# CHAPTER 1

**1.1** You are given the following differential equation with the initial condition, *v*(*t* = 0) = 0,



Multiply both sides by *m*/*cd*



Define 



Integrate by separation of variables,



A table of integrals can be consulted to find that



Therefore, the integration yields



If *v* = 0 at *t* = 0, then because tanh–1(0) = 0, the constant of integration *C* = 0 and the solution is



This result can then be rearranged to yield



**1.2** **(a)** For the case where the initial velocity is positive (downward), Eq. (1.21) is



Multiply both sides by *m*/*cd*



Define *a* = 



Integrate by separation of variables,



A table of integrals can be consulted to find that



Therefore, the integration yields



If *v* = *v*0 at *t* = 0, then



Substitute back into the solution



Multiply both sides by *a*, taking the hyperbolic tangent of each side and substituting *a* gives,

 (1)

**(b)** For the case where the initial velocity is negative (upward), Eq. (1.21) is



Multiplying both sides of Eq. (1.8) by *m*/*cd* and defining  yields



Integrate by separation of variables,



A table of integrals can be consulted to find that



Therefore, the integration yields



The initial condition, *v*(0) = *v*0 gives



Substituting this result back into the solution yields



Multiplying both sides by *a* and taking the tangent gives



or substituting the values for *a* and simplifying gives

 (2)

(c) We use Eq. (2) until the velocity reaches zero. Inspection of Eq. (2) indicates that this occurs when the argument of the tangent is zero. That is, when



The time of zero velocity can then be computed as



Thereafter, the velocities can then be computed with Eq. (1.9),

 (3)

Here are the results for the parameters from Example 1.2, with an initial velocity of –40 m/s.



Therefore, for *t* = 2, we can use Eq. (2) to compute



For *t* = 4, the jumper is now heading downward and Eq. (3) applies



The same equation is then used to compute the remaining values. The results for the entire calculation are summarized in the following table and plot:

|  |  |
| --- | --- |
| ***t* (s)** | ***v* (m/s)** |
| 0 | -40 |
| 2 | -14.8093 |
| 3.470239 | 0 |
| 4 | 5.17952 |
| 6 | 23.07118 |
| 8 | 35.98203 |
| 10 | 43.69242 |
| 12 | 47.78758 |



1.3 (a) Use the conservation of cash to compute the balance on 6∕1, 7∕1, 8∕1, and 9∕1 if the interest rate is 1% per month (*i* = 0.01∕month). Show each step in the computation.

(b) Write a differential equation for the cash balance in the form



where *t* = time (months), *D*(*t*) = deposits as a function of time ($/month), *W*(*t*) = withdrawals as a function of time ($/month). For this case, assume that interest is compounded continuously; that is, interest = *iB.*

(c) Use Euler’s method with a time step of 0.5 month to simulate the balance. Assume that the deposits and with­drawals are applied uniformly over the month.

(d) Develop a plot of balance versus time for **(a)** and **(c)**.

==========================================

**(*a*)** This is a transient computation. For the period ending June 1:

Balance = Previous Balance + Deposits – Withdrawals + Interest

Balance = 1512.33 + 220.13 – 327.26 + 0.01(1512.33) = 1420.32

The balances for the remainder of the periods can be computed in a similar fashion as tabulated below:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Date** | **Deposit** | **Withdrawal** | **Interest** | **Balance** |
| 1-May |  |  |  | $1,512.33 |
|  | $220.13 | $327.26 | $15.12 |  |
| 1-Jun |  |  |  | $1,420.32 |
|  | $216.80 | $378.61 | $14.20 |  |
| 1-Jul |  |  |  | $1,272.72 |
|  | $450.25 | $106.80 | $12.73 |  |
| 1-Aug |  |  |  | $1,628.89 |
|  | $127.31 | $350.61 | $16.29 |  |
| 1-Sep |  |  |  | **$1,421.88** |

(*b*) 

(*c*) for *t* = 0 to 0.5:





for *t* = 0.5 to 1:





The balances for the remainder of the periods can be computed in a similar fashion as tabulated below:

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **Date** | **Deposit** | **Withdrawal** | **Interest** | ***dB*/*dt*** | **Balance** |
| 1-May | $220.13 | $327.26 | $15.12 | -$92.01 | $1,512.33 |
| 16-May | $220.13 | $327.26 | $14.66 | -$92.47 | $1,466.33 |
| 1-Jun | $216.80 | $378.61 | $14.20 | -$147.61 | $1,420.09 |
| 16-Jun | $216.80 | $378.61 | $13.46 | -$148.35 | $1,346.29 |
| 1-Jul | $450.25 | $106.80 | $12.72 | $356.17 | $1,272.12 |
| 16-Jul | $450.25 | $106.80 | $14.50 | $357.95 | $1,450.20 |
| 1-Aug | $127.31 | $350.61 | $16.29 | -$207.01 | $1,629.18 |
| 16-Aug | $127.31 | $350.61 | $15.26 | -$208.04 | $1,525.67 |
| 1-Sep |  |  |  |  | **$1,421.65** |

(*d*) As in the plot below, the results of the two approaches are very close.



