برای دسترسی به نسخه کامل حل المسائل، روی لینک زیر کلیک کنید و یا به وبسایت "ایبوک یاب" مراجعه بفرمایید
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Problem 1.1

Given points P_1 (1, -1, 2) and P_2 (1, 0, -3)

(a) Computation of the unit vector in the direction of $\overrightarrow{P_1P_2}$

The position vectors of points P_1 and P_2 are

$$\overrightarrow{OP_1} = \underline{i}_1 - \underline{i}_2 + 2\underline{i}_3$$

$$\overrightarrow{OP_2} = \underline{i}_1 - 3\underline{i}_3$$

According to the parallelogram rule the vector $\overrightarrow{P_1P_2}$ is obtained as follows

$$\overrightarrow{P_1P_2} = \overrightarrow{OP_2} - \overrightarrow{OP_1} = \underline{i}_2 - 5\underline{i}_3$$

The unit vector in the direction of $\overrightarrow{P_1P_2}$ is

$$\underline{i}_{12} = \frac{\overrightarrow{P_1P_2}}{|P_1P_2|} = \frac{\underline{i}_2 - 5\underline{i}_3}{\sqrt{26}}$$

(b) Computation of the angle $< P_1OP_2$

The dot product of the vectors $\overrightarrow{OP_1}$ and $\overrightarrow{OP_2}$ is

$$(\overrightarrow{OP_1})(\overrightarrow{OP_2}) = 1 - 6 = 5$$

We have also that

$$(\overrightarrow{OP_1})(\overrightarrow{OP_2}) = |\overrightarrow{OP_1}||\overrightarrow{OP_2}|\cos\Theta = (\sqrt{6})(\sqrt{10})\cos\Theta$$

and

$$\cos\Theta = \frac{-5}{\sqrt{60}} = -0.6454$$

or

$$\Theta = 130.2^\circ$$

Computation of the angle $<OP_1P_2$

The vectors $\overrightarrow{P_1O}$ and $\overrightarrow{P_1P_2}$ are

$$\overrightarrow{P_1O} = -\underline{i}_1 + \underline{i}_2 - 2\underline{i}_3$$

$$\overrightarrow{P_1P_2} = \underline{i}_2 - 5\underline{i}_3$$

Their dot product is

$$(\overrightarrow{P_1O})(\overrightarrow{P_1P_2}) = 1 + 10 = 11$$

We have also that

$$(\overrightarrow{P_1O})(\overrightarrow{P_1P_2}) = |\overrightarrow{P_1O}||\overrightarrow{P_1P_2}|\cos\phi = (\sqrt{6})(\sqrt{26})\cos\phi$$

and

$$\cos\phi = \frac{11}{(\sqrt{6})(\sqrt{26})} = 0.8807$$

or

$$\phi = 28.3^\circ$$

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(c) Computation of the unit vector $\mathbf{i}_n$ normal to the plane specified by the points O, P_1 and P_2.

$$\overrightarrow{OP_1} = \underline{i}_1 - \underline{i}_2 + 2\underline{i}_3$$
$$\overrightarrow{OP_2} = \underline{i}_1 - 3\underline{i}_3$$

The cross product of these vectors is

$$(\overrightarrow{OP_1}) \times (\overrightarrow{OP_2}) = (3+0)\underline{i}_1 + (2+3)\underline{i}_2 + (0+1)\underline{i}_3 = 3\underline{i}_1 + 5\underline{i}_2 + \underline{i}_3$$

and

$$\underline{i}_n = \pm \frac{3\underline{i}_1 + 5\underline{i}_2 + \underline{i}_3}{\sqrt{35}}$$

(d) Computation of the volume of the parallelepiped whose edges are OP_1, OP_2 and OP_3.

$$\overrightarrow{OP_1} = \underline{i}_1 - \underline{i}_2 + 2\underline{i}_3$$
$$\overrightarrow{OP_2} = \underline{i}_1 - 3\underline{i}_3$$
$$\overrightarrow{OP_3} = -\underline{i}_1 + 2\underline{i}_2 + \underline{i}_3$$

As shown above

$$(\overrightarrow{OP_1}) \times (\overrightarrow{OP_2}) = 3\underline{i}_1 + 5\underline{i}_2 + \underline{i}_3$$

And the volume V of the parallelepiped is

$$V = (\overrightarrow{OP_3}) \left[(\overrightarrow{OP_1}) \times (\overrightarrow{OP_2}) \right] = -3 + 10 + 1 = 8\,m^3$$

Problem 1.2

Referring to relation (1.31) we have

$$a'_i = \sum_{j=1}^{3} \lambda_{ij} a_j = \lambda_{i1} a_1 + \lambda_{i2} a_2$$

Thus,

$$a'_1 = \lambda_{11} a_1 + \lambda_{12} a_2 = a_1 = 4$$
$$a'_2 = \lambda_{21} a_1 + \lambda_{22} a_2 = \frac{a_2}{\sqrt{2}} = \frac{3}{\sqrt{2}}$$
$$a'_3 = \lambda_{31} a_1 + \lambda_{32} a_2 = \frac{-a_2}{\sqrt{2}} = \frac{-3}{\sqrt{2}}$$

or

$$\underline{a} = 4\underline{i}_1 + \frac{3}{\sqrt{2}}\underline{i}_2 - \frac{3}{\sqrt{2}}\underline{i}_3 = 4\underline{i}_1 + \left(\frac{3\sqrt{2}}{2}\right)\underline{i}_2 - \left(\frac{3\sqrt{2}}{2}\right)\underline{i}_3$$

Another way of solving this problem is referring to relation (1.33a)

$$\begin{Bmatrix} a'_1 \\ a'_2 \\ a'_3 \end{Bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 0 & \frac{-1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix} \begin{Bmatrix} 4 \\ 3 \\ 0 \end{Bmatrix} = \begin{Bmatrix} 4 \\ \frac{3}{\sqrt{2}} \\ -\frac{3}{\sqrt{2}} \end{Bmatrix}$$

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Problem 1.3

(a) Calculation of the angle between the vectors **a** and **b**. We denote this angle by Θ

$$\cos\Theta = \frac{\underline{a}\cdot\underline{b}}{|\underline{a}||\underline{b}|} = \frac{12+8-12}{\sqrt{29}\sqrt{56}} = \frac{8}{\sqrt{(29)(56)}} = 0.1985$$

or

$$\Theta = 78.55^\circ$$

(b) The component of the vector **b** in the direction of vector **a** is given by the following dot product

$$\underline{b}\cdot\frac{\underline{a}}{|\underline{a}|} = \frac{8}{\sqrt{29}} = 1.49\,m$$

(c) We calculate the cross product of the vectors **a** and **b** referring to relation (1.19)

$$\underline{a}\times\underline{b} = (24+4)\underline{i}_1 + (8+18)\underline{i}_2 + (-6+16)\underline{i}_3 = 28\underline{i}_1 + 26\underline{i}_2 + 10\underline{i}_3$$

The area of the parallelogram whose sides are the vectors **a** and **b** is equal to the magnitude of the vector (**a** x **b**), thus

$$|(\underline{a}\times\underline{b})| = \sqrt{28^2+26^2+10^2} = 39.5\,m^2$$

(d) The volume of the parallelepiped specified by the vectors **a**, **b** and **c** is equal to the absolute value of the dot product of the vectors **c** and (**a** x **b**), thus

$$|\underline{c}\cdot(\underline{a}\times\underline{b})| = |(\underline{i}_1 - 6\underline{i}_2 + \underline{i}_3)\cdot(28\underline{i}_1 + 26\underline{i}_2 + 10\underline{i}_3)| = |28-156+10| =$$
$$|-118| = 118\,m^3$$

Problem 1.4

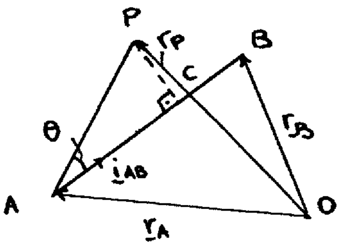

Figure a

The position vectors of points A, B and P are

$$\underline{r}_A = -2\underline{i}_1 + \underline{i}_2 + 3\underline{i}_3$$
$$\underline{r}_B = -\underline{i}_1 + 2\underline{i}_2 - \underline{i}_3$$
$$\underline{r}_P = -\underline{i}_1 + 2\underline{i}_2$$

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And the vector $\overrightarrow{AB}$ is

$$\overrightarrow{AB} = \underline{r}_B - \underline{r}_A = \underline{i}_1 + \underline{i}_2 - 4\underline{i}_3$$

The unit vector in the direction of the vector $\overrightarrow{AB}$ is

$$\underline{i}_{AB} = \frac{\underline{i}_1 + \underline{i}_2 - 4\underline{i}_3}{\sqrt{18}}$$

Referring to Fig. a the shortest distance from point *P* to the line passing through points *A* and *B* is equal to the magnitude of the vector PC. Thus,

$$|\overrightarrow{PC}| = |\overrightarrow{AP}| \sin\Theta \qquad \text{(a)}$$

where

$$\overrightarrow{AP} = \underline{r}_P - \underline{r}_A = -\underline{i}_1 + 2\underline{i}_2 + 2\underline{i}_1 - \underline{i}_2 - 3\underline{i}_3 = \underline{i}_1 + \underline{i}_2 - 3\underline{i}_3$$

We have also that

$$|\overrightarrow{PC}| = |\underline{i}_{AB} \times \overrightarrow{AP}| \qquad \text{(b)}$$

Thus,

$$\underline{i}_{AB} \times \overrightarrow{AP} = \frac{(\underline{i}_1 + \underline{i}_2 - 4\underline{i}_3)}{\sqrt{18}} \times (\underline{i}_1 + \underline{i}_2 - 3\underline{i}_3) = \frac{1}{\sqrt{18}}\left[(-3+4)\underline{i}_1 + (-4+3)\underline{i}_2 + (1-1)\underline{i}_3\right]$$

$$= \frac{1}{\sqrt{18}}(\underline{i}_1 - \underline{i}_2)$$

And the magnitude of the vector ($\mathbf{i}_{AB}$ x AP) is

$$|\underline{i}_{AB} \times \overrightarrow{AP}| = \sqrt{\frac{2}{18}} = \frac{1}{3} \qquad \text{(c)}$$

Substituting relation (c) into (b) we have

$$|\underline{i}_{AB} \times \overrightarrow{AP}| = |\overrightarrow{PC}| = \frac{1}{3}$$

Problem 1.5

(a) Referring to relations (1.25a) we have

$$\begin{aligned} &\lambda_{11}\lambda_{13} + \lambda_{21}\lambda_{23} + \lambda_{31}\lambda_{33} = 0 \\ &\lambda_{12}\lambda_{13} + \lambda_{22}\lambda_{23} + \lambda_{32}\lambda_{33} = 0 \\ &\lambda_{13}^2 + \lambda_{23}^2 + \lambda_{33}^2 = 1 \end{aligned} \qquad \text{(a)}$$

From the data of the problem we have

$$\begin{aligned} &\lambda_{11} = 0 \qquad \lambda_{12} = -\frac{4}{5} \qquad \lambda_{13} = \frac{3}{5} \\ &\lambda_{21} = 1 \qquad \lambda_{22} = 0 \qquad \lambda_{23} = 0 \end{aligned} \qquad \text{(b)}$$

Substituting (b) into (a) we obtain

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$$\lambda_{31}\lambda_{33} = 0$$
$$\left(-\frac{4}{5}\right)\left(\frac{3}{5}\right) + \lambda_{32}\lambda_{33} = 0$$
$$\left(\frac{3}{5}\right)^2 + \lambda_{33}^2 = 1$$

or

$$\lambda_{33} = \pm\frac{4}{5}, \quad \lambda_{31} = 0, \quad \lambda_{32} = \pm\frac{3}{5}$$

(b) We denote the given tensor by [A]. Referring to relation (1.37a) we have

$$[A'] = [\Lambda][A][\Lambda]^T =$$

$$\begin{bmatrix} 0 & -\frac{4}{5} & \frac{3}{5} \\ 1 & 0 & 0 \\ 0 & \frac{3}{5} & \frac{4}{5} \end{bmatrix} \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & -5 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ -\frac{4}{5} & 0 & \frac{3}{5} \\ \frac{3}{5} & 0 & \frac{4}{5} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{117}{25} & 0 & \frac{56}{25} \\ 0 & 2 & 0 \\ -\frac{69}{25} & 0 & -\frac{17}{25} \end{bmatrix} = \begin{bmatrix} 4.68 & 0 & 2.24 \\ 0 & 2 & 0 \\ -2.76 & 0 & -0.68 \end{bmatrix}$$

Problem 1.6

(a) The coordinates of point P with respect to the system of axes x_1', x_2', x_3' are

$$\begin{Bmatrix} x_1' \\ x_2' \\ x_3' \end{Bmatrix} = \begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{3}{5\sqrt{2}} & -\frac{4}{5\sqrt{2}} \\ 0 & \frac{4}{5} & -\frac{3}{5} \\ \frac{1}{\sqrt{2}} & \frac{3}{5\sqrt{2}} & \frac{4}{5\sqrt{2}} \end{bmatrix} \begin{Bmatrix} -2 \\ 1 \\ 0 \end{Bmatrix} = \begin{Bmatrix} -\frac{13}{5\sqrt{2}} \\ \frac{4}{5} \\ -\frac{7}{5\sqrt{2}} \end{Bmatrix} = \begin{Bmatrix} -1.84 \\ 0.8 \\ -0.99 \end{Bmatrix}$$

