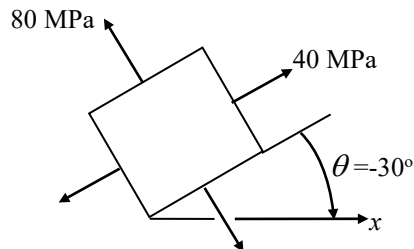
$$\tau_{x'y'} = \frac{50}{2} \sin 27^\circ + 49 \cos 27^\circ = 55 \text{ MPa}$$

Thus,

$$\theta_s' = 13.5^\circ$$



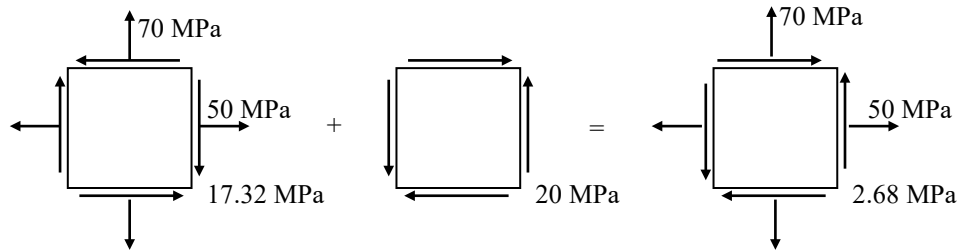
SOLUTION (1.26)



(CONT.)

1.26 (CONT.)

$$\sigma_x = \frac{40+80}{2} + \frac{40-80}{2} \cos(-60^\circ) = 60 - 10 = 50 \text{ MPa}$$
$$\sigma_y = 60 + 10 = 70 \text{ MPa}$$
$$\tau_{xy} = -\frac{40-80}{2} \sin(-60^\circ) = -17.32 \text{ MPa}$$

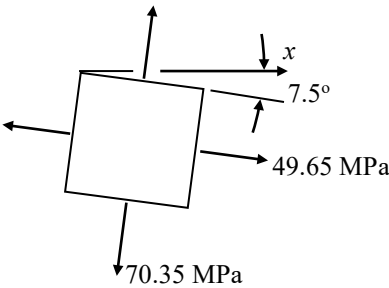


$$\sigma_{1,2} = \frac{50+70}{2} \pm \sqrt{\left(\frac{50-70}{2}\right)^2 + 2.68^2} = 60 \pm 10.35$$
$$\sigma_1 = 70.35 \text{ MPa} \quad \sigma_2 = 49.65 \text{ MPa}$$
$$2\theta_p = \tan^{-1}\left[\frac{2(2.68)}{50-70}\right] = -15^\circ$$

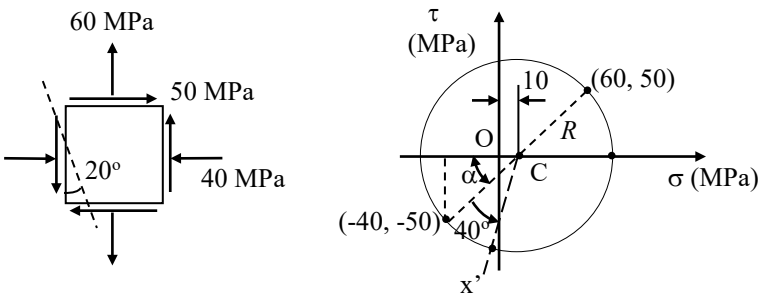
$$\sigma_{x'} = 60 + \frac{50-70}{2} \cos(-15^\circ) + 2.68 \sin(-15^\circ)$$
$$= 60 - 9.66 - 0.694 = 49.65 \text{ MPa}$$

Thus,

$$\theta_p'' = -7.5^\circ$$



SOLUTION (1.27)

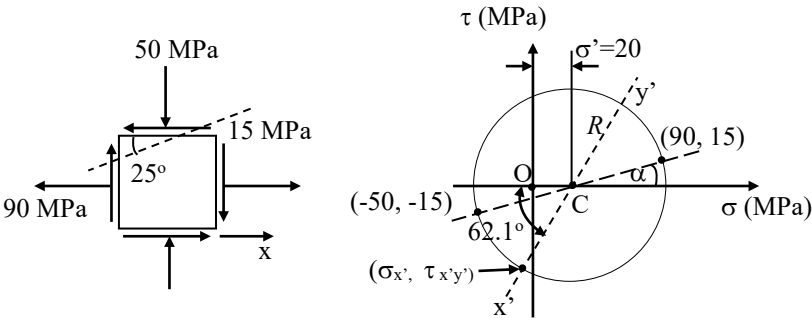


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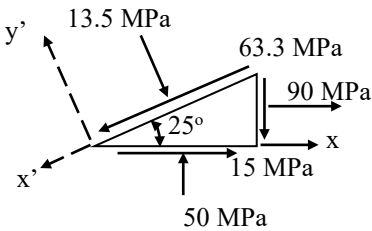
1.27 (CONT.)