**1.4** At *t* = 12 s, the analytical solution is 50.6175 (Example 1.1). The numerical results are:

|  |  |  |
| --- | --- | --- |
| **step** | **v(12)** | **absolute relative error** |
| 2 | 51.6008 | 1.94% |
| 1 | 51.2008 | 1.15% |
| 0.5 | 50.9259 | 0.61% |

where the relative error is calculated with



The error versus step size can be plotted as



Thus, halving the step size approximately halves the error.

**1.5** (*a*) The force balance is



Applying Laplace transforms,



Solve for

 (1)

The first term to the right of the equal sign can be evaluated by a partial fraction expansion,

 (2)



Equating like terms in the numerators yields



Therefore,



These results can be substituted into Eq. (2), and the result can be substituted back into Eq. (1) to give



Applying inverse Laplace transforms yields



or



where the first term to the right of the equal sign is the general solution and the second is the particular solution. For our case, *v*(0) = 0, so the final solution is



**Alternative solution:** Another way to obtain solutions is to use separation of variables,



The integrals can be evaluated as



where *C* = a constant of integration, which can be evaluated by applying the initial condition



which can be substituted back into the solution



This result can be rearranged algebraically to solve for *v*,



where the first term to the right of the equal sign is the general solution and the second is the particular solution. For our case, *v*(0) = 0, so the final solution is



**(b)** The numerical solution can be implemented as





The computation can be continued and the results summarized and plotted as:

|  |  |  |
| --- | --- | --- |
| ***t*** | ***v*** | ***dv*/*dt*** |
| 0 | 0 | 9.81 |
| 2 | 19.6200 | 6.4968 |
| 4 | 32.6136 | 4.3026 |
| 6 | 41.2187 | 2.8494 |
| 8 | 46.9176 | 1.8871 |
| 10 | 50.6917 | 1.2497 |
| 12 | 53.1911 | 0.8276 |
| ∞ | 58.0923 |  |



Note that the analytical solution is included on the plot for comparison.

**1.6** 

jumper #1: 

jumper #2: 







**1.7** Note that the differential equation should be formulated as



This ensures that the sign of the drag is correct when the parachutist has a negative upward velocity. Before the chute opens (*t* < 10), Euler’s method can be implemented as



After the chute opens (*t* ≥ 10), the drag coefficient is changed and the implementation becomes



Here is a summary of the results along with a plot:

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **Chute closed** | |  | **Chute opened** | |  |
| ***t*** | ***v*** | ***dv*/*dt*** | ***t*** | ***v*** | ***dv*/*dt*** |
| 0 | -20.0000 | 11.0600 | 10 | 51.5260 | -39.9698 |
| 1 | -8.9400 | 10.0598 | 11 | 11.5561 | 7.3060 |
| 2 | 1.1198 | 9.8061 | 12 | 18.8622 | 3.1391 |
| 3 | 10.9258 | 9.4370 | 13 | 22.0013 | 0.7340 |
| 4 | 20.3628 | 8.5142 | 14 | 22.7352 | 0.1183 |
| 5 | 28.8770 | 7.2041 | 15 | 22.8535 | 0.0172 |
| 6 | 36.0812 | 5.7417 | 16 | 22.8707 | 0.0025 |
| 7 | 41.8229 | 4.3439 | 17 | 22.8732 | 0.0003 |
| 8 | 46.1668 | 3.1495 | 18 | 22.8735 | 0.0000 |
| 9 | 49.3162 | 2.2097 | 19 | 22.8736 | 0.0000 |
|  |  |  | 20 | 22.8736 | 0.0000 |



**1.8 (a)** The first two steps are





The process can be continued to yield

|  |  |  |
| --- | --- | --- |
| ***t*** | ***c*** | ***dc*/*dt*** |
| 0 | 100.0000 | -17.5000 |
| 0.1 | 98.2500 | -17.1938 |
| 0.2 | 96.5306 | -16.8929 |
| 0.3 | 94.8413 | -16.5972 |
| 0.4 | 93.1816 | -16.3068 |
| 0.5 | 91.5509 | -16.0214 |
| 0.6 | 89.9488 | -15.7410 |
| 0.7 | 88.3747 | -15.4656 |
| 0.8 | 86.8281 | -15.1949 |
| 0.9 | 85.3086 | -14.9290 |
| 1 | 83.8157 | -14.6678 |

**(b)** The results when plotted on a semi-log plot yields a straight line



The slope of this line can be estimated as



Thus, the slope is approximately equal to the negative of the decay rate. If we had used a smaller step size, the result would be more exact.

