Chapter 1: The Hydrologic Cycle

PROBLEM 1.1

a) *Los Angeles gets its water from the Sierra Nevada Snowpack (79 %), the Colorado River (8 %), the local groundwater (12 %) and recycling (1 %).*

The Sierra Nevada snowpack and the Colorado River are the two sources of surface water and are transported via three aqueducts as seen in Figure 1.14. The Colorado aqueduct transports water from the Colorado River to Los Angeles and contributes to 8 % of the city water supply. The California aqueduct brings water from the bay area and contributes for 45 % of the supply. The Los Angeles aqueduct crosses the valley and contributes to 34 % of the water supply. (https://www.ladwp.com/ladwp/faces/ladwp/aboutus/a-water?).

b) *The long term average of the global hydrologic cycle is 1 m/yr, which means that:*

$$\overline{P}_{global} \cong 1 \ \ m/yr$$

It is important to keep in mind that the 1 m/yr is a long-term global average and precipitation and evaporation among other fluxes significantly change in both space and time. For example Cherrapunji in India (the wettest place on Earth) receives an annual average rainfall of 11.1 m/year and the driest place on Earth (the Atacama Desert in Chile) receives no precipitation (i.e. There has never been recorded rainfall). Los Angeles receives an average of approximately 0.38 m/year (15 inches) of rainfall per year making it a relatively dry place.

c) *The long-term average of the global hydrologic cycle is 1 m/yr that means*

$$\overline{P}_{global} \cong 1 \ \ m/yr = \overline{E}_{global}$$

where this is expressed in terms of a volumetric flux density (i.e. normalized over the surface of the Earth). The long-term average global precipitation and evaporation are equal in order to preserve the steady-state mass balance.

PROBLEM 1.2

a) *The latent heat of vaporization L_v is assumed to be constant with temperature and has the following value for water:*

$$L_v = 2.50 \cdot 10^6 \, J \ \ kg^{-1}$$

The fact that the latent heat of vaporization is so large for water, coupled with the fact that evaporation/condensation is a phase transformation quite common in the Earth system, result in significant energy sources and sinks in the hydrologic cycle. Specifically, latent heat of vaporization, along with latent heat of fusion, is critical in energy transfer between the surface and atmosphere and in global heat transport.

b)

$The\ water\ density\ at\ 30\ \deg.\ is: \rho_w = 995.67\ kg \cdot m^3$

$The\ latent\ of\ vaporization\ of\ water\ is: L_v = 2.5 \cdot 10^6 J \cdot kg^{-1}$

$The\ volume\ of\ water\ is: V_w = 5 \cdot 10^{-6} m^3$

$\rightarrow The\ mass\ of\ water\ is: m_w = V_w \times \rho_w = 5 \cdot 10^{-6} m^3 \times 995.67\ kg \cdot m^3 = 4.98 \cdot 10^{-3} kg$

$The\ energy\ absorbed\ by\ the\ water\ is:$

$E_{water} = L_v \times m_w = 2.5 \cdot 10^6 J \cdot kg^{-1} \times 4.98 \cdot 10^{-3} kg = 12450\ J$

c)

$The\ energy\ released\ by\ the\ air\ is:$

$E_{air} = -\ E_{water}$

$The\ volume\ of\ air\ to\ be\ cooled\ is: V_{air} = 1\ m^3$

$\rightarrow The\ mass\ of\ air\ is: m_{air} = V_{air} \times \rho_{air} = 1\ m^3 \times 1.16\ kg \cdot m^3 = 1.16\ kg$

$The\ temperature\ change\ is:$

$$\Delta_{Tair} = \frac{E_{air}}{m_{air} \times c_{p_{air}}} = \frac{-12450\ J}{1.16\ kg \times 0.24 \times 4177\ J \cdot kg^{-1} \cdot K^{-1}} = -10.71\ K$$

The water absorbs energy from that air inside the cooler, which decreases the air temperature from 30 degrees Celsius to 19.29 degree Celsius.