(b) Expanding relation (1.70) we obtain

$$A'_{ij} = \sum_{k=1}^{3}\sum_{m=1}^{3} \lambda_{ik}\lambda_{jm}A_{km} = \sum_{k=1}^{3}(\lambda_{ik}\lambda_{j1}A_{k1} + \lambda_{ik}\lambda_{j2}A_{k2} + \lambda_{ik}\lambda_{j3}A_{k3})$$
$$= \lambda_{i1}\lambda_{j2}A_{12} + \lambda_{i2}\lambda_{j2}A_{22} + \lambda_{i3}\lambda_{j2}A_{32}$$

Thus substituting the given values in this equation we have

$$A'_{11} = (\lambda_{11}\lambda_{12})A_{12} + \lambda_{12}^2A_{22} + (\lambda_{13}\lambda_{12})A_{32}$$
$$= -\frac{3}{5} - \frac{9}{50} - \frac{24}{50} = \frac{-63}{50} = -1.26$$

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$$A'_{12} = (\lambda_{11}\lambda_{22})A_{12} + (\lambda_{12}\lambda_{22})A_{22} + (\lambda_{13}\lambda_{22})A_{32}$$
$$= \frac{8}{5\sqrt{2}} + \frac{12}{25\sqrt{2}} + \frac{32}{25\sqrt{2}} = \frac{84}{25\sqrt{2}} = 2.376$$

$$A'_{13} = (\lambda_{11}\lambda_{32})A_{12} + (\lambda_{12}\lambda_{32})A_{22} + (\lambda_{13}\lambda_{32})A_{32}$$
$$= \frac{3}{5} + \frac{9}{50} + \frac{24}{50} = \frac{63}{50} = 1.260$$

$$A'_{21} = (\lambda_{22}\lambda_{12})A_{22} + (\lambda_{23}\lambda_{12})A_{32}$$
$$= \frac{12}{25\sqrt{2}} - \frac{18}{25\sqrt{2}} = \frac{-6}{25\sqrt{2}} = -0.170$$

$$A'_{22} = \lambda_{22}^2 A_{22} + (\lambda_{23}\lambda_{22})A_{32}$$
$$= \frac{-16}{25} + \frac{24}{25} = \frac{8}{25} = 0.320$$

$$A'_{23} = (\lambda_{22}\lambda_{32})A_{22} + (\lambda_{23}\lambda_{32})A_{32}$$
$$= \frac{-12}{25\sqrt{2}} + \frac{18}{25\sqrt{2}} = \frac{6}{25\sqrt{2}} = 0.170$$

$$A'_{31} = (\lambda_{31}\lambda_{12})A_{12} + (\lambda_{32}\lambda_{12})A_{22} + (\lambda_{33}\lambda_{12})A_{32}$$
$$= -\frac{3}{5} + \frac{9}{50} + \frac{24}{50} = \frac{3}{50} = 0.06$$

$$A'_{32} = (\lambda_{31}\lambda_{22})A_{12} + (\lambda_{32}\lambda_{22})A_{22} + (\lambda_{33}\lambda_{22})A_{32}$$
$$= \frac{8}{5\sqrt{2}} - \frac{12}{25\sqrt{2}} - \frac{32}{25\sqrt{2}} = \frac{-4}{25\sqrt{2}} = -0.113$$

$$A'_{33} = (\lambda_{31}\lambda_{32})A_{12} + (\lambda_{32}\lambda_{32})A_{22} + (\lambda_{33}\lambda_{32})A_{32}$$
$$= \frac{3}{5} - \frac{9}{50} - \frac{24}{50} = -\frac{3}{50} = -0.06$$

The same results may be obtained using matrix multiplication. That is

$$[A'] = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{-3}{5\sqrt{2}} & -\frac{4}{5\sqrt{2}} \\ 0 & \frac{4}{5} & -\frac{3}{5} \\ \frac{1}{\sqrt{2}} & \frac{3}{5\sqrt{2}} & \frac{4}{5\sqrt{2}} \end{bmatrix} \begin{bmatrix} 0 & 2 & 0 \\ 0 & -1 & 0 \\ 0 & -2 & 0 \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ -\frac{3}{5\sqrt{2}} & \frac{4}{5} & \frac{3}{5\sqrt{2}} \\ -\frac{4}{5\sqrt{2}} & -\frac{3}{5} & \frac{4}{5\sqrt{2}} \end{bmatrix}$$

$$= \begin{bmatrix} -\frac{63}{50} & \frac{84}{25\sqrt{2}} & \frac{63}{50} \\ -\frac{6}{25\sqrt{2}} & \frac{8}{25} & \frac{6}{25\sqrt{2}} \\ \frac{3}{50} & -\frac{4}{25\sqrt{2}} & \frac{-3}{50} \end{bmatrix} = \begin{bmatrix} -1.26 & 2.376 & 1.26 \\ -0.17 & 0.32 & 0.17 \\ 0.06 & -0.113 & -0.06 \end{bmatrix}$$

(c) The equation of the plane $x_1 - x_2 + 2x_3 = 1$ can be rewritten as

$$[1 \quad -1 \quad 2]\begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = 1 \qquad \text{(a)}$$

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Referring to the equation (1.34a) we have

$$\begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = [\Lambda_s]^T \begin{Bmatrix} x_1' \\ x_2' \\ x_3' \end{Bmatrix} = \begin{bmatrix} \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ \frac{-3}{5\sqrt{2}} & \frac{4}{5} & \frac{3}{5\sqrt{2}} \\ \frac{-4}{5\sqrt{2}} & \frac{-3}{5} & \frac{4}{5\sqrt{2}} \end{bmatrix} \begin{Bmatrix} x_1' \\ x_2' \\ x_3' \end{Bmatrix} \qquad \text{(b)}$$

Substituting equation (b) into (a) we have

$$\begin{bmatrix} 1 & -1 & 2 \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ \frac{-3}{5\sqrt{2}} & \frac{4}{5} & \frac{3}{5\sqrt{2}} \\ \frac{-4}{5\sqrt{2}} & \frac{-3}{5} & \frac{4}{5\sqrt{2}} \end{bmatrix} \begin{Bmatrix} x_1' \\ x_2' \\ x_3' \end{Bmatrix} = 1$$

or

$$-2x_2' + \sqrt{2}\, x_3' = 1$$

Problem 1.7

(a) Referring to relation (1.33a) the coordinates of point P in the system of axes x_i' are

$$\begin{Bmatrix} x_1' \\ x_2' \\ x_3' \end{Bmatrix} = [\Lambda_s] \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \begin{bmatrix} \frac{1}{\sqrt{3}} & 0 & -\frac{2}{\sqrt{6}} \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} \\ \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} \end{bmatrix} \begin{vmatrix} 3 \\ 2 \\ -4 \end{vmatrix}$$

$$= \begin{bmatrix} \frac{3\sqrt{2}+8}{\sqrt{6}} \\ \frac{3\sqrt{2}+2\sqrt{3}-4}{\sqrt{6}} \\ \frac{3\sqrt{2}-2\sqrt{3}-4}{\sqrt{6}} \end{bmatrix} = \begin{bmatrix} 4.998 \\ 1.513 \\ -1.315 \end{bmatrix}$$

(b) Referring to equation (1.73b) we have

$$[A'] = [\Lambda_s][A][\Lambda_s]^T$$

Thus,

$$[A'] = \begin{bmatrix} \frac{1}{\sqrt{3}} & 0 & -\frac{2}{\sqrt{6}} \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} \\ \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} \end{bmatrix} \begin{bmatrix} 0 & -2 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \\ 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ -\frac{2}{\sqrt{6}} & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} \end{bmatrix}$$

$$= \begin{bmatrix} -0.471 & -1.288 & 0.345 \\ 1.390 & -0.658 & -0.025 \\ -0.919 & -0.503 & 2.130 \end{bmatrix}$$

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(c) The equation of the plane in the system of axes x_1, x_2, x_3 is

$$2x_1 + 3x_2 - x_3 = 1$$

or

$$\begin{bmatrix} 2 & 3 & -1 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = 1 \qquad \text{(a)}$$

From relation (1.34a) we have

$$\begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = [\Lambda]^T \begin{Bmatrix} x_1' \\ x_2' \\ x_3' \end{Bmatrix} = \begin{bmatrix} \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \\ 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ -\frac{2}{\sqrt{6}} & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} \end{bmatrix} \begin{Bmatrix} x_1' \\ x_2' \\ x_3' \end{Bmatrix} \qquad \text{(b)}$$

Substituting equation (b) into (a) we have

$$\begin{bmatrix} 2 & 3 & -1 \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \\ 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ -\frac{2}{\sqrt{6}} & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} \end{bmatrix} \begin{Bmatrix} x_1' \\ x_2' \\ x_3' \end{Bmatrix} = 1$$

Or

$$1.9712\, x_1' + 2.8678\, x_2' - 1.3749\, x_3' = 1$$

Problem 1.8A

Given the symmetric tensor of the second rank

$$[A] = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 4 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

(a) Referring to relation (1.124) we have the following stationary values for the tensor $[A]$

$$\tilde{A}_1 = \frac{1}{2}(A_{11} + A_{22}) + \sqrt{\left(\frac{A_{11} - A_{22}}{2}\right)^2 + A_{12}^2} = 3 + \sqrt{2}$$

$$\tilde{A}_2 = \frac{1}{2}(A_{11} + A_{22}) - \sqrt{\left(\frac{A_{11} - A_{22}}{2}\right)^2 + A_{12}^2} = 3 - \sqrt{2}$$

$$\tilde{A}_3 = 3$$

From relation (1.122) we have

$$\tan 2\tilde{\Phi}_{11} = \frac{2A_{12}}{A_{11} - A_{22}} = -1 \qquad \text{and} \qquad 2\tilde{\Phi}_{11} = 135^\circ$$

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Or

$\tilde{\phi}_{11} = 67.5^\circ$ counter-clockwise

In Figure a. we show the principal axes $\tilde{x}_1$, $\tilde{x}_2$

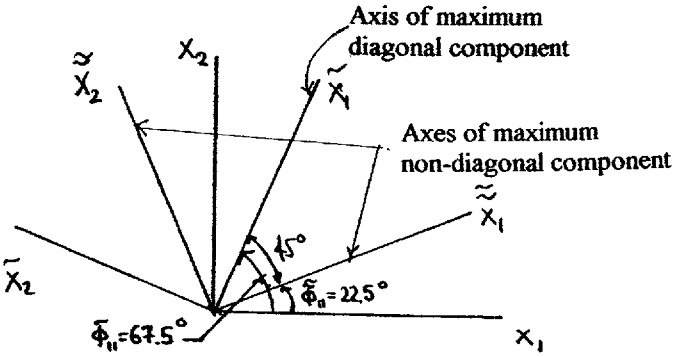

Figure a

(b) Referring to relation (1.129) the maximum value of the non-diagonal components of the tensor [A] is

$$(A'_{12})_{max} = \left[\left(\frac{A_{11} - A_{12}}{2}\right)^2 + A_{12}^2\right]^{1/2} = \left[\left(\frac{2-4}{2}\right)^2 + 1^2\right]^{1/2} = \sqrt{2}$$

From relation (1.127) we have

$$\tan 2\tilde{\tilde{\phi}}_{11} = -\frac{A_{11} - A_{22}}{2A_{12}} = -\frac{2-4}{2} = 1 \quad \text{and} \quad 2\tilde{\tilde{\phi}}_{11} = 45^\circ$$

Or

$\tilde{\tilde{\phi}}_{11} = 22.5^\circ$ counter-clockwise

In Fig. a we also show the axes $\tilde{\tilde{x}}_1$, $\tilde{\tilde{x}}_2$ with respect of which this maximum value of the non-diagonal components occurs.

(c) Referring to relation (1.61) the diagonal component of the tensor [A] in the direction of the unit vector $\mathbf{i}_n$ is

$$A_{nn} = \{n\}^T [A] \{n\} = \frac{1}{\sqrt{3}} \begin{bmatrix} 1 & 1 & -1 \end{bmatrix} \begin{bmatrix} 2 & 1 & 0 \\ 1 & 4 & 0 \\ 0 & 0 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix} \frac{1}{\sqrt{3}} = \frac{11}{3}$$

Problem 1.8B

Given the symmetric tensor of the second rank

$$[B] = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 3 & -1 \\ 0 & -1 & 2 \end{bmatrix}$$

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(a) Referring to relation (1.124) we have the following stationary values for the tensor [B]

$$\tilde{B}_1 = 0$$

$$\tilde{B}_2 = \frac{1}{2}(B_{22} + B_{33}) + \sqrt{\left(\frac{B_{22} - B_{33}}{2}\right)^2 + B_{23}^2} = \frac{5+\sqrt{5}}{2}$$

$$\tilde{B}_3 = \frac{1}{2}(B_{22} + B_{33}) - \sqrt{\left(\frac{B_{22} - B_{33}}{2}\right)^2 + B_{23}^2} = \frac{5-\sqrt{5}}{2}$$

From relation (1.122) we have

$$\tan 2\tilde{\phi}_{22} = \frac{2B_{23}}{B_{22} - B_{33}} = \frac{-2}{3-2} = -2 \qquad \text{and} \qquad 2\tilde{\phi}_{22} = -63.43^\circ$$

Or

$$\tilde{\phi}_{22} = 31.72^\circ \quad \text{clockwise}$$

In Figure a we show the principal axes $\tilde{x}_2$, $\tilde{x}_3$

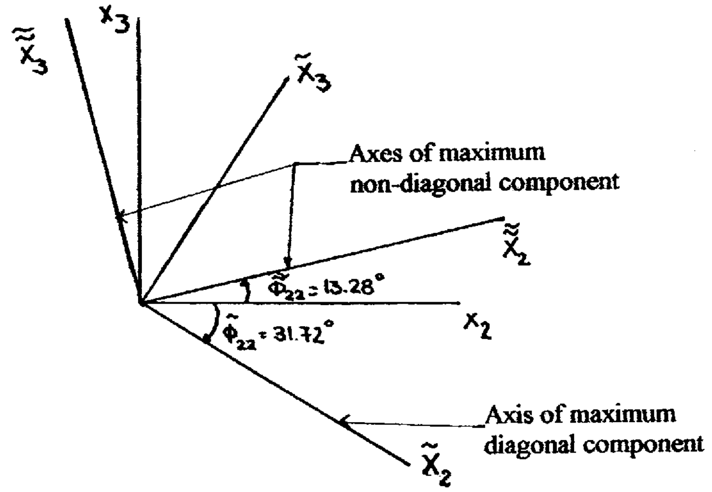

Figure a

(b) Referring to relation (1.129) the maximum value of the non-diagonal components of the tensor [B] is

$$(B'_{23})_{max} = \left[\left(\frac{B_{22} - B_{33}}{2}\right)^2 + B_{23}^2\right]^{1/2} = \left[\left(\frac{3-2}{2}\right)^2 + (-1)^2\right]^{1/2} = \frac{\sqrt{5}}{2}$$

From relation (1.127) we have

$$\tan 2\tilde{\tilde{\phi}}_{22} = -\frac{B_{22} - B_{33}}{2B_{23}} = -\frac{3-2}{(-2)} = \frac{1}{2} \qquad \text{and} \qquad 2\tilde{\tilde{\phi}}_{22} = 26.56^\circ$$

Or

$$\tilde{\tilde{\phi}}_{22} = 13.28^\circ \quad \text{counter- clockwise}$$

In Fig. a we also show the axes $\tilde{\tilde{x}}_2$ and $\tilde{\tilde{x}}_3$ with respect of which this maximum value of the non-diagonal components occurs.