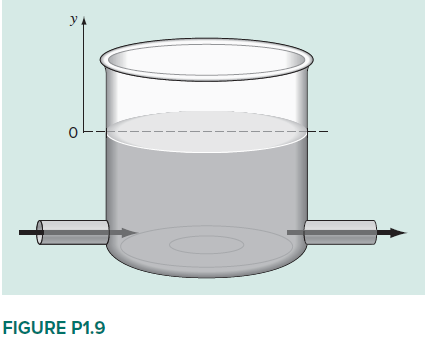
**1.9** A storage tank (Fig. P1.9) contains a liquid at depth *y* where *y* = 0 when the tank is half full. Liquid is withdrawn at a constant flow rate *Q* to meet demands. The contents are resupplied at a sinusoidal rate 3*Q* sin2(*t*). Equation (1.14) can be written for this system as



or, since the surface area *A* is constant



Use Euler’s method to solve for the depth *y* from *t* = 0 to 10 d with a step size of 0.5 d. The parameter values are *A* = 1250 m2 and *Q* = 450 m3/d. Assume that the initial condition is *y* = 0.



==========================================

The first two steps yield





The process can be continued to give the following table and plot:

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| ***t*** | ***y*** | ***dy*/*dt*** | ***t*** | ***y*** | ***dy*/*dt*** |
| 0 | 0.00000 | -0.36000 | 5.5 | 1.10271 | 0.17761 |
| 0.5 | -0.18000 | -0.11176 | 6 | 1.19152 | -0.27568 |
| 1 | -0.23588 | 0.40472 | 6.5 | 1.05368 | -0.31002 |
| 1.5 | -0.03352 | 0.71460 | 7 | 0.89866 | 0.10616 |
| 2 | 0.32378 | 0.53297 | 7.5 | 0.95175 | 0.59023 |
| 2.5 | 0.59026 | 0.02682 | 8 | 1.24686 | 0.69714 |
| 3 | 0.60367 | -0.33849 | 8.5 | 1.59543 | 0.32859 |
| 3.5 | 0.43443 | -0.22711 | 9 | 1.75972 | -0.17657 |
| 4 | 0.32087 | 0.25857 | 9.5 | 1.67144 | -0.35390 |
| 4.5 | 0.45016 | 0.67201 | 10 | 1.49449 | -0.04036 |
| 5 | 0.78616 | 0.63310 |  |  |  |



**1.10** The first two steps yield





The process can be continued to give

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| ***t*** | ***y*** | ***dy*/*dt*** | ***t*** | ***y*** | ***dy*/*dt*** |
| 0 | 0.00000 | -0.12000 | 5.5 | 1.61981 | 0.02876 |
| 0.5 | -0.06000 | 0.13887 | 6 | 1.63419 | -0.42872 |
| 1 | 0.00944 | 0.64302 | 6.5 | 1.41983 | -0.40173 |
| 1.5 | 0.33094 | 0.89034 | 7 | 1.21897 | 0.06951 |
| 2 | 0.77611 | 0.60892 | 7.5 | 1.25372 | 0.54423 |
| 2.5 | 1.08058 | 0.02669 | 8 | 1.52584 | 0.57542 |
| 3 | 1.09392 | -0.34209 | 8.5 | 1.81355 | 0.12227 |
| 3.5 | 0.92288 | -0.18708 | 9 | 1.87468 | -0.40145 |
| 4 | 0.82934 | 0.32166 | 9.5 | 1.67396 | -0.51860 |
| 4.5 | 0.99017 | 0.69510 | 10 | 1.41465 | -0.13062 |
| 5 | 1.33772 | 0.56419 |  |  |  |



**1.11** When the water level is above the outlet pipe, the volume balance can be written as



In order to solve this equation, we must relate the volume to the level. To do this, we recognize that the volume of a cone is given by *V* = π*r*2*y*/3. Defining the side slope as *s* = *y*top/*r*top, the radius can be related to the level (*r* = *y*/*s*) and the volume can be reexpressed as



which can be solved for

 (1)

and substituted into the volume balance

 (2)

For the case where the level is below the outlet pipe, outflow is zero and the volume balance simplifies to

 (3)

These equations can then be used to solve the problem. Using the side slope of *s* = 4/2.5 = 1.6, the initial volume can be computed as



For the first step, *y* < *y*out and Eq. (3) gives



and Euler’s method yields



For the second step, Eq. (3) still holds and





Equation (1) can then be used to compute the new level,



Because this level is now higher than the outlet pipe, Eq. (2) holds for the next step