PROBLEM 1.3

a) *The table below lists the SI units associated with each of these fluxes.*

Quantity	***SI units***
Volumetric flux	$m^3\ s^{-1}$
Mass flux	$kg\ s^{-1}$
Volumetric flux density	$m\ s^{-1}$
Mass flux density	$kg\ m^{-2}\ s^{-1}$
Energy flux density	$W\ m^{-2} (= J\ m^{-2}\ s^{-1})$

The cross-sectional area A *over which the flux occurs is used to convert between a volumetric flux and a volumetric flux density as shown below:*

$Volumetric\ flux\ density = 1/A \cdot (Volumetric\ flux)$

$Volumetric\ flux = A \cdot (Volumetric\ flux\ density)$

b)

$$37\frac{in}{yr}\cdot\frac{2.54\,cm}{1\,in}\cdot\frac{1\,m}{100\,cm}=0.94\frac{m}{yr}$$

Los Angeles receives about 15 inches (0.38 m/yr) of rainfall per year. These figures explain the contrast between the wet climate of Seattle and the dry climate of Los Angeles.

c)

$$37\,in\cdot\frac{2.54\,cm}{1\,in}\cdot\frac{1\,m}{100\,cm}\cdot\frac{1\,km}{1000\,m}\cdot 1000\,km^2= \;\; 0.94\,km^3$$

d)

$$1\,AF\cdot\frac{43,560\;ft^2}{1\,ac}=43,560\;ft^3$$

$$43,560\;ft^3\cdot\left(\frac{1\,m}{3.28\,ft}\right)^3=1,234\,m^3$$

e) *Top loading washing machine:*

$$\left(\frac{8\;\;loads}{week\;\;family}\right)\cdot\left(\frac{4.5\;\;ft^3}{load}\right)\cdot\left(\frac{30.48\;\;cm}{1\;\;ft}\right)^3= \quad 1,019,406\;\;cm^3\;\;week^{-1}\,family^{-1}$$

Front loading washing machine:

$$\left(\frac{8\;\;loads}{week\;\;family}\right)\cdot\left(\frac{28.4\;\;gal}{load}\right)\cdot\left(\frac{3.785\times10^{-3}\;\;m^3}{1\;\;gal}\right)\cdot\left(\frac{100\;\;cm}{1\;\;m}\right)^3=859,952\;\;cm^3 week^{-1}\,family^{-1}$$

Water savings:

$$\left(8,000,000\;\;families\right)\left(1,019,406-859,952\;\;cm^{-3}week^{-1}\,family^{-1}\right)\cdot\left(\frac{1\;\;m}{100\;\;cm}\right)^3$$

$$=1.28.10^6\,m^3 week^{-1}$$

PROBLEM 1.4

Long-term water balance equation: $P + S_{w_{in}} - S_{w_{out}} - E + G_{w_{in}} - G_{w_{out}} = 0$

G_{wout}=0 because the pond is lined with cement.

$$A_{pond} = 40,000 ft^2 \cdot \left(\frac{1m}{3.28 ft}\right)^2 = 3,718 m^2$$

$$P = 260 mm yr^{-1} \cdot \frac{1m}{1000mm} \cdot 3,718 m^2 \quad 966.7 m^3 yr^{-1}$$

$$E = 105 in yr^{-1} \cdot \frac{2.54cm}{1in} \cdot \frac{1m}{100cm} \cdot 3,718 m^2 = 9,915.9 m^3 yr^{-1}$$

$$S_{w_{in}} - S_{w_{out}} = (0.25 - 0.30) m^3 s^{-1} \cdot \frac{(3600 \times 24 \times 365)s}{1yr} = 1,576,800 m^3 yr^{-1}$$

So $G_{w_{in}} = E - P - (S_{w_{in}} - S_{w_{out}}) = 9,915.9 - 966.7 + 1,576,800 = 1,585,749.2 m^3 yr^{-1}$

1,585,749.2m³ (4345m³ per day or 0.0503m³ per second) must be added to the pond each year to keep its level constant.