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(c) Referring to relation (1.61) the diagonal component of the tensor [B] in the direction of the unit vector $\mathbf{i}_n$ is

$$B_{nn} = \{n\}^T [B] \{n\} = \frac{1}{\sqrt{3}} \begin{bmatrix} 1 & 1 & -1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 3 & -1 \\ 0 & -1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix} \frac{1}{\sqrt{3}} = \frac{7}{3}$$

Problem 1.8C

Given the symmetric tensor of the second rank

$$[C] = \begin{bmatrix} 2 & 0 & -3 \\ 0 & 0 & 0 \\ -3 & 0 & 4 \end{bmatrix}$$

(a) Referring to relation (1.124) we have the following stationary values for the tensor C

$$\tilde{C}_3 = \frac{1}{2}(C_{33}+C_{11}) + \sqrt{\left(\frac{C_{33}-C_{11}}{2}\right)^2 + C_{13}^2} = 3+\sqrt{10}$$

$$\tilde{C}_1 = \frac{1}{2}(C_{33}+C_{11}) - \sqrt{\left(\frac{C_{33}-C_{11}}{2}\right)^2 + C_{13}^2} = 3-\sqrt{10}$$

$$\tilde{C}_2 = 0$$

From relation (1.122) we have

$$\tan 2\tilde{\phi}_{33} = \frac{2C_{13}}{C_{33}-C_{11}} = \frac{-6}{4-2} = -3 \qquad \text{and} \quad 2\tilde{\phi}_{33} = -71.56^\circ$$

Or

$$\tilde{\phi}_{33} = 35.78^\circ \qquad \text{clockwise}$$

In Figure a, we show the principal axes $\tilde{x}_3$, $\tilde{x}_1$

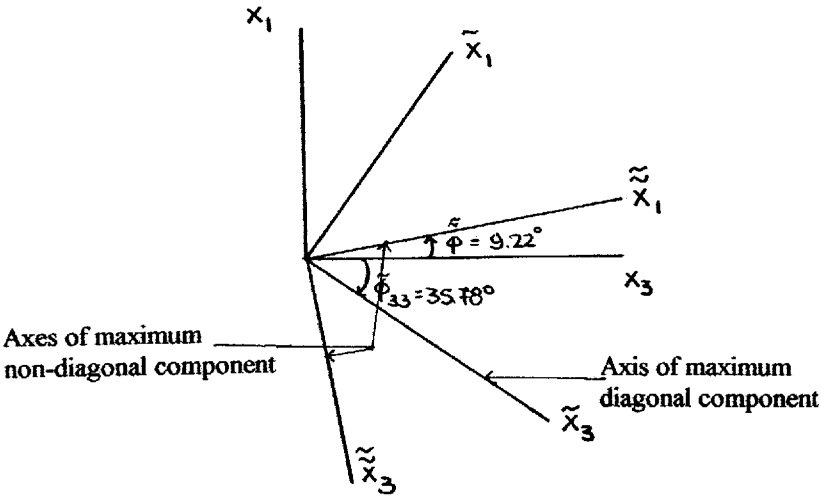

Figure a

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(b) Referring to relation (1.129) the maximum value of the non-diagonal components of the tensor [C] is

$$(C'_{13})_{max} = \left[\left(\frac{C_{33}-C_{11}}{2}\right)^2 + C_{13}^2\right]^{1/2} = \left[1^2 + (-3)^2\right]^{1/2} = \sqrt{10}$$

From relation (1.127) we have

$$\tan 2\bar{\bar{\phi}}_{33} = -\frac{C_{33}-C_{11}}{2C_{13}} = \frac{1}{3} \qquad \text{and} \qquad 2\bar{\bar{\phi}}_{33} = 18.43^\circ$$

Or

$$\bar{\bar{\phi}}_{33} = 9.22^\circ \quad \text{counter- clockwise}$$

In Fig. a we show also the axes $\bar{\bar{x}}_3$, $\bar{\bar{x}}_1$ with respect of which this maximum value of the non-diagonal components occurs.

(c) Referring to relation (1.61) the diagonal component of the tensor [C] in the direction of the unit vector $\mathbf{i}_n$ is

$$C_{nn} = \{n\}^T [C]\{n\} = \frac{1}{\sqrt{3}}\begin{bmatrix} 1 & 1 & -1 \end{bmatrix}\begin{bmatrix} 2 & 0 & -3 \\ 0 & 0 & 0 \\ -3 & 0 & 4 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}\frac{1}{\sqrt{3}} = \frac{12}{3} = 4$$

Problem 1.9 A

Given the symmetric tensor of the second rank

$$[A] = \begin{bmatrix} -3 & -2 & 0 \\ -2 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

(a) We plot Mohr's circle using the two points on the same diameter X_1 (-3, -2) and X_2 (2, 2). Referring to Figure a. we see that the center, C, of the circle is located on the A'_{11} axis at a distance $1/2\,(A_{11} + A_{22}) = -0.5$ from the origin O.

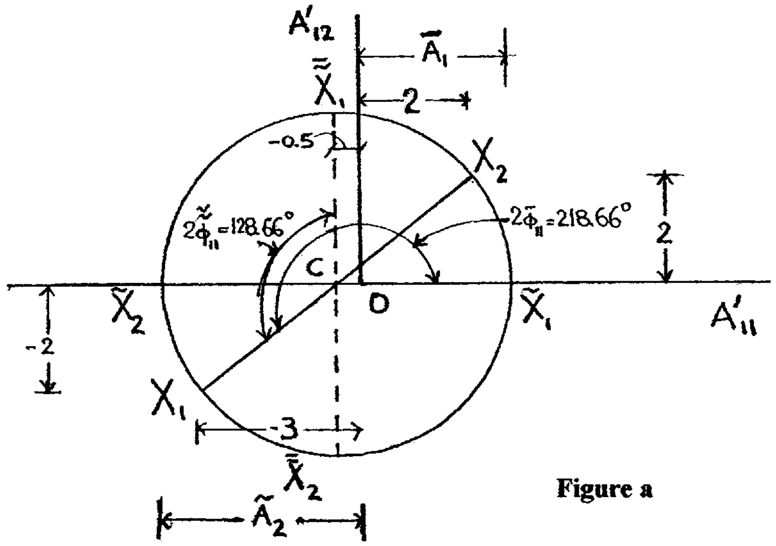

Figure a

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From geometric considerations the radius, R, of the circle is

$$R = [(2.5)^2 + 2^2]^{1/2} = 3.2$$

Moreover as shown in Fig. a the maximum value of the diagonal components of the tensor [A] is the abscissa of point $\tilde{X}_1$, while the minimum value is the abscissa of point $\tilde{X}_2$. Thus

$$(A'_{11})_{max} = \tilde{A}_1 = \overline{C\tilde{X}_1} - \overline{OC} = 3.2 - 0.5 = 2.7$$

$$(A'_{11})_{min} = \tilde{A}_2 = -(\overline{OC} + \overline{C\tilde{X}_2}) = -3.2 - 0.5 = -3.7$$

Referring to Fig. a. we have

$$\tan 2\tilde{\varphi}_{11} = \tan \angle X_1C\tilde{X}_1 = 2/2.5$$

Hence

$$2\tilde{\varphi}_{11} = 218.66° \qquad \text{or} \qquad \tilde{\varphi}_{11} = 109.33° \text{ counter- clockwise.}$$

One of the principal directions is the i_3 whereas the other two lie in the x_1x_2 plane. So we see in Fig. a that point $\tilde{X}_1$ is located 218.66° clockwise from point X_1. Consequently, the principal axis $\tilde{x}_1$ associated with the principal value $\tilde{A}_1$ of the diagonal component of the tensor is located $\tilde{\varphi}_{11}$ = 109.33° counter- clockwise from the x_1 axis. In Fig. b. we show the principal axes of the tensor $\tilde{x}_1$, $\tilde{x}_2$.

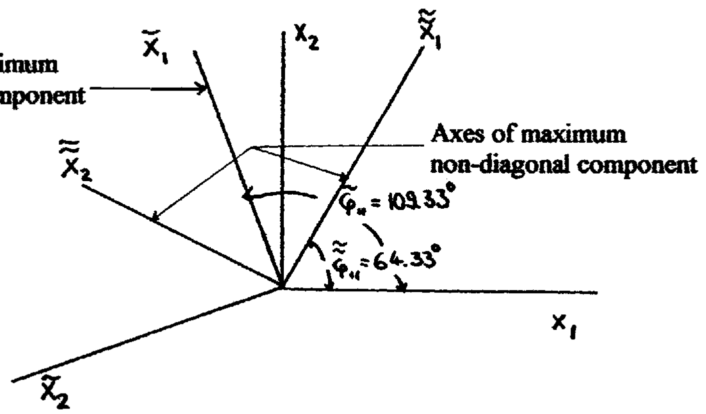

Figure b

(b) Referring to Fig. a the maximum non-diagonal component of the tensor is the ordinate of point $\tilde{\tilde{X}}_1$. Thus,

$$(A'_{12})_{max} = 3.2$$

From Fig. a we can see also that the point $\tilde{\tilde{X}}_1$ is located 128.66° clockwise from point X_1. Consequently, the axes $\tilde{\tilde{x}}_1$ and $\tilde{\tilde{x}}_2$, where the maximum value of the non-diagonal components of tensor [A] occur, are located as shown in Fig. b.

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(c) From relation (1.61) the diagonal component of the tensor [A] in the direction of the unit vector $\mathbf{i}_n$ is

$$A_{nn} = \{n\}^T [A]\{n\} = \frac{1}{\sqrt{3}}[1 \quad 1 \quad -1]\begin{bmatrix} -3 & -2 & 0 \\ -2 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}\frac{1}{\sqrt{3}} = -\frac{4}{3}$$

Problem 1.9B
Given the symmetric tensor of the second rank

$$[B] = \begin{bmatrix} 6 & 0 & -3 \\ 0 & 0 & 0 \\ -3 & 0 & -2 \end{bmatrix}$$

(a) We plot Mohr's circle using the two points on the same diameter X_3 (-2, -3) and X_1 (6, 3). Referring to Figure a. we see that the center, C, of the circle is located on the B'_{33} axis at a distance 1/2 $(B_{11} + B_{22}) = 2$ from the origin O.

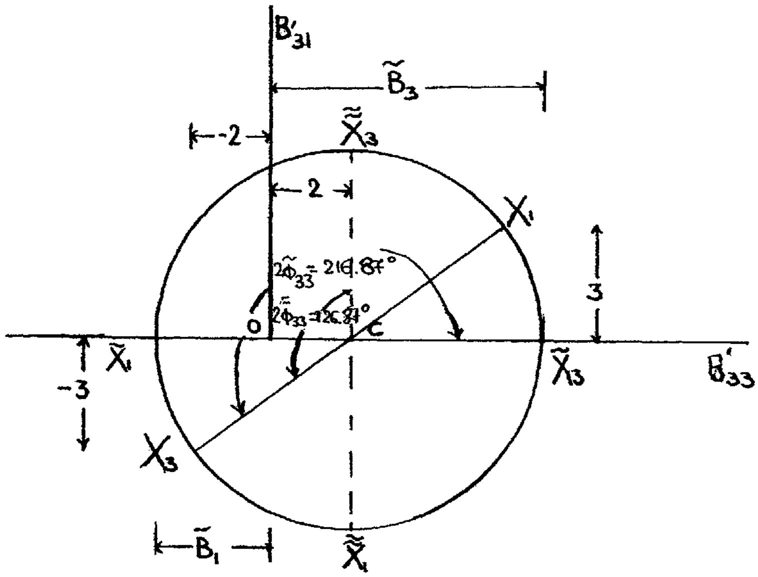

Figure a

From geometric considerations the radius, R, of the circle is

$$R = (3^2 + 4^2)^{1/2} = 5$$

Moreover as shown in Fig. a the maximum value of the diagonal components of the tensor [B] is the abscissa of point $\tilde{X}_3$, while the minimum value is the abscissa of point $\tilde{X}_1$. Thus,

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$(B'_{33})_{max} = \tilde{B}_3 = \overline{CX}_3 + \overline{OC} = 5 + 2 = 7.0$

$(B'_{11})_{min} = \tilde{B}_1 = -\overline{CX}_1 + \overline{OC} = -5 + 2 = -3.0$

Referring to Fig. a. we have

$\tan 2\tilde{\varphi}_{33} = \tan < X_3C\tilde{X}_3 = 3/4$

Hence

$2\tilde{\varphi}_{33} = 216.87^\circ$ or $\tilde{\varphi}_{33} = 108.43^\circ$ counter- clockwise.