$$\alpha = \tan^{-1} \frac{50}{50} = 45^\circ$$
$$R = (50^2 + 50^2)^{\frac{1}{2}} = 70.7$$
$$\tau_{x'y'} = \sin 85^\circ (70.7) = 70.4 \text{ MPa}$$
$$\sigma_{x'} = 10 - \cos 85^\circ (70.7) = 3.84 \text{ MPa}$$

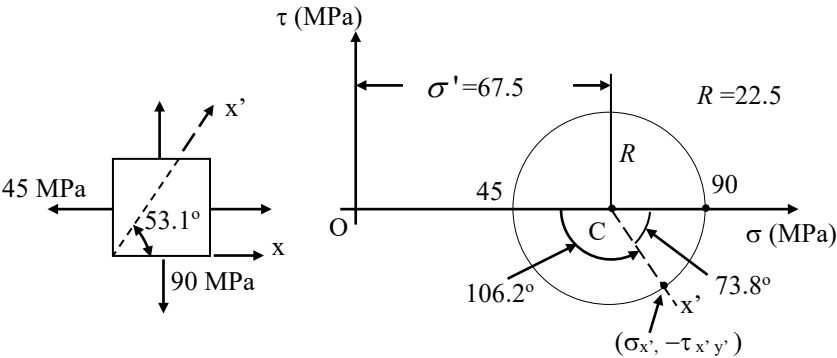
SOLUTION (1.28)



$$\alpha = \tan^{-1} \frac{15}{70} = 12.1^\circ$$
$$R = (15^2 + 70^2)^{\frac{1}{2}} = 71.6$$
$$\tau_{x'y'} = 71.6 \sin 62.1^\circ = 63.3 \text{ MPa}$$
$$\sigma_{x'} = -71.6 \cos 62.1^\circ + 20$$
$$= -13.5 \text{ MPa}$$



SOLUTION (1.29)



(CONT.)

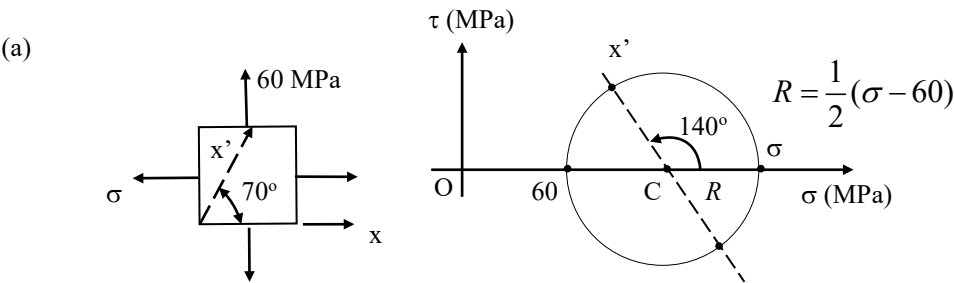
1.29 (CONT.)

$\tau_{x'y'} = 22.5 \sin 73.8^\circ = 21.6 \text{ MPa}$ ▶

$\sigma_{x'} = 67.5 + 22.5 \cos 73.8^\circ = 73.8 \text{ MPa}$

Sketch of results is as shown in solution of Prob. 1.20.