The remainder of the calculation is summarized in the following table and figure.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| ***t*** | ***Q*in** | ***V*** | ***y*** | ***Q*out** | ***dV*/*dt*** |
| 0 | 0 | 0.20944 | 0.8 | 0 | 0 |
| 0.5 | 0.689547 | 0.20944 | 0.8 | 0 | 0.689547 |
| 1 | 2.12422 | 0.554213 | 1.106529 | 0.104309 | 2.019912 |
| 1.5 | 2.984989 | 1.564169 | 1.563742 | 1.269817 | 1.715171 |
| 2 | 2.480465 | 2.421754 | 1.809036 | 2.183096 | 0.29737 |
| 2.5 | 1.074507 | 2.570439 | 1.845325 | 2.331615 | -1.25711 |
| 3 | 0.059745 | 1.941885 | 1.680654 | 1.684654 | -1.62491 |
| 3.5 | 0.369147 | 1.12943 | 1.40289 | 0.767186 | -0.39804 |
| 4 | 1.71825 | 0.93041 | 1.31511 | 0.530657 | 1.187593 |
| 4.5 | 2.866695 | 1.524207 | 1.55031 | 1.224706 | 1.641989 |
| 5 | 2.758607 | 2.345202 | 1.78977 | 2.105581 | 0.653026 |
| 5.5 | 1.493361 | 2.671715 | 1.869249 | 2.431294 | -0.93793 |
| 6 | 0.234219 | 2.202748 | 1.752772 | 1.95937 | -1.72515 |
| 6.5 | 0.13883 | 1.340173 | 1.48522 | 1.013979 | -0.87515 |
| 7 | 1.294894 | 0.902598 | 1.301873 | 0.497574 | 0.79732 |
| 7.5 | 2.639532 | 1.301258 | 1.470703 | 0.968817 | 1.670715 |
| 8 | 2.936489 | 2.136616 | 1.735052 | 1.890596 | 1.045893 |
| 8.5 | 1.912745 | 2.659563 | 1.866411 | 2.419396 | -0.50665 |
| 9 | 0.509525 | 2.406237 | 1.805164 | 2.167442 | -1.65792 |
| 9.5 | 0.016943 | 1.577279 | 1.568098 | 1.284566 | -1.26762 |
| 10 | 0.887877 | 0.943467 | 1.321233 | 0.5462 | 0.341677 |



**1.12** 



****

Thus, the rise in temperature during the 20 minutes of the class is 14.86571 K.

Therefore, the final temperature is (20 + 273.15) + 14.86571 = 308.01571 K**.**

### 1.13

### 

### 1.14 (a) The force balance can be written as:



Dividing by mass gives



**(b)** Recognizing that *dx*/*dt* = *v*, the chain rule is



Setting drag to zero and substituting this relationship into the force balance gives



**(c)** Using separation of variables



Integrating gives



Applying the initial condition yields



which can be solved for *C* = *v*02/2 – *g*(0)*R*, which can be substituted back into the solution to give



or



Note that the plus sign holds when the object is moving upwards and the minus sign holds when it is falling.

**(d)** Euler’s method can be developed as



The first step can be computed as



The remainder of the calculations can be implemented in a similar fashion as in the following table

|  |  |  |  |
| --- | --- | --- | --- |
| ***x*** | ***v*** | ***dv*/*dx*** | ***v*-analytical** |
| 0 | 1500.000 | -0.00654 | 1500.000 |
| 10000 | 1434.600 | -0.00682 | 1433.216 |
| 20000 | 1366.433 | -0.00713 | 1363.388 |
| 30000 | 1295.089 | -0.00750 | 1290.023 |
| 40000 | 1220.050 | -0.00794 | 1212.476 |
| 50000 | 1140.644 | -0.00847 | 1129.885 |
| 60000 | 1055.974 | -0.00912 | 1041.050 |
| 70000 | 964.800 | -0.00995 | 944.208 |
| 80000 | 865.319 | -0.01106 | 836.581 |
| 90000 | 754.745 | -0.01264 | 713.303 |
| 100000 | 628.364 | -0.01513 | 564.203 |

For the analytical solution, the value at 10,000 m can be computed as



The remainder of the analytical values can be implemented in a similar fashion as in the last column of the above table. The numerical and analytical solutions can be displayed graphically.



**1.15** The volume of the droplet is related to the radius as

 (1)

This equation can be solved for radius as

 (2)

The surface area is

 (3)

Equation (2) can be substituted into Eq. (3) to express area as a function of volume



This result can then be substituted into the original differential equation,

 (4)

The initial volume can be computed with Eq. (1),



Euler’s method can be used to integrate Eq. (4). Here are the beginning and last steps

|  |  |  |
| --- | --- | --- |
| ***t*** | ***V*** | ***dV*/*dt*** |
| 0 | 65.44985 | -6.28319 |
| 0.25 | 63.87905 | -6.18225 |
| 0.5 | 62.33349 | -6.08212 |
| 0.75 | 60.81296 | -5.98281 |
| 1 | 59.31726 | -5.8843 |
| •  •  • |  |  |
| 9 | 23.35079 | -3.16064 |
| 9.25 | 22.56063 | -3.08893 |
| 9.5 | 21.7884 | -3.01804 |
| 9.75 | 21.03389 | -2.94795 |
| 10 | 20.2969 | -2.87868 |

A plot of the results is shown below. We have included the radius on this plot (dashed line and right scale):



Eq. (2) can be used to compute the final radius as



Therefore, the average evaporation rate can be computed as



which is approximately equal to the given evaporation rate of 0.08 mm/min.