One of the principal directions is the $\mathbf{i}_2$ whereas the other two lie in the x_3x_1 plane. So we see in Fig. a that point $\tilde{X}_3$ is located 216.87° clockwise from point X_3. Consequently, the principal axis $\tilde{x}_3$ associated with the principal value $\tilde{B}_3$ of the diagonal component of the tensor is located $\tilde{\varphi}_{33} = 108.43^\circ$ counter- clockwise from the x_3 axis. In Fig. b. we show the principal axes of the tensor $\tilde{x}_3$, $\tilde{x}_1$.

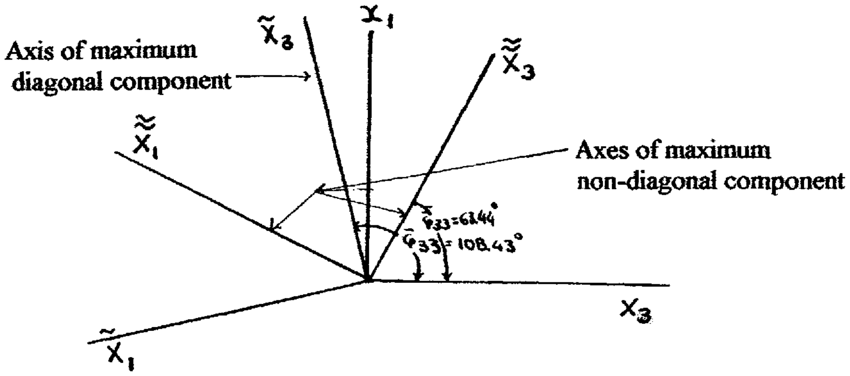

Figure b

(b) Referring to Fig. a the maximum non-diagonal component of the tensor is the ordinate of point X_3. Thus,

$(B'_{31})_{max} = 5$

From Fig. a we can also see that point $\tilde{\tilde{X}}_3$ is located 126.87° clockwise from point X_3. Consequently, the axes $\tilde{\tilde{x}}_3$ and $\tilde{\tilde{x}}_1$, where the maximum value of the non-diagonal components of tensor [B] occur, are located as shown in Fig. b.

(c) From relation (1.61) the diagonal component of the tensor [B] in the direction of the unit vector $\mathbf{i}_n$ is

$$B_{nn} = \{n\}^T[B]\{n\} = \frac{1}{\sqrt{3}}\begin{bmatrix}1 & 1 & -1\end{bmatrix}\begin{bmatrix}6 & 0 & -3\\0 & 0 & 0\\-3 & 0 & -2\end{bmatrix}\begin{bmatrix}1\\1\\-1\end{bmatrix}\frac{1}{\sqrt{3}} = \frac{10}{3}$$

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Problem 1.9 C

Given the symmetric tensor of the second rank

$$[C] = \begin{bmatrix} 0 & 0 & 0 \\ 0 & -2 & 4 \\ 0 & 4 & 6 \end{bmatrix}$$

(a) We plot Mohr's circle using the two points on the same diameter X_2 (-2, 4) and X_3 (6, -4). Referring to Figure a. we see that the center, C, of the circle is located on the C'_{22} axis at a distance $1/2\,(C_{22} + C_{33}) = 2$ from the origin O.

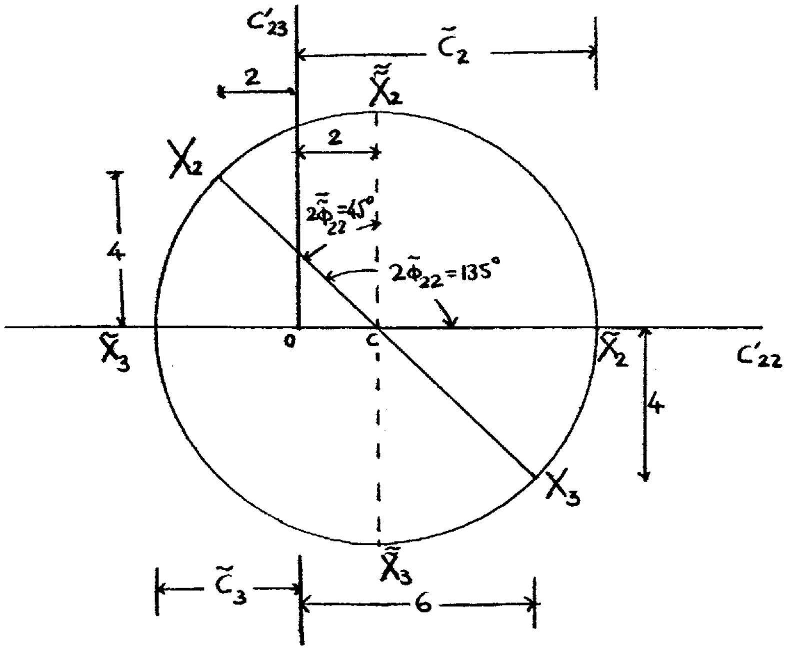

Figure a

From geometric considerations the radius, R, of the circle is

$$R = (4^2 + 4^2)^{1/2} = 5.657$$

Moreover as shown in Fig. a the maximum value of the diagonal components of the tensor [C] is the abscissa of point $\tilde{X}_2$, while the minimum value is the abscissa of point $\tilde{X}_3$. Thus

$$(C'_{22})_{max} = \tilde{C}_2 = \overline{OC} + \overline{C\tilde{X}_2} = 5.657 + 2 = 7.657$$

$$(C'_{22})_{min} = \tilde{C}_3 = -\,\overline{C\tilde{X}_3} + \overline{OC} = -5.657 + 2 = -3.657$$

Referring to Fig. a. we have

$$2\tilde{\varphi}_{22} = \text{angle } X_2 C \tilde{X}_2$$

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Hence

$$2\tilde{\varphi}_{22} = 135^\circ \qquad \text{or} \qquad \tilde{\varphi}_{22} = 67.5^\circ \text{ counter-clockwise.}$$

One of the principal directions is the $\mathbf{i}_1$ whereas the other two lie in the x_2x_3 plane. So we see in Fig. a that point $\tilde{X}_2$ is located 135° clockwise from point X_2. Consequently, the principal axis $\tilde{x}_2$ associated with the principal value $\tilde{C}_2$ of the diagonal component of the tensor is located $\tilde{\varphi}_{22} = 67.5^\circ$ counter- clockwise from the x_2 axis. In Fig. b. we show the principal axes of the tensor $\tilde{x}_2, \tilde{x}_3$.

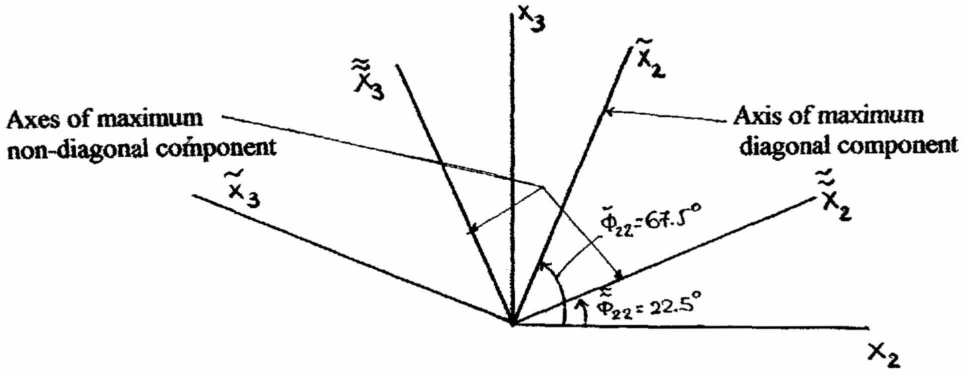

Figure b

(b) Referring to Fig. a the maximum non-diagonal component of the tensor is the ordinate of point X_2. Thus,

$$(C'_{23})_{max} = 5.657$$

From Fig. a we can also see that point $\tilde{\tilde{X}}_2$ is located 45° clockwise from point X_2. Consequently, the axes $\tilde{\tilde{x}}_2$ and $\tilde{\tilde{x}}_3$, where the maximum value of the non-diagonal components of tensor [C] occur, are located as shown in Fig. b.

(c) From relation (1.61) the diagonal component of the tensor [C] in the direction of the unit vector $\mathbf{i}_n$ is

$$C_{nn} = \{n\}^T[C]\{n\} = \frac{1}{\sqrt{3}}[1 \quad 1 \quad -1]\begin{bmatrix} 0 & 0 & 0 \\ 0 & -2 & 4 \\ 0 & 4 & 6 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}\frac{1}{\sqrt{3}} = -\frac{4}{3}$$

Problem 1.9D

Given the symmetric tensor of the second rank

$$[D] = \begin{bmatrix} -4 & 2 & 0 \\ 2 & -8 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

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(a) We plot Mohr's circle using the two points on the same diameter X_1 (-4, 2) and X_2 (-8, -2). Referring to Figure a. we see that the center, C, of the circle is located on the D'_{11} axis at a distance 1/2 $(D_{11} + D_{22}) = -6$ from the origin O.

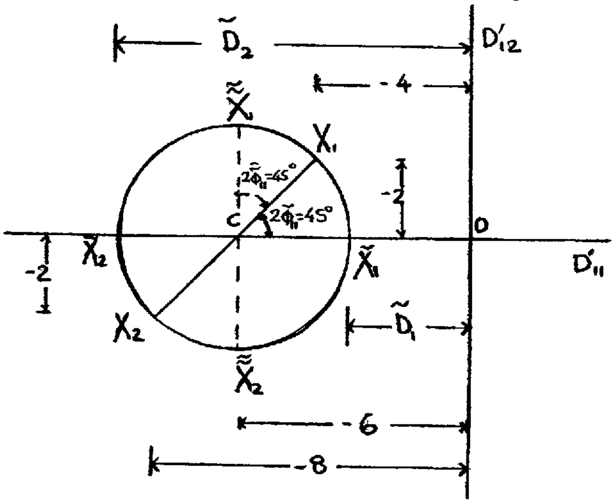

Figure a

From geometric considerations the radius, R, of the circle is

$$R = [2^2 + (-2)^2]^{1/2} = 2.828$$

Moreover as shown in Fig. a the maximum value of the diagonal components of the tensor [D] is the abscissa of point X_1, while the minimum value is the abscissa of point X_2. Thus

$$(D'_{11})_{max} = \tilde{D}_1 = -\overline{CO} + \overline{C\tilde{X}_1} = -6 + 2.828 = -3.172$$

$$(D'_{11})_{min} = \tilde{D}_2 = -\overline{CO} - \overline{C\tilde{X}_2} = -6 - 2.828 = -8.828$$

Referring to Fig. a. we have

$$\tan 2\tilde{\varphi}_{11} = \tan \angle X_1 C\tilde{X}_1 = 1$$

Hence

$$2\tilde{\varphi}_{11} = 45° \qquad \text{or} \qquad \tilde{\varphi}_{11} = 22.5° \text{ counter clockwise.}$$

One of the principal directions is the $\mathbf{i}_3$ whereas the other two lie in the x_1x_2 plane. So we see in Fig. a that point $\tilde{X}_1$ is located 45° clockwise from point X_1. Consequently, the principal axis $\tilde{x}_1$ associated with the principal value $\tilde{D}_1$ of the diagonal component of the tensor is located $\tilde{\varphi}_{11} = 22.5°$ counter- clockwise from the x_1 axis. In Fig. b we show the principal axes of the tensor $\tilde{x}_1$, $\tilde{x}_2$.

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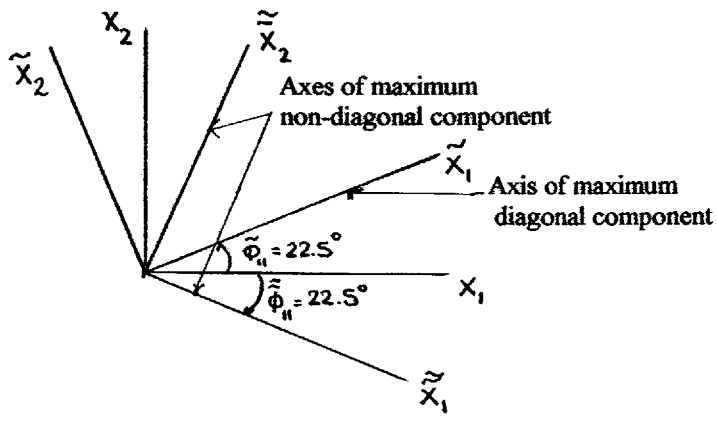

Figure b

(b) Referring to Fig. a the maximum non-diagonal component of the tensor is the ordinate of point X_1. Thus,

$$(D'_{12})_{max} = 2.828$$

From Fig. a we can see also that the point $\tilde{\tilde{X}}_1$ is located 45° clockwise from point X_1. Consequently, the axes $\tilde{\tilde{x}}_1$ and $\tilde{\tilde{x}}_2$, where the maximum value of the non-diagonal components of tensor [D] occur, are located as shown in Fig. b.