SOLUTION (1.30)



$\tau_{x'y'} = -30 = \frac{\sigma - 60}{2} \sin(-40^\circ); \quad \sigma = 153.3 \text{ MPa}$ ▲

(b) $\sigma_{x'} = 80 = 60 + \frac{\sigma - 60}{2} [1 - \cos(-40^\circ)]$ ▲
 $\sigma = 231 \text{ MPa}$

SOLUTION (1.31)

(a) From Mohr's circle, Fig. (a):
 $\sigma_1 = 121 \text{ MPa} \quad \sigma_2 = -71 \text{ MPa} \quad \tau_{\max} = 96 \text{ MPa}$ ▲
 $\theta_p' = -19.3^\circ \quad \theta_s' = 25.7^\circ$

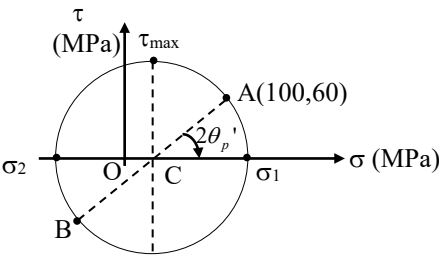


Figure (a)

By applying Eq. (1.20):

$\sigma_{1,2} = \frac{50}{2} \pm \left[\frac{22,500}{4} + 3600 \right]^{\frac{1}{2}} = 25 \pm 96$

or $\sigma_1 = 121 \text{ MPa} \quad \sigma_2 = -71 \text{ MPa}$ ▲

Using Eq. (1.19):

$\tan 2\theta_p = -\frac{12}{15} = -0.8$ ▲

$\theta_p' = -19.3^\circ \quad \theta_s' = 25.7^\circ$

(CONT.)

1.31 (CONT.)

(b) From Mohr's circle, Fig. (b):

$$\sigma_1 = 200 \text{ MPa} \quad \sigma_2 = -50 \text{ MPa} \quad \tau_{\max} = 125 \text{ MPa}$$

$$\theta_p' = 26.55^\circ \quad \theta_s' = 71.55^\circ$$

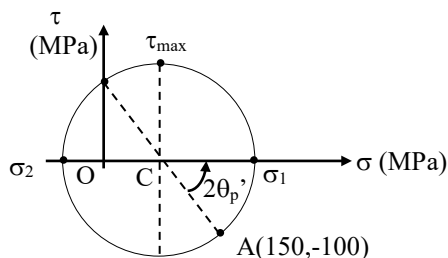


Figure (b)

Through the use of Eq. (1.20),

$$\sigma_{1,2} = 75 \pm \left[\frac{22,500}{4} + 10,000 \right]^{\frac{1}{2}} = 75 \pm 125$$

or

$$\sigma_1 = 200 \text{ MPa} \quad \sigma_2 = -50 \text{ MPa}$$

Using Eq. (1.19), $\tan 2\theta_p = 4/3$:

$$\theta_p' = 26.57^\circ \quad \theta_s' = 71.57^\circ$$

SOLUTION (1.32)

Referring to Mohr's circle, Fig. 1.15:

$$\sigma_{x'} = \frac{\sigma_1 + \sigma_2}{2} + \frac{\sigma_1 - \sigma_2}{2} \cos 2\theta \quad (a)$$

$$\sigma_{y'} = \frac{\sigma_1 + \sigma_2}{2} - \frac{\sigma_1 - \sigma_2}{2} \cos 2\theta$$

$$\tau_{x'y'} = \frac{\sigma_1 - \sigma_2}{2} \sin 2\theta \quad (b)$$

From Eqs. (a),

$$\sigma_{x'} + \sigma_{y'} = \sigma_1 + \sigma_2$$

By using $\cos^2 2\theta + \sin^2 2\theta = 1$, and Eqs. (a) and (b), we have

$$\sigma_{x'} \cdot \sigma_{y'} - \tau_{x'y'}^2 = \sigma_1 \cdot \sigma_2 = \text{const.}$$

SOLUTION (1.33)

We have

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{2(-70)}{50 - (-190)} = -0.583$$

$$2\theta_p = -30.24^\circ \quad \text{and} \quad \theta_p = -15.12^\circ$$

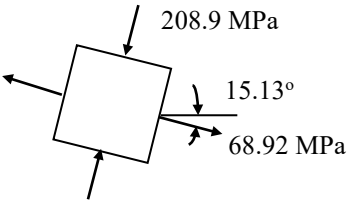
(CONT.)

1.33 (CONT.)