**1.16** Continuity at the nodes can be used to determine the flows as follows:













Therefore, the final results are



**1.17** The first two steps can be computed as



The remaining results are displayed below along with a plot of the results.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| ***t*** | ***T*** | ***dT*/*dt*** | ***t*** | ***T*** | ***dT*/*dt*** |
| 0 | 70.00000 | -0.95000 | 12.00000 | 59.62967 | -0.75296 |
| 2 | 68.10000 | -0.91390 | 14.00000 | 58.12374 | -0.72435 |
| 4 | 66.27220 | -0.87917 | 16.00000 | 56.67504 | -0.69683 |
| 6 | 64.51386 | -0.84576 | 18.00000 | 55.28139 | -0.67035 |
| 8 | 62.82233 | -0.81362 | 20.00000 | 53.94069 | -0.64487 |
| 10 | 61.19508 | -0.78271 |  |  |  |



**1.18** **(a)** For the constant temperature case, Newton’s law of cooling is written as



The first two steps of Euler’s methods are



The remaining calculations are summarized in the following table:

|  |  |  |  |
| --- | --- | --- | --- |
| ***t*** | ***Ta*** | ***T*** | ***dT*/*dt*** |
| 0:00 | 10 | 37.0000 | -3.2400 |
| 0:30 | 10 | 35.3800 | -3.0456 |
| 1:00 | 10 | 33.8572 | -2.8629 |
| 1:30 | 10 | 32.4258 | -2.6911 |
| 2:00 | 10 | 31.0802 | -2.5296 |
| 2:30 | 10 | 29.8154 | -2.3778 |
| 3:00 | 10 | 28.6265 | -2.2352 |
| 3:30 | 10 | 27.5089 | -2.1011 |
| 4:00 | 10 | 26.4584 | -1.9750 |
| 4:30 | 10 | 25.4709 | -1.8565 |
| 5:00 | 10 | 24.5426 | -1.7451 |

**(b)** For this case, the room temperature can be represented as



where *t* = time (hrs). Newton’s law of cooling is written as



The first two steps of Euler’s methods are



The remaining calculations are summarized in the following table:

|  |  |  |  |
| --- | --- | --- | --- |
| ***t*** | ***Ta*** | ***T*** | ***dT*/*dt*** |
| 0:00 | 20 | 37.0000 | -2.0400 |
| 0:30 | 19 | 35.9800 | -2.0376 |
| 1:00 | 18 | 34.9612 | -2.0353 |
| 1:30 | 17 | 33.9435 | -2.0332 |
| 2:00 | 16 | 32.9269 | -2.0312 |
| 2:30 | 15 | 31.9113 | -2.0294 |
| 3:00 | 14 | 30.8966 | -2.0276 |
| 3:30 | 13 | 29.8828 | -2.0259 |
| 4:00 | 12 | 28.8699 | -2.0244 |
| 4:30 | 11 | 27.8577 | -2.0229 |
| 5:00 | 10 | 26.8462 | -2.0215 |

Comparison with (a) indicates that the effect of the room air temperature has a significant effect on the expected temperature at the end of the 5-hr period (difference = 26.8462 – 24.5426 = 2.3036oC).

**(c)** The solutions for (a) Constant *Ta*, and (b) Cooling *Ta* are plotted below:



**1.19** The two equations to be solved are



Euler’s method can be applied for the first step as



For the second step:



The remaining steps can be computed in a similar fashion as tabulated and plotted below:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| ***t*** | ***x*** | ***v*** | ***dx*/*dt*** | ***dv*/*dt*** |
| 0 | 0.0000 | 0.0000 | 0.0000 | 9.8100 |
| 2 | 0.0000 | 19.6200 | 19.6200 | 8.3968 |
| 4 | 39.2400 | 36.4137 | 36.4137 | 4.9423 |
| 6 | 112.0674 | 46.2983 | 46.2983 | 1.9409 |
| 8 | 204.6640 | 50.1802 | 50.1802 | 0.5661 |
| 10 | 305.0244 | 51.3123 | 51.3123 | 0.1442 |



**1.20 (a)** The force balance with buoyancy can be written as



Divide both sides by mass,



**(b)** For a sphere, the mass is related to the volume as in *m* = *ρsV* where *ρs* = the sphere’s density (kg/m3). Substituting this relationship gives



The formulas for the volume and projected area can be substituted to give



(c) At steady state (*dv*/*dt* = 0),



which can be solved for the terminal velocity



Substituting the values, the terminal velocity is found to be,



(d) Before implementing Euler’s method, the parameters can be substituted into the differential equation to give



The first two steps for Euler’s method are



The remaining steps can be computed in a similar fashion as tabulated and plotted below:

|  |  |  |
| --- | --- | --- |
| ***t*** | ***v*** | ***dv*/*dt*** |
| 0 | 0.000000 | 6.176667 |
| 0.03125 | 0.193021 | 5.690255 |
| 0.0625 | 0.370841 | 4.381224 |
| 0.09375 | 0.507755 | 2.810753 |
| 0.125 | 0.595591 | 1.545494 |
| 0.15625 | 0.643887 | 0.763953 |
| 0.1875 | 0.667761 | 0.355136 |
| 0.21875 | 0.678859 | 0.160023 |
| 0.25 | 0.683860 | 0.071055 |