(c) From relation (1.61) the diagonal component of the tensor [D] in the direction of the unit vector i_n is

$$D_{\eta\eta} = \{\eta\}^T [D] \{\eta\} = \frac{1}{\sqrt{3}} [1 \; 1 \; -1] \begin{bmatrix} -4 & 2 & 0 \\ 2 & -8 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix} \frac{1}{\sqrt{3}} = -\frac{8}{3}$$

Problem 1.9E

Given the symmetric tensor of the second rank

$$E = \begin{bmatrix} 0 & 3 & 0 \\ 3 & -9 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

(a) We plot Mohr's circle using the two points on the same diameter X_1 (0, 3) and X_2 (-9, -3). Referring to Figure a. we see that the center, C, of the circle is located on the E'_{11} axis at a distance 1/2 ($E_{11} + E_{22}$) = -4.5 from the origin O. From geometric considerations the radius, R, of the circle is

$$R = [3^2 + (-4.5)^2]^{1/2} = 5.41$$

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As shown in Fig. a the maximum value of the diagonal components of the tensor [E] is the abscissa of point $\tilde{X}_1$, while the minimum value is the abscissa of point $\tilde{X}_2$. Thus

$$(E'_{11})_{max} = \tilde{E}_1 = \overline{C\tilde{X}_1} - \overline{CO} = 5.41 - 4.5 = 0.91$$

$$(E'_{11})_{min} = \tilde{E}_2 = -\overline{CO} - \overline{C\tilde{X}_2} = -4.5 - 5.41 = -9.91$$

Referring to Fig. a. we have

$$\tan 2\tilde{\varphi}_{11} = \tan < X_1C\tilde{X}_1 = 3/4.5$$

Hence, $2\tilde{\varphi}_{11} = 33.69°$ or $\tilde{\varphi}_{11} = 16.85°$ counter clockwise.

One of the principal directions is the i_3 whereas the other two lie in the x_1x_2 plane. So we see in Fig. a that point $\tilde{X}_1$ is located 33.69° clockwise from point X_1. Consequently, the principal axis $\tilde{x}_1$ associated with the principal value $\tilde{E}_1$ of the diagonal component of the tensor is located $\tilde{\varphi}_{11} = 16.85°$ counter- clockwise from the x_1 axis. In Fig. b. we show the principal axes of the tensor $\tilde{x}_1, \tilde{x}_2$.

(b) Referring to Fig. a the maximum non-diagonal component of the tensor is the ordinate of point $\tilde{\tilde{X}}_1$. Thus,

$$(E'_{12})_{max} = 5.41$$

From Fig. a we also can see that the point $\tilde{\tilde{X}}_1$ is located 56.31° counter-clockwise from point X_1. Consequently, the axes $\tilde{\tilde{x}}_1$ and $\tilde{\tilde{x}}_2$, where the maximum value of the non-diagonal components of tensor [E] occur, are located as shown in Fig. b.

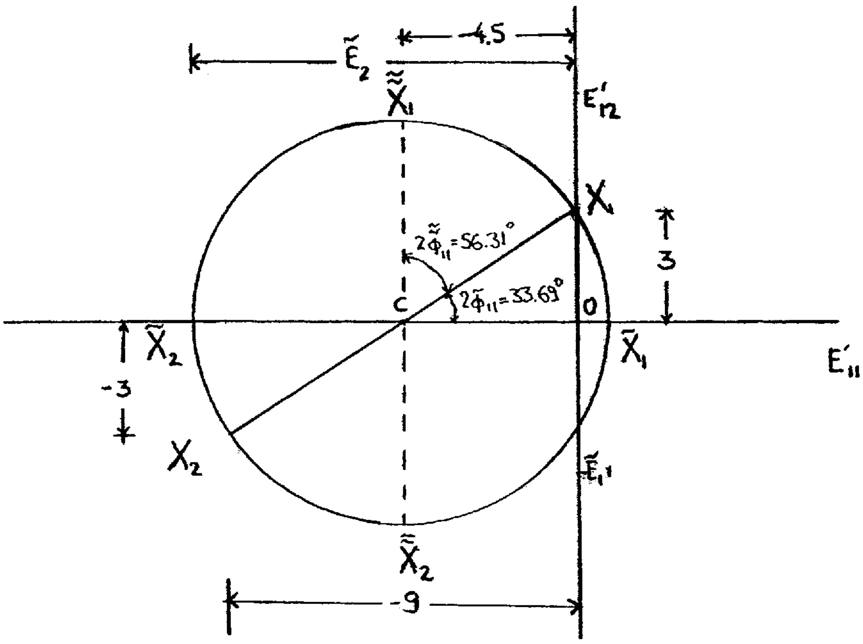

Figure a

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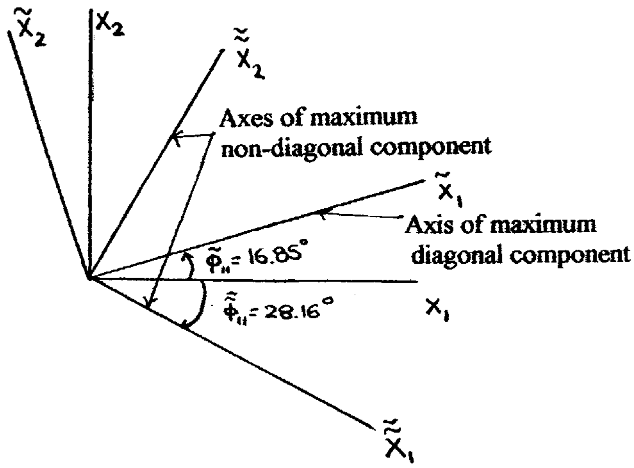

Figure b

(c) From relation (1.61) the diagonal component of the tensor [E] in the direction of the unit vector $\mathbf{i}_n$ is

$$E_{nn} = \{n\}^T [E] \{n\} = \frac{1}{\sqrt{3}} [1 \quad 1 \quad -1] \begin{bmatrix} 0 & 3 & 0 \\ 3 & -9 & 0 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix} = -\frac{1}{3}$$

Problem 1.9F

Given the symmetric tensor of the second rank

$$[F] = \begin{bmatrix} 0 & 0 & 0 \\ 0 & -4 & -3 \\ 0 & -3 & -12 \end{bmatrix}$$

(a) We plot Mohr's circle using the two points on the same diameter X_2 (-4, -3) and X_3 (-12, 3). Referring to Figure a we see that the center, C, of the circle is located on the F'_{22} axis at a distance $1/2\,(F_{22} + F_{33}) = -8$ from the origin O.

From geometric considerations the radius, R, of the circle is

$$R = [(-3)^2 + (-4)^2]^{1/2} = 5$$

Moreover as shown in Fig. a the maximum value of the diagonal components of the tensor [F] is the abscissa of point $\tilde{X}_2$, while the minimum value is the abscissa of point $\tilde{X}_3$. Thus

$$(F'_{22})_{max} = \tilde{F}_2 = -\overline{CO} + \overline{C\tilde{X}_2} = -8 + 5 = -3$$

$$(F'_{22})_{min} = \tilde{F}_3 = -\overline{C\tilde{X}_3} - \overline{CO} = -5 - 8 = -13$$

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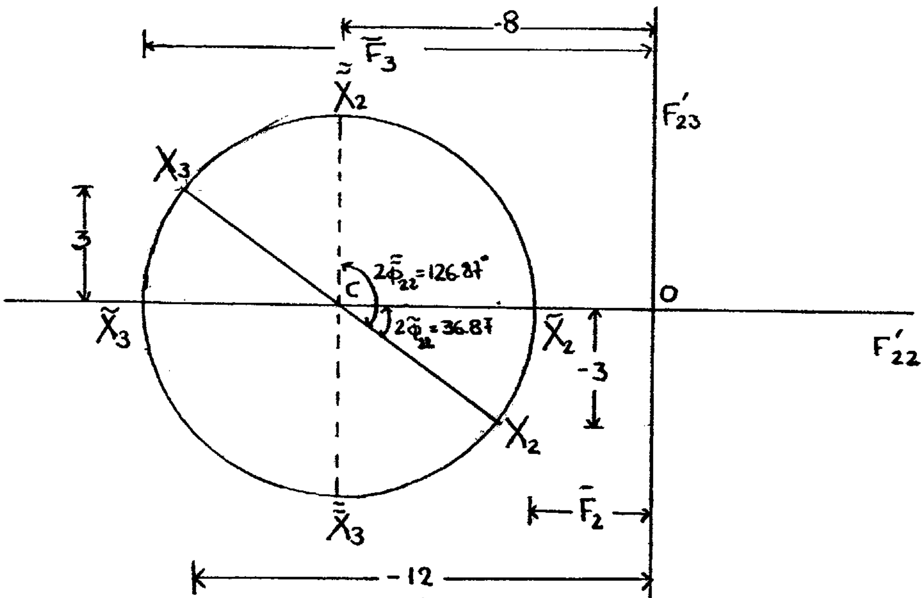

Figure a

Referring to Fig. a. we have

$$\tan 2\tilde{\varphi}_{22} = \tan\angle X_2 C \tilde{X}_2 = 3/4$$

Hence

$$2\tilde{\varphi}_{22} = 36.87° \qquad \text{or} \qquad \tilde{\varphi}_{22} = 18.43° \text{ clockwise.}$$

One of the principal directions is the $\mathbf{i}_1$ whereas the other two lie in the x_2x_3 plane. So we see in Fig. a that point $\tilde{X}_2$ is located 36.87° counter-clockwise from point X_2. Consequently, the principal axis $\tilde{x}_2$ associated with the principal value $\tilde{F}_2$ of the diagonal component of the tensor is located $\tilde{\varphi}_{22}$ = 18.43° clockwise from the x_2 axis. In Fig. b. we show the principal axes of the tensor $\tilde{x}_2$, $\tilde{x}_3$.

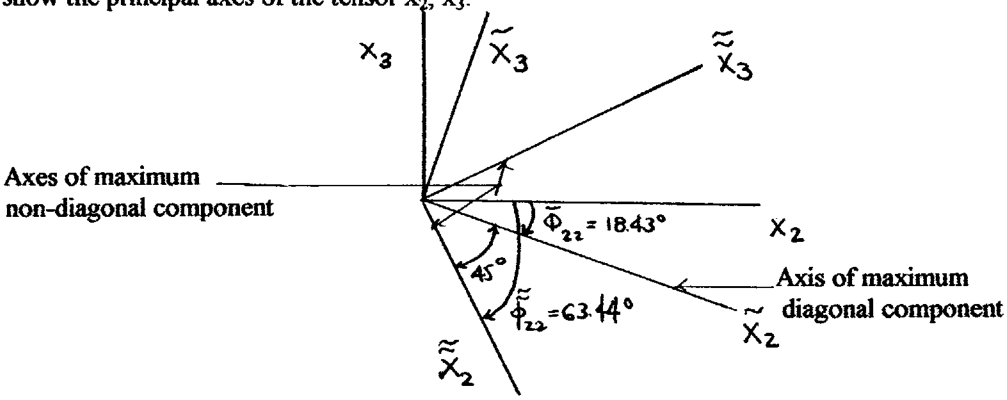

Figure b

(b) Referring to Fig. a the maximum non-diagonal component of the tensor is the ordinate of point $\tilde{\tilde{X}}_2$. Thus,

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$(F'_{23})_{max} = 5.0$

From Fig. a we can also see that point $\bar{\bar{X}}_2$ is located 126.87° counter-clockwise from point X_2. Consequently, the axes $\tilde{\tilde{x}}_2$ and $\tilde{\tilde{x}}_3$, where the maximum value of the non-diagonal components of tensor [C] occur, are located as shown in Fig. b.

(c) From relation (1.61) the diagonal component of the tensor [F] in the direction of the unit vector $\mathbf{i}_n$ is

$$F_{nn} = \{n\}^T [F] \{n\} = \frac{1}{\sqrt{3}} \begin{bmatrix} 1 & 1 & -1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 \\ 0 & -4 & -3 \\ 0 & -3 & -12 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix} \frac{1}{\sqrt{3}} = -\frac{10}{3}$$

Problem 1.10A

Referring to the solution of problem 1.9A we plot Mohr's circle in Fig. a. Moreover we plot line $X'_1CX'_2$ making an angle of 60° clockwise with line X_1CX_2. Referring to Fig. a we have that

angle $ACX'_1 = 21.34°$ and

$$A'_{11} = -\overline{OC} - \overline{CA} = -0.5 - R\cos(21.34°) = -3.481$$

$$A'_{12} = \overline{AX'_1} = R\sin(21.34°) = 3.2\sin(21.34°) = 1.164$$

$$A'_{22} = \overline{OB} = \overline{CB} - \overline{CO} = R\cos(21.34°) - 0.5 = 2.481$$

Or

$$[A'] = \begin{bmatrix} -3.481 & 1.164 & 0 \\ 1.164 & 2.481 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