Equations (1.18):

$$\sigma_{x'} = \frac{50-190}{2} + \frac{50+190}{2} \cos(-30.26^\circ) - 70 \sin(-30.26^\circ)$$
$$= -70 + 103.65 + 35.275 = 68.93 \text{ MPa} = \sigma_1 \quad \blacktriangle$$

$$\sigma_{y'} = \sigma_x + \sigma_y - \sigma_{x'} = -208.9 \text{ MPa} = \sigma_2$$



SOLUTION (1.34)

$$\tau_{\max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

Substituting the given values

$$140^2 = \left(\frac{60+100}{2}\right)^2 + \tau_{xy}^2$$

or

$$\tau_{xy, \max} = 114.89 \text{ MPa} \quad \blacktriangle$$

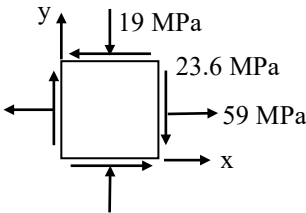
SOLUTION (1.35)

Transform from $\theta = 60^\circ$ to $\theta = 0^\circ$ with $\sigma_{x'} = -20 \text{ MPa}$, $\sigma_{y'} = 60 \text{ MPa}$, $\tau_{x'y'} = -22 \text{ MPa}$, and $\theta = -60^\circ$. Use Eqs. (1.18):

$$\sigma_x = \frac{-20+60}{2} + \frac{-20-60}{2} \cos 2(-60^\circ) - 22 \sin 2(-60^\circ) = 59 \text{ MPa} \quad \blacktriangleleft$$

$$\sigma_y = \sigma_{x'} + \sigma_{y'} - \sigma_x = -19 \text{ MPa}$$

$$\tau_{xy} = -23.6 \text{ MPa}$$



SOLUTION (1.36)

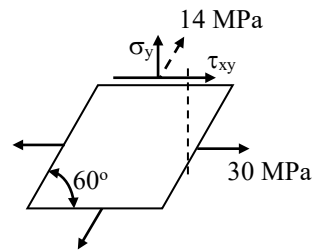


Figure (a)

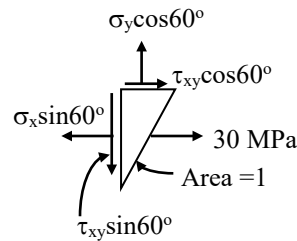


Figure (b)

(a) Figure (a):

$$\sigma_y = 14 \sin 60^\circ = 12.12 \text{ MPa}$$

$$\tau_{xy} = 14 \cos 60^\circ = 7 \text{ MPa}$$

Figure (b):

$$\sum F_y = 12.12 \cos 60^\circ - \tau_{xy} \sin 60^\circ = 0$$

or

$$\tau_{xy} = 7 \text{ MPa (as before)}$$

$$\sum F_x = -\sigma_x \sin 60^\circ + 30 + 7 \cos 60^\circ = 0$$

or

$$\sigma_x = 38.68 \text{ MPa}$$

(b) Equation (1.20) is therefore:

$$\sigma_{1,2} = \frac{38.68+12.12}{2} \pm \left[\left(\frac{38.68-12.12}{2} \right)^2 + 7^2 \right]^{\frac{1}{2}}$$

or

$$\sigma_1 = 40.41 \text{ MPa}, \quad \sigma_2 = 10.39 \text{ MPa}$$

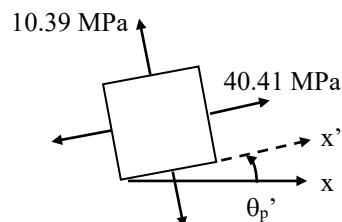
Also,

$$\theta_p = \frac{1}{2} \tan^{-1} \frac{2(7)}{38.68-12.12} = 13.9^\circ$$

Note: Eq. (1.18a) gives, $\sigma_{x'} = 40.41 \text{ MPa}$.

Thus,

$$\theta_p' = 13.9^\circ$$



SOLUTION (1.37)

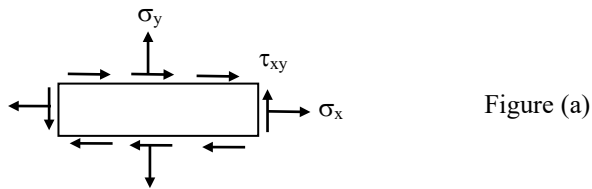
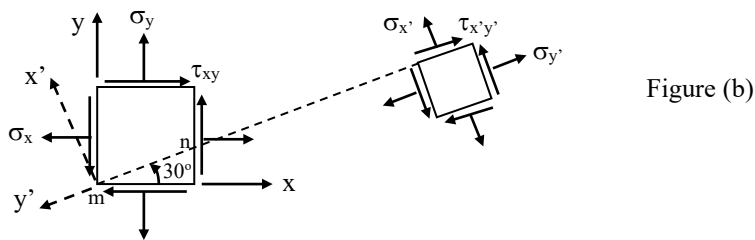