PROBLEM 1.5

a) *The heat capacity of a substance is the amount of energy (heat) absorbed/released by a mass of a substance when its temperature is raised/lowered. Latent heat is the amount of energy released or absorbed when a given mass of a substance changes phases. In particular, the latent heat of fusion is the amount of energy added/released when a unit mass of substance melts/freezes. The latent heat of vaporization is the quantity of energy absorbed/released when a unit mass of substance vaporizes/condenses.*

The latent heat of a substance is used in the context of evaporation to convert mass flux to energy. This is in fact expressed in energy over mass units.

The typical values for heat capacity (c_p), latent heat of fusion (L_f), and latent heat of vaporization (L_v) for water are as follows:

$$c_p = 4216 J\ kg^{-1} K^{-1}$$

$$L_f = 3.34 \cdot 10^5 J\ kg^{-1}$$

$$L_v = 2.50 \cdot 10^6 J\ kg^{-1}$$

b) *The nominal density of liquid water is bigger than the one of ice. Their nominal densities are 1000 kg/m³ and 917 kg/m³ respectively.*

c)

$$Density = \rho_i = 916.7kg\ m^{-3}$$

$$Volume = V_i = \left(2cm \cdot \frac{1m}{100cm}\right)^3 = 8.0 \cdot 10^{-6} m^3$$

$$Mass = m_i = V_i \cdot \rho_i = \left(8.0 \cdot 10^{-6} m^3\right) \cdot \left(916.7kg\ m^{-3}\right) = 7.3 \cdot 10^{-3} kg$$

$$Energy = E_{melt} = L_f \cdot m_i = \left(3.34 \cdot 10^5 J\ kg^{-1}\right) \cdot \left(7.3 \cdot 10^{-3} kg\right) = 2449J\ \ (absorbed)$$

d)

$$\text{Energy released by tea} = \text{-}\left(\text{Energy absorbed by ice}\right)$$

$$\therefore E_{tea} = -3E_{melt} = -3 \cdot 2449 \text{ J}$$

$$\Delta T_{tea} = \frac{E_{tea}}{m_{tea} \cdot c_p} = \frac{-3 \cdot 2449 \text{ J}}{\left(0.5 \cdot 10^{-3} \text{m}^3 \cdot 1000 \text{ kg m}^{-3}\right) \cdot \left(4216 \text{ J kg}^{-1}\text{K}^{-1}\right)} = -3.5K = -3.5^\circ C$$

$$T_{tea} = 75 - 3.5 = 71.5^\circ C$$

In order to cool its temperature the tea releases energy, which is absorbed by the ice cubes providing the energy for them to melt.

e)

$$m_{tea,left} = V_{tea,left} \cdot \rho_w = 5 \cdot 10^{-5} \text{m}^3\ \ 1000 \text{ kg m}^{-3}\ \ = 0.05kg$$

$$E_{evap} = L_v \cdot m_{tea,left} = \left(2.5 \cdot 10^6 J\ kg^{-1}\right) \cdot \left(0.05kg\right) = 125000J\ \ (released)$$

$$\text{Energy released by air} = \text{-}\left(\text{Energy absorbed by the tea to evaporate}\right)$$

$$\therefore E_{air} = -E_{evap} = -125000J$$

$$\Delta T_{air} = \frac{E_{air}}{m_{air} \cdot c_p(air)} = \frac{-125000J}{\left(5 \cdot 5 \cdot 3m^3 \cdot 1.2kgm^{-3}\right) \cdot \left(1011.84J\ kg^{-1}K^{-1}\right)} = -1.4K = -1.4^\circ C$$

To evaporate the water, energy is transferred from the air to the water. In this case, the water absorbs energy while the air provides it the same amount of energy. As a result, the air temperature would decrease.

Chapter 2: Atmospheric Thermodynamics

PROBLEM 2.1

a) *Density ρ, temperature T, humidity (vapor pressure) e, and pressure p are related to each other via the ideal gas law (IGL) for moist air.*

$$IGL:\ p \cdot \left[1 - (1-\varepsilon) \cdot \frac{e}{p}\right] = \rho R_d T \rightarrow \rho = \frac{p}{R_d T} \cdot \left[1 - (1-\varepsilon) \cdot \frac{e}{p}\right]$$