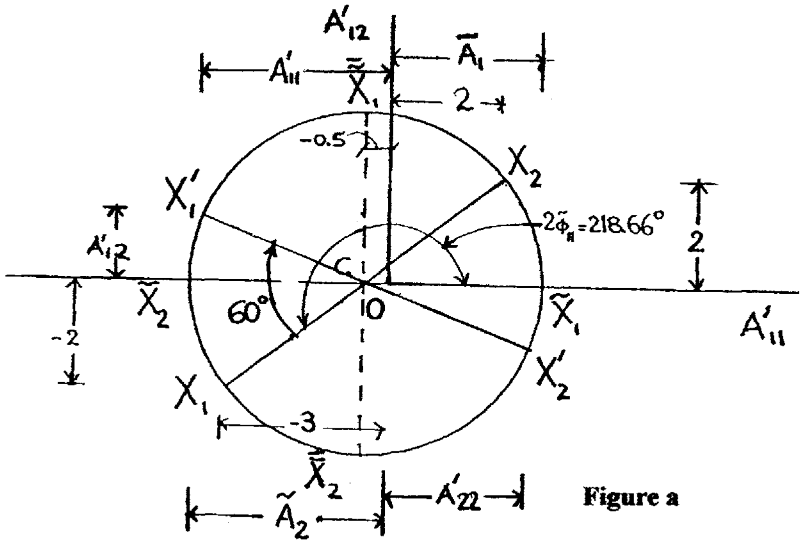

Figure a

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Problem 1.10B

Referring to the solution of problem 1.9B we plot Mohr's circle in Fig. a. Moreover we plot line $X'_3CX'_1$ making an angle of 60° clockwise with line X_3CX_1. Referring to Fig. a we have that

angle $X'_3CA = 23.13°$ and

$$B'_{33} = -\overline{CA} + \overline{CO} = -R\cos(23.13°) + 2 = -2.598$$

$$B'_{31} = \overline{AX'} = R\sin(23.13°) = 5\sin(23.13°) = 1.964$$

$$B'_{11} = \overline{OB} = \overline{OC} + \overline{CB} = 2 + R\cos(23.13°) = 6.598$$

Or

$$[B'] = \begin{bmatrix} 6.598 & 0 & 1.964 \\ 0 & 0 & 0 \\ 1.964 & 0 & -2.598 \end{bmatrix}$$

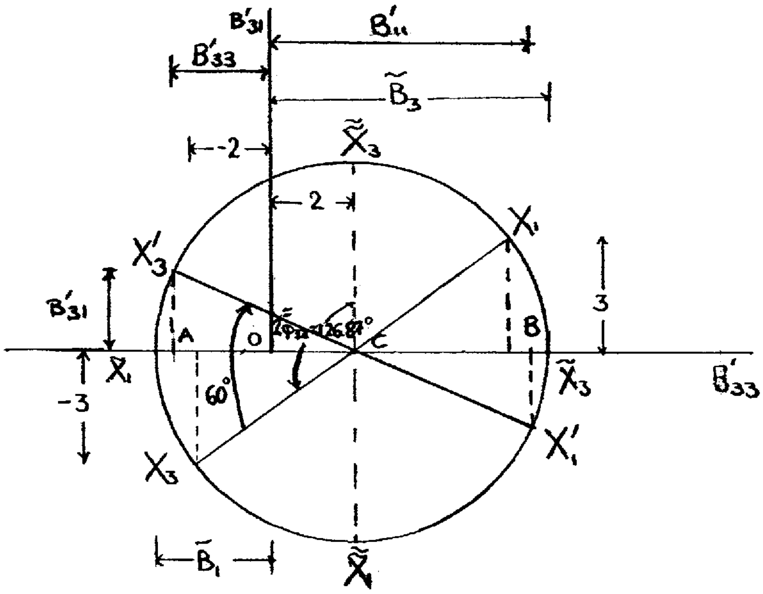

Figure a

Problem 1.10D

Referring to the solution of problem 1.9D we plot Mohr's circle in Fig. a. Moreover we plot line $X'_1CX'_2$ making an angle of 60° clockwise with line X_1CX_2. Referring to Fig. a we have that

angle $X'_1CA = 15°$ and

$$D'_{11} = -\overline{CO} + R\cos(15°) = -6 + 2.828\cos(15°) = -3.268$$

$$D'_{12} = -\overline{AX'_1} = -R\sin(15°) = -2.828\sin(15°) = -0.732$$

$$D'_{22} = -\overline{BO} = -(\overline{CO} + \overline{CB}) = -6 - R\cos(15°) = -8.732$$

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Or

$$[D'] = \begin{bmatrix} 3.268 & -0.732 & 0 \\ -0.732 & -8.732 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

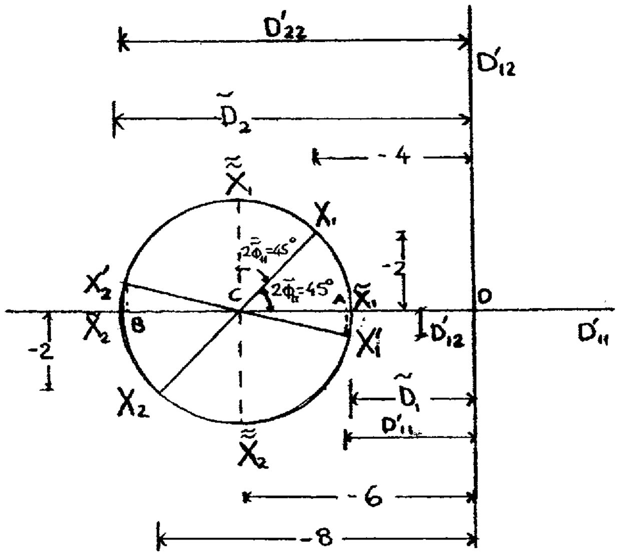

Figure a

Problem 1.10E

Referring to the solution of problem 1.9E we plot Mohr's circle in Fig. a. Moreover we plot line $X'_1CX'_2$ making an angle of 60° clockwise with line X_1CX_2. Referring to Fig. a we have that

angle $X'_1CA = 26.31°$ and

$$E'_{11} = \overline{OA} = \overline{CA} - \overline{CO} = R\cos(26.31°) - 4.5 = 0.35$$

$$E'_{12} = -\overline{AX_1} = -R\sin(26.31°) = -5.41\sin(26.31°) = -2.40$$

$$E'_{22} = -\overline{OB} = -\overline{BC} - \overline{CO} = -R\cos(26.31°) - 4.5 = -9.35$$

Or

$$[E'] = \begin{bmatrix} 0.35 & -2.40 & 0 \\ -2.40 & -9.35 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

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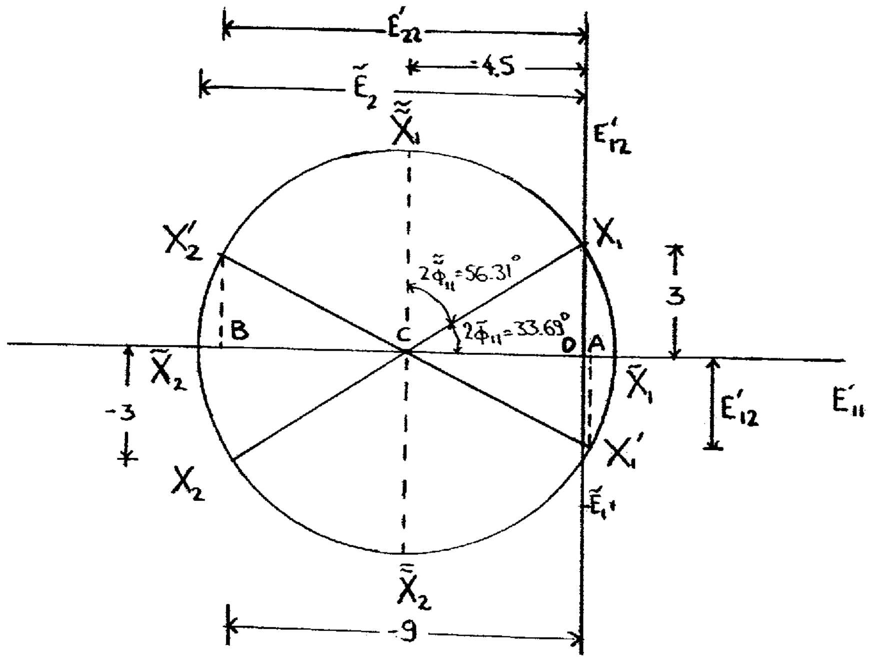

Figure a

Problem 1.11A

Given the symmetric tensor of the second rank

$$[A] = \begin{bmatrix} 3 & -2 & 0 \\ -2 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Referring to relation (1.73a) we have

$$[A'] = [\Lambda_s][A][\Lambda_s]^T$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{\sqrt{3}}{2} & \frac{1}{2} \\ 0 & -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} 3 & -2 & 0 \\ -2 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ 0 & \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$$

$$= \frac{1}{4} \begin{bmatrix} -12 & -4\sqrt{3} & 4 \\ -4\sqrt{3} & 7 & -\sqrt{3} \\ 4 & -\sqrt{3} & 5 \end{bmatrix}$$

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Problem 1.11E

Given the symmetric tensor of the second rank

$$[\varepsilon] = \begin{bmatrix} 0 & 3 & 0 \\ 3 & -9 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

Referring to relation (1.73a) we have

$$[\varepsilon'] = [\Lambda_s][\varepsilon][\Lambda_s]^T$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{\sqrt{3}}{2} & \frac{1}{2} \\ 0 & -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} 0 & 3 & 0 \\ 3 & -9 & 0 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ 0 & \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$$

$$= \frac{1}{4} \begin{bmatrix} 0 & 6\sqrt{3} & -6 \\ 6\sqrt{3} & -25 & 11\sqrt{3} \\ -6 & 11\sqrt{3} & -3 \end{bmatrix}$$

Problem 1.12

The state of stress of the particle of Fig. 1P12 is

$$[\tau] = \begin{bmatrix} 60 & 0 & 30 \\ 0 & 0 & 0 \\ 30 & 0 & -20 \end{bmatrix}$$

(a) The unit vector $\mathbf{i}_s$ normal to $\mathbf{i}_n$, which lies in the x_1, x_3 plane, is

$$\mathbf{i}_s = \frac{4}{5}\mathbf{i}_1 - \frac{3}{5}\mathbf{i}_3$$

Thus referring to relation (1.61) we have

$$\tau_{nn} = \{n\}^T[\tau]\{n\} = \frac{1}{5}[3 \; 0 \; 4] \begin{bmatrix} 60 & 0 & 30 \\ 0 & 0 & 0 \\ 30 & 0 & -20 \end{bmatrix} \begin{bmatrix} 3 \\ 0 \\ 4 \end{bmatrix} \frac{1}{5} = 37.6\,\text{MPa}$$

Referring to relation (1.64a) we have

$$\tau_{ns} = \{n\}^T[\tau]\{s\} = \frac{1}{5}[3 \; 0 \; 4] \begin{bmatrix} 60 & 0 & 30 \\ 0 & 0 & 0 \\ 30 & 0 & -20 \end{bmatrix} \begin{bmatrix} 4 \\ 0 \\ -3 \end{bmatrix} \frac{1}{5} = 46.80\,\text{MPa}$$

The positive sign indicates that τ_{ns} is in the direction of the unit vector $\mathbf{i}_s$ as we show in Fig. a.

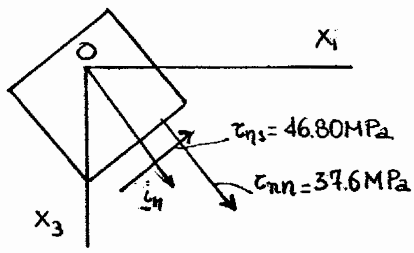

Figure a Results

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Another way of solving this problem is using Mohr's circle. We plot the circle using two points X_3 (-20, 30) and X_1 (60, -30). These points are located on the same diameter of the circle. Referring to Fig. b we see that the center, C of the circle is located on the τ'_{33} axis at a distance $\overline{OC} = 1/2\ (\tau_{33} + \tau_{11}) = 20$ from the origin O. From geometric considerations we have also that the radius of the circle is

$$R = (40^2 + 30^2)^{1/2} = 50 \qquad \text{and} \qquad \overline{AC} = \overline{AO} + \overline{OC} = 20 + 20 = 40$$

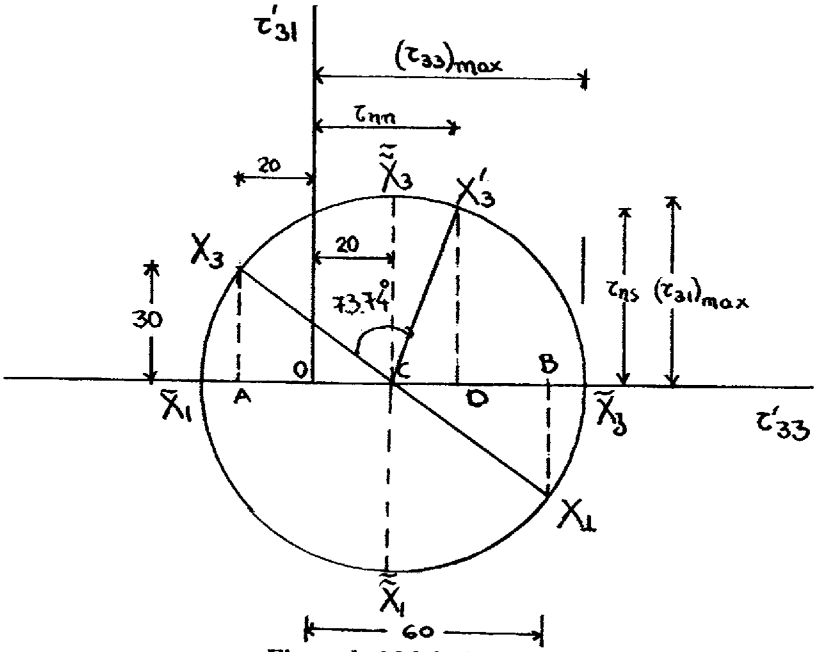

Figure b Mohr's circle

The point of Mohr's circle whose coordinates are the components of stress acting on the plane normal to the unit vector $\mathbf{i}_n$, referring to Fig. c, is located 2x36.87° = 73.74° clockwise from point X_3.