Figure (a):

$$\sigma_x = 100 \cos 45^\circ = 70.7 \text{ MPa}$$
$$\sigma_y = 100 \sin 45^\circ = 70.7 \text{ MPa}$$
$$\tau_{xy} = 100 \cos 45^\circ = 70.7 \text{ MPa}$$

Now, Eqs. (1.18) give (Fig. b):

$$\sigma_{x'} = 70.7 + 0 + 70.7 \sin 240^\circ = 9.47 \text{ MPa}$$
$$\tau_{x'y'} = -0 + 70.7 \cos 240^\circ = -35.35 \text{ MPa}$$
$$\sigma_{y'} = 70.7 - 0 - 70.7 \sin 240^\circ = 131.9 \text{ MPa}$$



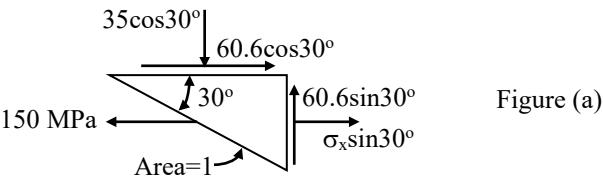
SOLUTION (1.38)

$$\sigma_y = -70 \sin 30^\circ = -35 \text{ MPa}$$
$$\tau_{xy} = 70 \cos 30^\circ = 60.6 \text{ MPa}$$

(a) Figure (a):

$$\sum F_x = -150 + 0.5\sigma_x + 60.6(0.866) = 0$$

or $\sigma_x = 195 \text{ MPa}$



(CONT.)

1.38 (CONT.)

(b) Equation (1.20):

$$\sigma_{1,2} = \frac{195-35}{2} \pm \left[\left(\frac{195+35}{2} \right)^2 + 60.6^2 \right]^{\frac{1}{2}}$$

or $\sigma_1 = 210 \text{ MPa} \quad \sigma_2 = -50 \text{ MPa}$ ▶

Also,

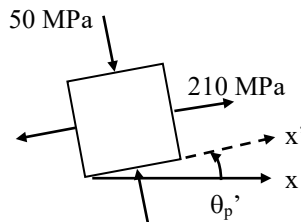
$$\theta_p = \frac{1}{2} \tan^{-1} \frac{2(60.6)}{195+35} = 13.89^\circ$$

Equation (1.18a):

$$\sigma_{x'} = 80 + 115 \cos 2(13.89^\circ) + 60.6 \sin 2(13.89^\circ) = 210 \text{ MPa}$$

Thus,

$$\theta_p' = 13.89^\circ$$



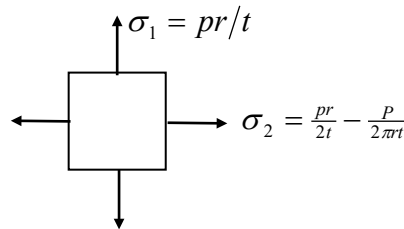
SOLUTION (1.39)

For pure shear, $\sigma_1 = -\sigma_2$:

$$\frac{pr}{t} = -\frac{pr}{2t} + \frac{P}{2\pi t}$$

from which

$$P = 3\pi pr^2$$
 ◀

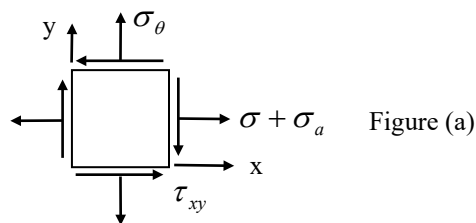


SOLUTION (1.40)

Table D.4:

$$A = 2\pi r t$$

$$J = 2\pi r^3 t$$



Stresses are (Fig. a):

$$\sigma = \frac{-P}{A} = \frac{-30(10^3)\pi}{2\pi(0.12)(0.005)} = -25 \text{ MPa}$$

$$\sigma_a = \frac{pr}{2t} = \frac{4(10^6)120}{2(5)} = 48 \text{ MPa}$$

$$\sigma_\theta = 2\sigma_a = 96 \text{ MPa}$$

$$\tau_{xy} = \frac{-Tr}{J} = \frac{-10\pi(10^3)}{2\pi(0.12^2)(0.005)} = -69.4 \text{ MPa}$$

(CONT.)