**1.21** (a) The force balance can be written as



Dividing by mass gives

 (1)

The mass of the sphere is *ρsV* where *V* = volume (m3). The projected area and volume of a sphere are π*d*2/4 and π*d*3/6, respectively. Substituting these relationships gives



(b) The first step for Euler’s method is





The remaining steps are shown in the following table:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| ***t*** | ***x*** | ***v*** | ***dx*/*dt*** | ***dv*/*dt*** |
| 0 | 100.0000 | -40.0000 | -40.0000 | 10.0363 |
| 2 | 20.0000 | -19.9274 | -19.9274 | 9.8662 |
| 4 | -19.8548 | -0.1951 | -0.1951 | 9.8100 |
| 6 | -20.2450 | 19.4249 | 19.4249 | 9.7566 |
| 8 | 18.6049 | 38.9382 | 38.9382 | 9.5956 |
| 10 | 96.4813 | 58.1293 | 58.1293 | 9.3321 |
| 12 | 212.7399 | 76.7935 | 76.7935 | 8.9759 |
| 14 | 366.3269 | 94.7453 | 94.7453 | 8.5404 |

(c) The results can be graphed as (notice that we have reversed the axis for the distance, *x*, so that the negative elevations are upwards.





(d) Inspecting the differential equation for velocity (Eq. 1) indicates that the bulk drag coefficient is



Therefore, for this case, because *A* = π(1.2)2/4 = 1.131 m2, the bulk drag coefficient is



**1.22** (a) A force balance on a sphere can be written as:



where

Substituting the individual terms into the force balance yields



Divide by *m*



Note that *m* = *ρsV*, so



The volume can be represented in terms of more fundamental quantities as *V* = π*d*3/6. Substituting this relationship into the differential equation gives the final differential equation



**(b)** At steady-state, the equation is



which can be solved for the terminal velocity



This equation is sometimes called *Stokes Settling Law*.

(c) Before computing the result, it is important to convert all the parameters into consistent units. For the present problem, the necessary conversions are

The terminal velocity can then computed as



(d) The Reynolds number can be computed as



This is far below 1, so the flow is very laminar.

(e) Before implementing Euler’s method, the parameters can be substituted into the differential equation to give



The first two steps for Euler’s method are





The remaining steps can be computed in a similar fashion as tabulated and plotted below:

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| ***t*** | ***v*** | ***dv*/*dt*** | ***t*** | ***v*** | ***dv*/*dt*** |
| 0 | 0 | 6.108113 | 2.29×10–5 | 5.99E-05 | 0.409017 |
| 3.81×10–6 | 2.33E-05 | 3.892358 | 2.67×10–5 | 6.15E-05 | 0.260643 |
| 7.63×10–6 | 3.81E-05 | 2.480381 | 3.05×10–5 | 6.25E-05 | 0.166093 |
| 1.14×10–5 | 4.76E-05 | 1.580608 | 3.43×10–5 | 6.31E-05 | 0.105842 |
| 1.53×10–5 | 5.36E-05 | 1.007233 | 3.81×10–5 | 6.35E-05 | 0.067447 |
| 1.91×10–5 | 5.75E-05 | 0.641853 |  |  |  |



**1.23** Substituting the parameters into the differential equation gives



The first step of Euler’s method is



The second step is



The remainder of the calculations along with the analytical solution are summarized in the following table and plot. Note that the results of the numerical and analytical solutions are close.

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| ***x*** | ***y-*Euler** | ***dy*/*dx*** | ***y-*analytical** | ***x*** | ***y-*Euler** | ***dy/dx*** | ***y-*analytical** |
| 0 | 0 | 0 | 0 | 2.125 | 0.001832 | 0.001472 | 0.001925 |
| 0.125 | 0 | 0.000149 | 9.42E-06 | 2.25 | 0.002016 | 0.001504 | 0.002111 |
| 0.25 | 1.86E-05 | 0.000289 | 3.69E-05 | 2.375 | 0.002204 | 0.001531 | 0.002301 |
| 0.375 | 5.47E-05 | 0.00042 | 8.13E-05 | 2.5 | 0.002395 | 0.001554 | 0.002494 |
| 0.5 | 0.000107 | 0.000542 | 0.000141 | 2.625 | 0.00259 | 0.001574 | 0.00269 |
| 0.625 | 0.000175 | 0.000655 | 0.000216 | 2.75 | 0.002787 | 0.001591 | 0.002887 |
| 0.75 | 0.000257 | 0.000761 | 0.000305 | 2.875 | 0.002985 | 0.001605 | 0.003087 |
| 0.875 | 0.000352 | 0.000859 | 0.000406 | 3 | 0.003186 | 0.001615 | 0.003288 |
| 1 | 0.000459 | 0.000949 | 0.000519 | 3.125 | 0.003388 | 0.001624 | 0.003491 |
| 1.125 | 0.000578 | 0.001032 | 0.000643 | 3.25 | 0.003591 | 0.00163 | 0.003694 |
| 1.25 | 0.000707 | 0.001108 | 0.000777 | 3.375 | 0.003795 | 0.001635 | 0.003898 |
| 1.375 | 0.000845 | 0.001177 | 0.00092 | 3.5 | 0.003999 | 0.001638 | 0.004103 |
| 1.5 | 0.000992 | 0.00124 | 0.001071 | 3.625 | 0.004204 | 0.00164 | 0.004308 |
| 1.625 | 0.001147 | 0.001298 | 0.00123 | 3.75 | 0.004409 | 0.001641 | 0.004513 |
| 1.75 | 0.00131 | 0.001349 | 0.001395 | 3.875 | 0.004614 | 0.001641 | 0.004718 |
| 1.875 | 0.001478 | 0.001395 | 0.001567 | 4 | 0.004819 | 0.001641 | 0.004923 |
| 2 | 0.001653 | 0.001436 | 0.001744 |  |  |  |  |