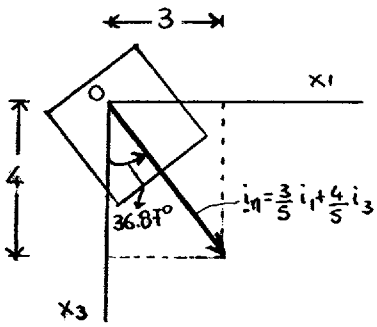

Figure c

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Thus, referring to Fig. b, we have that

$$\tau_{nn} = \overline{OC} + \overline{CD} = 20 + R\cos(69.39^\circ) = 37.6 \text{ MPa}$$

$$\tau_{ns} = R\sin(69.39^\circ) = 46.80 \text{ MPa}$$

(b) Referring to Fig. b we see that the maximum normal component of stress is the abscissa of point $\tilde{X}_3$. Thus,

$$(\tau'_{33})_{max} = \overline{OC} + R = 20 + 50 = 70 \text{ MPa}$$

Moreover, $\tan 2\tilde{\varphi}_{33} = -30/40 = -0.75$ and

$$2\tilde{\varphi}_{33} = 143.14^\circ \qquad \text{or} \qquad \tilde{\varphi}_{33} = 71.57^\circ \text{ counter -clockwise}$$

In Fig. d we show the plane on which the maximum normal component of stress acts.

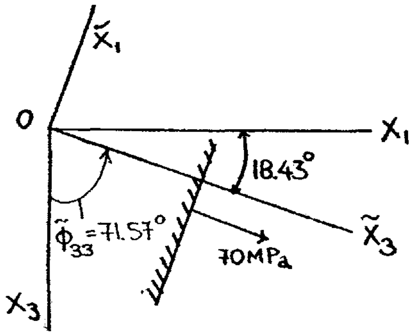

Figure d Principal directions & maximum component of stress

(c) The maximum shearing component of stress is the ordinate of point $\tilde{\tilde{X}}_3$. Thus,

$$(\tau'_{31})_{max} = R = 50 \text{ MPa}$$

Moreover $\quad < X_3 C \tilde{\tilde{X}}_3 = 143.14^\circ - 90^\circ = 53.14^\circ$ and $\tilde{\tilde{\varphi}}_{33} = 26.57^\circ$ counter - clockwise

In Fig. e we show the plane on which the maximum shearing component of stress acts.

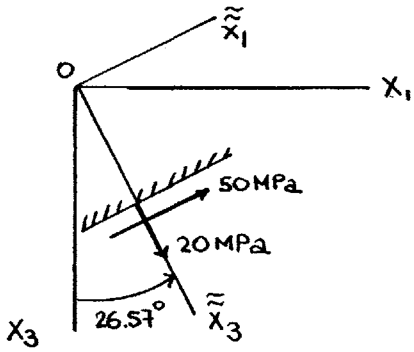

Figure e Plane of maximum shear stress

(d) Referring to relation (1.61) the normal component of stress acting on the plane normal to unit vector $\mathbf{i}_n = 1/3\,(\mathbf{i}_1 - \mathbf{i}_2 + \mathbf{i}_3)$ is

$$\tau_{nn} = \{n\}^T[\tau]\{n\} = \frac{1}{\sqrt{3}}[1 \ \ -1 \ \ 1]\begin{bmatrix} 60 & 0 & 30 \\ 0 & 0 & 0 \\ 30 & 0 & -20 \end{bmatrix}\begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}\frac{1}{\sqrt{3}} = \frac{100}{3} \text{ MPa}$$

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Problem 1.13

The state of stress of the particle of Fig. 1P13 is

$$[\tau] = \begin{bmatrix} -120 & 0 & -30 \\ 0 & 0 & 0 \\ -30 & 0 & -40 \end{bmatrix}$$

(a) The unit vector $\mathbf{i}_s$ normal to $\mathbf{i}_n$, which lies in the x_1, x_3 plane, is

$$\mathbf{i}_s = \frac{4}{5}\mathbf{i}_1 - \frac{3}{5}\mathbf{i}_3$$

Thus referring to relation (1.61) we have

$$\tau_{nn} = \{n\}^T[\tau]\{n\} = \frac{1}{5}\begin{bmatrix} 3 & 0 & 4 \end{bmatrix}\begin{bmatrix} -120 & 0 & -30 \\ 0 & 0 & 0 \\ -30 & 0 & -40 \end{bmatrix}\begin{bmatrix} 3 \\ 0 \\ 4 \end{bmatrix}\frac{1}{5} = -97.6 \text{ MPa}$$

Referring to relation (1.64a) we have

$$\tau_{ns} = \{n\}^T[\tau]\{s\} = \frac{1}{5}\begin{bmatrix} 3 & 0 & 4 \end{bmatrix}\begin{bmatrix} -120 & 0 & -30 \\ 0 & 0 & 0 \\ -30 & 0 & -40 \end{bmatrix}\begin{bmatrix} 4 \\ 0 \\ -3 \end{bmatrix}\frac{1}{5} = -46.8 \text{ MPa}$$

The negative sign indicates that τ_{ns} is in the opposite direction of the unit vector $\mathbf{i}_s$. That is as we show in Fig. a.

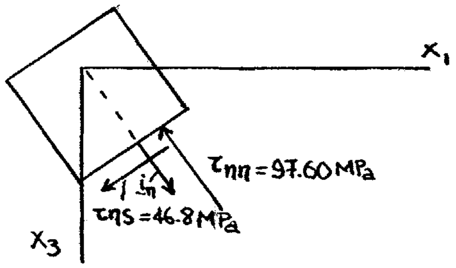

Figure a Results

Another way of solving this problem is using Mohr's circle. We plot the circle using two points X_3 (-40, -30) and X_1 (-120, 30). These points are located on the same diameter of the circle. Referring to Fig. b we see that the center, C of the circle is located on the τ'_{33} axis at a distance $1/2\ (\tau_{33} + \tau_{11}) = -80$ from the origin O. From geometric considerations we have also that the radius of the circle is

$$R = (40^2 + 30^2)^{1/2} = 50$$

The point of Mohr's circle whose coordinates are the components of stress acting on the plane normal to the unit vector $\mathbf{i}_n$, which referring to Fig. c of the solution of problem 1.12, is located $2\text{x}36.87° = 73.74°$ clockwise from point X_3. So from Fig. b we have that

$$\tau_{nn} = -(\overline{OC} + \overline{CD}) = -[80 + R\cos(69.39°)] = -97.60 \text{ MPa}$$

$$\tau_{ns} = -R\sin(69.39°) = -46.80 \text{ MPa}$$

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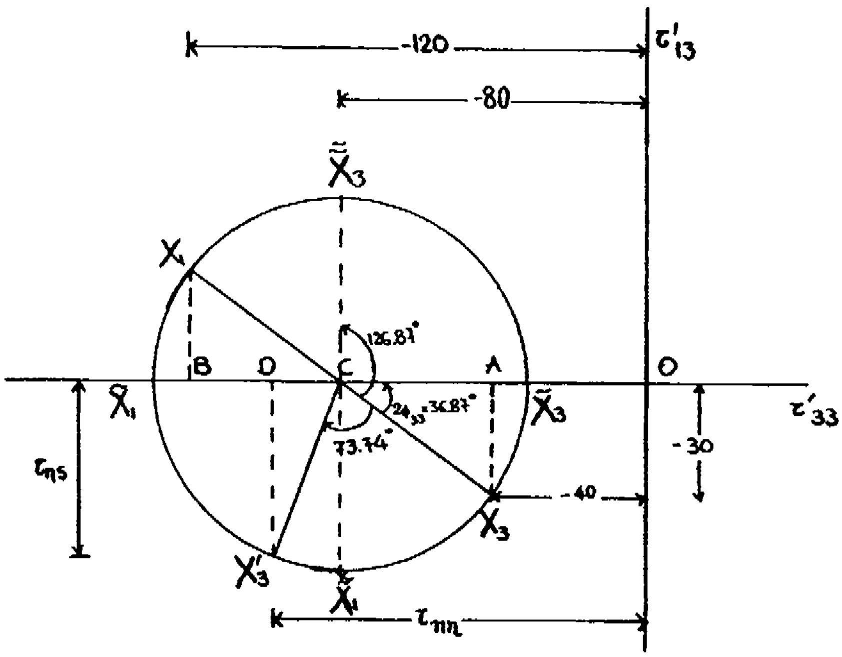

Figure b Mohr's circle

(b) Referring to Fig. b we have that the maximum normal component of stress is the abscissa of point X'_3. Thus

$$(\tau'_{33})_{max} = -\overline{CO} + R = -80 + 50 = -30 \text{ MPa}$$

In following Fig. c we show the plane on which it acts

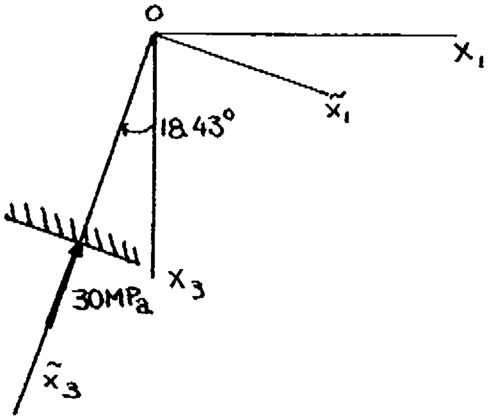

Figure c Principal directions & maximum component of stress

(c) The maximum shearing component of stress is the ordinate of point X_3. Thus

$$(\tau'_{31})_{max} = 50 \text{ MPa}$$

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In Fig. d we show the plane on which the maximum shearing component of stress acts

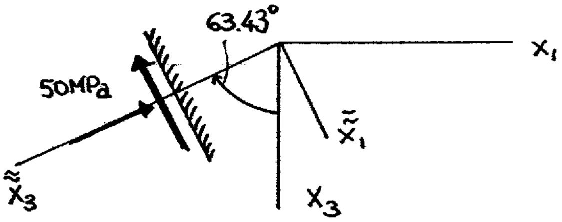

Figure d Plane of maximum shear stress

(d) Referring to relation (1.61) the normal component of stress acting on the plane normal to unit vector $\mathbf{i}_n = 1/3\,(\mathbf{i}_1 - \mathbf{i}_2 + \mathbf{i}_3)$ is

$$\tau_{nn} = \{n\}^T [\tau] \{n\} = \frac{1}{\sqrt{3}} \begin{bmatrix} 1 & -1 & 1 \end{bmatrix} \begin{bmatrix} -120 & 0 & -30 \\ 0 & 0 & 0 \\ -30 & 0 & -40 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} \frac{1}{\sqrt{3}} = -\frac{220}{3}$$

Problem 1.14

(a) Referring to relations (1.78) to (1.80) the invariants of the tensor [A] are

$$\mathrm{II}_1 = 3 \qquad \mathrm{II}_2 = -21 \qquad \mathrm{II}_3 = 4$$

Thus, relation (1.105) becomes

$$A_K^3 - 3A_K^2 - 21A_K - 4 = 0$$

The roots of this equation are

$$A_1 = 6.3863 \qquad A_2 = -0.1963 \qquad A_3 = -3.19$$

Substituting $A_1 = 6.3863$ into the first two of relations (1.103) we obtain

$$(3 - 6.3863)\lambda_{11}^{(1)} + 4\lambda_{12}^{(1)} + \lambda_{13}^{(1)} = 0$$

$$4\lambda_{11}^{(1)} + (0 - 6.3863)\lambda_{12}^{(1)} + 2\lambda_{13}^{(1)} = 0$$

Thus,

$$\lambda_{12}^{(1)} = 0.7488\,\lambda_{11}^{(1)}$$
$$\lambda_{13}^{(1)} = 0.3911\,\lambda_{11}^{(1)} \tag{a}$$

Referring to equation (1.96), from the orthogonality condition $\mathbf{i}_1^{(1)} \cdot \mathbf{i}_1^{(1)} = 1$ we have

$$(\lambda_{11}^{(1)})^2 + (\lambda_{12}^{(1)})^2 + (\lambda_{13}^{(1)})^2 = 1 \tag{b}$$

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Solving relations (a) and (b) we get

$$\lambda_{11}^{(1)} = \pm 0.7639 \qquad \lambda_{12}^{(1)} = \pm 0.5720 \qquad \lambda_{13}^{(1)} = \pm 0.2988$$

The set of direction cosines with the upper sign specifies the same direction as the set of direction cosines with the lower sign but opposite sense.

Substituting $A_2 = -0.1963$ into the first of relations (1.103) we get

$$(3+0.1963)\lambda_{21}^{(2)} + 4\lambda_{22}^{(2)} + \lambda_{23}^{(2)} = 0$$
$$4\lambda_{21}^{(2)} + 0.1963\lambda_{22}^{(2)} + 2\lambda_{23}^{(2)} = 0$$

Thus,

$$\lambda_{22}^{(2)} = -0.3066\,\lambda_{21}^{(2)} \qquad \text{(c)}$$
$$\lambda_{23}^{(2)} = -1.9699\,\lambda_{21}^{(2)}$$

and

$$(\lambda_{21}^{(2)})^2 + (\lambda_{22}^{(2)})^2 + (\lambda_{23}^{(2)})^2 = 1 \qquad \text{(d)}$$

Solving relations (c) and (d) we have

$$\lambda_{21}^{(2)} = \pm 0.4484 \qquad \lambda_{22}^{(2)} = \mp 0.1375 \qquad \lambda_{23}^{(2)} = \mp 0.8833$$

Substituting $A_3 = -3.19$ into the first two of relations (1.103) we have

$$(3+3.19)\lambda_{31}^{(3)} + 4\lambda_{32}^{(3)} + \lambda_{33}^{(3)} = 0$$
$$4\lambda_{31}^{(3)} + 3.19\lambda_{32}^{(3)} + 2\lambda_{33}^{(3)} = 0$$

Thus,

$$\lambda_{32}^{(3)} = -1.7422\,\lambda_{31}^{(3)} \qquad \text{(e)}$$
$$\lambda_{33}^{(3)} = 0.7788\,\lambda_{31}^{(3)}$$

and

$$(\lambda_{31}^{(3)})^2 + (\lambda_{32}^{(3)})^2 + (\lambda_{33}^{(3)})^2 = 1 \qquad \text{(f)}$$

Solving relations (e) and (f) we get

$$\lambda_{31}^{(3)} = \pm 0.4641 \qquad \lambda_{32}^{(3)} = \mp 0.8086 \qquad \lambda_{33}^{(3)} = \pm 0.3614$$

Thus, the principal directions of the tensor are

$$\tilde{i}_1^{(1)} = 0.7639\,\underset{\sim}{i}_1 + 0.5720\,\underset{\sim}{i}_2 + 0.2988\,\underset{\sim}{i}_3$$
$$\tilde{i}_2^{(2)} = 0.4484\,\underset{\sim}{i}_1 - 0.1375\,\underset{\sim}{i}_2 - 0.8833\,\underset{\sim}{i}_3 \qquad \text{(g)}$$
$$i_3^{(3)} = 0.4641\,\underset{\sim}{i}_1 - 0.8086\,\underset{\sim}{i}_2 + 0.3614\,\underset{\sim}{i}_3$$

As a check we apply the orthogonality conditions $\mathbf{i}_1^{(1)} \cdot \mathbf{i}_2^{(2)} = 0$ and $\mathbf{i}_1^{(1)} \cdot \mathbf{i}_3^{(3)} = 0$. Referring to relations (1.25a) we have

$$\lambda_{11}^{(1)}\lambda_{12}^{(2)} + \lambda_{21}^{(1)}\lambda_{22}^{(2)} + \lambda_{31}^{(1)}\lambda_{32}^{(2)} = 0$$
$$\lambda_{11}^{(1)}\lambda_{13}^{(3)} + \lambda_{21}^{(1)}\lambda_{23}^{(3)} + \lambda_{31}^{(1)}\lambda_{33}^{(3)} = 0$$

or

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$$0.7639 \cdot 0.4484 - 0.5720 \cdot 0.1375 - 0.2988 \cdot 0.8833 \simeq 0$$

$$0.7639 \cdot 0.4641 - 0.5720 \cdot 0.8086 + 0.2988 \cdot 0.3614 \simeq 0$$

(b) Referring to relations (1.143) the maximum values of the non-diagonal components of the tensor are

$$(A'_{12})_{max} = \frac{A_1 - A_2}{2} = 3.2913$$

$$(A'_{13})_{max} = \frac{A_1 - A_3}{2} = 4.7882$$

$$(A'_{23})_{max} = \frac{A_2 - A_3}{2} = 1.4969$$

Problem 1.15

(a) Referring to relations (1.78) to (1.80) the invariants of the tensor [A] are

$$\text{II}_1 = 7 \qquad \text{II}_2 = 0 \qquad \text{II}_3 = -25$$

Thus, relation (1.105) becomes

$$A_K^3 - 7A_K^2 + 25 = 0$$

The roots of this equation are

$$A_1 = 6.3872 \qquad A_2 = 2.3084 \qquad A_3 = -1.6956$$

Substituting $A_1 = 6.3872$ into the first two of relations (1.103) we obtain

$$(1 - 6.3872)\lambda_{11}^{(1)} - 2\lambda_{12}^{(1)} + \lambda_{13}^{(1)} = 0$$

$$-2\lambda_{11}^{(1)} + (4 - 6.3872)\lambda_{12}^{(1)} + 3\lambda_{13}^{(1)} = 0$$

Thus,

$$\lambda_{12}^{(1)} = -3.9198\lambda_{11}^{(1)} \qquad \text{(a)}$$

$$\lambda_{13}^{(1)} = -2.4526\lambda_{11}^{(1)}$$

Referring to equation (1.96), from the orthogonality condition $\mathbf{i}_1^{(1)} \cdot \mathbf{i}_1^{(1)} = 1$ we have

$$(\lambda_{11}^{(1)})^2 + (\lambda_{12}^{(1)})^2 + (\lambda_{13}^{(1)})^2 = 1 \qquad \text{(b)}$$

Solving relations (a) and (b) we get

$$\lambda_{11}^{(1)} = \pm 0.2114 \qquad \lambda_{12}^{(1)} = \mp 0.8285 \qquad \lambda_{13}^{(1)} = \mp 0.5185$$

The set of direction cosines with the upper sign specifies the same direction as the set of direction cosines with the lower sign but opposite sense.

Substituting $A_2 = 2.3084$ into the first of relations (1.103) we get

$$(1 - 2.3084)\lambda_{21}^{(2)} - 2\lambda_{22}^{(2)} + \lambda_{23}^{(2)} = 0$$

$$-2\lambda_{21}^{(2)} + (4 - 2.3084)\lambda_{22}^{(2)} + 3\lambda_{23}^{(2)} = 0$$

Thus,

$$\lambda_{22}^{(2)} = -0.2503\lambda_{21}^{(2)} \qquad \text{(c)}$$

$$\lambda_{23}^{(2)} = 0.8078\lambda_{21}^{(2)}$$

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and

$$(\lambda_{21}^{(2)})^2+(\lambda_{22}^{(2)})^2+(\lambda_{23}^{(2)})^2=1 \tag{d}$$

Solving relations (c) and (d) we have

$$\lambda_{21}^{(2)}=\pm 0.7636 \qquad \lambda_{22}^{(2)}=\mp 0.1911 \qquad \lambda_{23}^{(2)}=\pm 0.6168$$

Substituting A_3 = -1.6956 into the first two of relations (1.103) we have

$$(1+1.6956)\lambda_{31}^{(3)}-2\lambda_{32}^{(3)}+\lambda_{33}^{(3)}=0$$
$$-2\lambda_{31}^{(3)}+(4+1.6956)\lambda_{32}^{(3)}+3\lambda_{33}^{(3)}=0$$

Thus,

$$\lambda_{32}^{(3)}=0.8624\,\lambda_{31}^{(3)} \tag{e}$$
$$\lambda_{33}^{(3)}=-0.9708\,\lambda_{31}^{(3)}$$

and

$$(\lambda_{31}^{(3)})^2+(\lambda_{32}^{(3)})^2+(\lambda_{33}^{(3)})^2=1 \tag{f}$$

Solving relations (e) and (f) we get

$$\lambda_{31}^{(3)}=\pm 0.6101 \qquad \lambda_{32}^{(3)}=\pm 0.5262 \qquad \lambda_{33}^{(3)}=\mp 0.5923$$

Thus, the principal directions of the tensor are

$$\tilde{i}_1^{(1)}=0.2114\,\underset{\sim}{i}_1-0.8285\,\underset{\sim}{i}_2-0.5185\,\underset{\sim}{i}_3 \tag{g}$$
$$\tilde{i}_2^{(2)}=0.7636\,\underset{\sim}{i}_1-0.1911\,\underset{\sim}{i}_2+0.6168\,\underset{\sim}{i}_3$$
$$\tilde{i}_3^{(3)}=0.6101\,\underset{\sim}{i}_1+0.5262\,\underset{\sim}{i}_2-0.5923\,\underset{\sim}{i}_3$$

(b) Referring to relations (1.143) the maximum values of the non-diagonal components of the tensor are

$$(A'_{12})_{max}=\frac{A_1-A_2}{2}=2.0394$$
$$(A'_{13})_{max}=\frac{A_1-A_3}{2}=4.0414$$
$$(A'_{23})_{max}=\frac{A_2-A_3}{2}=2.002$$

Problem 1.16

(a) Referring to relations (1.78) to (1.80) the invariants of the tensor are

$$\mathrm{II}_1=0 \qquad \mathrm{II}_2=-25 \qquad \mathrm{II}_3=0$$

Thus, relation (1.105) becomes

$$A_K^3-25A_K=0$$

The roots of this equation are

$$A_1=0 \qquad A_2=5 \qquad A_3=-5$$

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Substituting $A_1 = 0$ into the first two of relations (1.103) we obtain

$$4\lambda_{12}^{(1)} + 3\lambda_{13}^{(1)} = 0$$
$$4\lambda_{11}^{(1)} = 0$$

Thus,

$$\lambda_{12}^{(1)} = -0.75\lambda_{13}^{(1)} \qquad \text{(a)}$$
$$\lambda_{11}^{(1)} = 0$$

Referring to equation (1.96), from the orthogonality condition $i_1^{(1)} \cdot i_1^{(1)} = 1$ we have

$$(\lambda_{11}^{(1)})^2 + (\lambda_{12}^{(1)})^2 + (\lambda_{13}^{(1)})^2 = 1 \qquad \text{(b)}$$

Solving relations (a) and (b) we get

$$\lambda_{11}^{(1)} = 0 \qquad \lambda_{12}^{(1)} = \mp\frac{3}{5} \qquad \lambda_{13}^{(1)} = \pm\frac{4}{5}$$

The set of direction cosines with the upper sign specifies the same direction as the set of direction cosines with the lower sign but opposite sense.

Substituting $A_2 = 5$ into the first of relations (1.103) we get

$$-5\lambda_{21}^{(2)} + 4\lambda_{22}^{(2)} + 3\lambda_{23}^{(2)} = 0$$
$$4\lambda_{21}^{(2)} - 5\lambda_{22}^{(2)} = 0$$

Thus,

$$\lambda_{21}^{(2)} = \frac{5}{4}\lambda_{22}^{(2)}$$
$$\lambda_{23}^{(2)} = \frac{3}{4}\lambda_{22}^{(2)} \qquad \text{(c)}$$

and

$$(\lambda_{21}^{(2)})^2 + (\lambda_{22}^{(2)})^2 + (\lambda_{23}^{(2)})^2 = 1 \qquad \text{(d)}$$

Solving relations (c) and (d) we have

$$\lambda_{21}^{(2)} = \pm\frac{1}{\sqrt{2}} \qquad \lambda_{22}^{(2)} = \pm\frac{4}{5\sqrt{2}} \qquad \lambda_{23}^{(2)} = \pm\frac{3}{5\sqrt{2}}$$

Substituting $A_3 = -5$ into the first two of relations (1.103) we have

$$5\lambda_{31}^{(3)} + 4\lambda_{32}^{(3)} + 3\lambda_{33}^{(3)} = 0$$
$$4\lambda_{31}^{(3)} + 5\lambda_{32}^{(3)} = 0$$

Thus,

$$\lambda_{31}^{(3)} = -\frac{5}{4}\lambda_{32}^{(3)} \qquad \text{(e)}$$
$$\lambda_{33}^{(3)} = \frac{3}{4}\lambda_{32}^{(3)}$$

and

$$(\lambda_{31}^{(3)})^2 + (\lambda_{32}^{(3)})^2 + (\lambda_{33}^{(3)})^2 = 1 \qquad \text{(f)}$$

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Solving relations (e) and (f) we get

$$\lambda_{31}^{(3)} = \mp \frac{1}{\sqrt{2}} \qquad \lambda_{32}^{(3)} = \pm \frac{4}{5\sqrt{2}} \qquad \lambda_{33}^{(3)} = \pm \frac{3}{5\sqrt{2}}$$

Thus, the principal directions of the tensor are

$$\tilde{i}_1^{(1)} = -\frac{3}{5} i_2 + \frac{4}{5} i_3$$

$$\tilde{i}_2^{(2)} = \frac{1}{\sqrt{2}} i_1 + \frac{4}{5\sqrt{2}} i_2 + \frac{3}{5\sqrt{2}} i_3$$

$$\tilde{i}_3^{(3)} = -\frac{1}{\sqrt{2}} i_1 + \frac{4}{5\sqrt{2}} i_2 + \frac{3}{5\sqrt{2}} i_3 \qquad \text{(g)}$$

As a check we apply the orthogonality conditions $i_1^{(1)} \cdot i_2^{(2)} = 0$ and $i_1^{(1)} \cdot i_3^{(3)} = 0$.

Referring to relations (1.25a) we have

$$\lambda_{11}^{(1)}\lambda_{12}^{(2)} + \lambda_{21}^{(1)}\lambda_{22}^{(2)} + \lambda_{31}^{(1)}\lambda_{32}^{(2)} = 0$$

$$\lambda_{11}^{(1)}\lambda_{13}^{(3)} + \lambda_{21}^{(1)}\lambda_{23}^{(3)} + \lambda_{31}^{(1)}\lambda_{33}^{(3)} = 0$$

or

$$0 \cdot \frac{1}{\sqrt{2}} - \frac{3}{5} \cdot \frac{4}{5\sqrt{2}} + \frac{4}{5} \cdot \frac{3}{5\sqrt{2}} = 0$$

$$0 \cdot \left(-\frac{1}{\sqrt{2}}\right) - \frac{3}{5} \cdot \frac{4}{5\sqrt{2}} + \frac{4}{5} \cdot \frac{3}{5\sqrt{2}} = 0$$

(b) Referring to Fig. 1P16 the transformation matrix is

$$[\Lambda_s] = \begin{bmatrix} 0.866 & 0.5 & 0 \\ -0.5 & 0.866 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad \text{(h)}$$

Substituting (h) into relation (1.73a) we have

$$[A'] = \begin{bmatrix} 0.866 & 0.5 & 0 \\ -0.5 & 0.866 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 4 & 3 \\ 4 & 0 & 0 \\ 3 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0.866 & -0.5 & 0 \\ 0.5 & 0.866 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 3.464 & 2 & 2.598 \\ 2 & -3.464 & -1.5 \\ 2.598 & -1.5 & 0 \end{bmatrix}$$