**1.24** [Note that students can easily get the underlying equations for this problem off the web]. The volume of a sphere can be calculated as



The portion of the sphere above water (the “cap”) can be computed as



Therefore, the volume below water is



Thus, the steady-state force balance can be written as



Cancelling common terms gives



Collecting terms yields



**1.25** [Note that students can easily get the underlying equations for this problem off the web]. The total volume of a right circular cone can be calculated as



The volume of the frustum below the earth’s surface can be computed as



Archimedes’ principle says that, at steady state, the downward force of the whole cone must be balanced by the upward buoyancy force of the below ground frustum,

 (1)

Before proceeding we have too many unknowns: *r*1 and *h*1. So before solving, we must eliminate *r*1 by recognizing that using similar triangles (*r*1/*h*1 = *r*2/*H*)



which can be substituted into Eq. (1) (and cancelling the *g*’s)



Therefore, the equation now has only 1 unknown: *h*1, and the steady-state force balance can be written as



Cancelling common terms gives



and collecting terms yields



**1.26** (a) The pair of differential equations to be solved are





At *t* = 0,







At *t* = 0.01



The calculation can be continued to give

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| ***t*** | ***i*** | ***q*** | ***di*/*dt*** | ***dq*/*dt*** |
| 0 | 0 | 1 | -2000 | 0 |
| 0.01 | -20 | 1 | -1200 | -20 |
| 0.02 | -32 | 0.8 | -320 | -32 |
| 0.03 | -35.2 | 0.48 | 448 | -35.2 |
| 0.04 | -30.72 | 0.128 | 972.8 | -30.72 |
| 0.05 | -20.992 | -0.1792 | 1198.08 | -20.992 |
| 0.06 | -9.0112 | -0.38912 | 1138.688 | -9.0112 |
| 0.07 | 2.37568 | -0.47923 | 863.4328 | 2.37568 |
| 0.08 | 11.01001 | -0.45547 | 470.5396 | 11.01005 |
| 0.09 | 15.71553 | -0.34537 | 62.1236 | 15.71541 |
| 0.1 | 16.33665 | -0.18822 | -277.026 | 16.33665 |

(b)



**1.27 (a)**

**(b)** Substituting the parameters

First step:

*vx* = 180 – 32.1429(1) =147.8571

*vy* = 0 + 9.81(1) = 9.81





The calculation can be continued to give

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| ***t*** | ***vx*** | ***vy*** | ***x*** | ***y*** | ***dvx*/*dt*** | ***dvy*/*dt*** | ***dx*/*dt*** | ***dy*/*dt*** |
| 0 | 180 | 0 | 0 | 0 | -32.1429 | 9.81 | 180 | 0 |
| 1 | 147.8571 | 9.81 | 180 | 0 | -26.4031 | 8.058214 | 147.8571 | 9.81 |
| 2 | 121.4541 | 17.86821 | 327.8571 | 9.81 | -21.6882 | 6.619247 | 121.4541 | 17.86821 |
| 3 | 99.76585 | 24.48746 | 449.3112 | 27.67821 | -17.8153 | 5.437239 | 99.76585 | 24.48746 |
| 4 | 81.95052 | 29.9247 | 549.0771 | 52.16568 | -14.634 | 4.466303 | 81.95052 | 29.9247 |
| 5 | 67.3165 | 34.391 | 631.0276 | 82.09038 | -12.0208 | 3.668749 | 67.3165 | 34.391 |
| 6 | 55.2957 | 38.05975 | 698.3441 | 116.4814 | -9.87423 | 3.013615 | 55.2957 | 38.05975 |
| 7 | 45.42147 | 41.07337 | 753.6398 | 154.5411 | -8.11098 | 2.47547 | 45.42147 | 41.07337 |
| 8 | 37.31049 | 43.54884 | 799.0613 | 195.6145 | -6.66259 | 2.033422 | 37.31049 | 43.54884 |
| 9 | 30.6479 | 45.58226 | 836.3718 | 239.1633 | -5.47284 | 1.670311 | 30.6479 | 45.58226 |
| 10 | 25.17506 | 47.25257 | 867.0197 | 284.7456 | -4.49555 | 1.372041 | 25.17506 | 47.25257 |
| 11 | 20.67952 | 48.62461 | 892.1947 | 331.9982 | -3.69277 | 1.127034 | 20.67952 | 48.62461 |
| 12 | 16.98674 | 49.75165 | 912.8742 | 380.6228 | -3.03335 | 0.925778 | 16.98674 | 49.75165 |
| 13 | 13.9534 | 50.67742 | 929.861 | 430.3744 | -2.49168 | 0.76046 | 13.9534 | 50.67742 |
| 14 | 11.46172 | 51.43788 | 943.8144 | 481.0519 | -2.04674 | 0.624664 | 11.46172 | 51.43788 |
| 15 | 9.414984 | 52.06255 | 955.2761 | 532.4897 | -1.68125 | 0.513117 | 9.414984 | 52.06255 |
| 16 | 7.733737 | 52.57566 | 964.6911 | 584.5523 | -1.38102 | 0.421489 | 7.733737 | 52.57566 |
| 17 | 6.352712 | 52.99715 | 972.4248 | 637.1279 | -1.13441 | 0.346223 | 6.352712 | 52.99715 |
| 18 | 5.218299 | 53.34338 | 978.7775 | 690.1251 | -0.93184 | 0.284397 | 5.218299 | 53.34338 |
| 19 | 4.28646 | 53.62777 | 983.9958 | 743.4685 | -0.76544 | 0.233612 | 4.28646 | 53.62777 |
| 20 | 3.521021 | 53.86138 | 988.2823 | 797.0962 | -0.62875 | 0.191896 | 3.521021 | 53.86138 |
| 21 | 2.892267 | 54.05328 | 991.8033 | 850.9576 | -0.51648 | 0.157629 | 2.892267 | 54.05328 |
| 22 | 2.375791 | 54.21091 | 994.6956 | 905.0109 | -0.42425 | 0.129481 | 2.375791 | 54.21091 |
| 23 | 1.951542 | 54.34039 | 997.0714 | 959.2218 | -0.34849 | 0.106359 | 1.951542 | 54.34039 |
| 24 | 1.603053 | 54.44675 | 999.0229 | 1013.562 | -0.28626 | 0.087366 | 1.603053 | 54.44675 |
| 25 | 1.316793 | 54.53411 | 1000.626 | 1068.009 | -0.23514 | 0.071765 | 1.316793 | 54.53411 |

**(c)**

Plot of the four variables versus time



Plot of *y* versus *x*



Inspecting these figures and the numerical results indicates that the individual would hit the ground at a little over 24 seconds if the chute did not open.

**1.28** (a)



 (1)



(Note: Only the balloon’s volume is used to calculate the buoyant force since it is much larger than the payload volume.)









**(b)** Using Eq. (1) at steady state





**(c)**

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Cd | 0.47 |  |  | P | 101300 | Pa |  | Volume | 2711 |
| Mp | 265 | kg |  | R | 287 | J/kg.K |  | Area | 235 |
| d | 17.3 | m |  | T | 373 | K |  |  |  |
| g | 9.81 |  |  | row\_a | 1.2 | kg/m3 |  |  |  |
| mg | 2564.3349 | N |  | row\_g | 0.9459 | kg/m3 |  |  |  |
| **vterm** | **7.92118** | m/s |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |
|  |  |  | Total mass | 2829.3349 |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |
| FB | 31914.44 |  | FG | 25166.61 |  |  |  |  |  |
| Fp | 2599.65 |  |  |  |  |  |  |  |  |

|  |  |  |
| --- | --- | --- |
| ***t*** | ***v*** | ***dv/dt*** |
| 0 | 0 | 1.466133 |
| 2 | 2.932265 | 1.264689 |
| 4 | 5.461643 | 0.767267 |
| 6 | 6.996178 | 0.319383 |
| 8 | 7.634944 | 0.100422 |
| 10 | 7.835789 | 0.027624 |
| 12 | 7.891038 | 0.007268 |
| 14 | 7.905573 | 0.001888 |
| 16 | 7.909349 | 0.000489 |
| 18 | 7.910327 | 0.000127 |
| 20 | 7.91058 | 0.000033 |
| 22 | 7.910646 | 0.000008 |
| 24 | 7.910663 | 0.000002 |
| 26 | 7.910667 | 0.000001 |
| 28 | 7.910668 | 0 |
| 30 | 7.910669 | 0 |
| . | . | . |
| . | . | . |
| . | . | . |
| 56 | 7.910669 | 0 |
| 58 | 7.910669 | 0 |
| 60 | 7.910669 | 0 |

The values can be plotted as

