# Chapter 5: Network Design in the Supply Chain

Exercise Solutions

## 1.

### (a)

The objective of this model is to decide optimal locations of home offices, and number of trips from each home office, so as to minimize the overall network cost. The overall network cost is a combination of fixed costs of setting up home offices and the total trip costs.

There are two constraint sets in the model. The first constraint set requires that a specified number of trips be completed to each state *j* and the second constraint set prevents trips from a home office *i* unless it is open. Also, note that there is no capacity restriction at each of the home offices. While a feasible solution can be achieved by locating a single home office for all trips to all states, it is easy to see that this might not save on trip costs, since trip rates vary between home offices and states. We need to identify better ways to plan trips from different home offices to different states so that the trip costs are at a minimum. Thus, we need an optimization model to handle this.

*Optimization model:*

|  |
| --- |
| *n = 4: possible home office locations.*  *m = 16: number of states.*  *Dj = Annual trips needed to state j*  *Ki = number of trips that can be handled from a home office*  *As explained, in this model there is no restriction*  *fi = Annualized fixed cost of setting up a home office*  *cij = Cost of a trip from home office i to state j*  *yi = 1 if home office i is open, 0 otherwise*  *xij = Number of trips from home office i to state j.*  *It should be integral and non-negative* |

*Please note that (5.2) is not active in this model since K is as large as needed. However, it will be used in answering (b).*

|  |  |  |
| --- | --- | --- |
| SYMBOL | INPUT | CELL |
| *Dj* | *Annual trips needed to state j* | E7:E22 |
| *cij* | *Transportation cost from office i to state j* | G7:G22,I7:I22,  K7:K22,M7:M22 |
| *fi* | *fixed cost of setting up office i* | G26,I26,K26,M26 |
| *xij* | *number of consultants from office i to state j.* | F7:F22,H7:H22,  J7:J22,L7:L22 |
| obj. | objective function | M31 |
| 5.1 | demand constraints | N7:N22 |

(Sheet SC consulting in workbook exercise5.1.xls)

With this we solve the model to obtain the following results:

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| State | **Total # of trips** | **Trips from LA** | **Cost from LA** | **Trips from Tulsa** | **Cost from Tulsa** | **Trips from Denver** | **Cost From Denver** | **Trips from Seattle** | **Cost from Seattle** |
| *Washington* | 40 | - | 150 | - | 250 | - | 200 | 40 | 25 |
| *Oregon* | 35 | - | 150 | - | 250 | - | 200 | 35 | 75 |
| *California* | 100 | 100 | 75 | - | 200 | - | 150 | - | 125 |
| *Idaho* | 25 | - | 150 | - | 200 | - | 125 | 25 | 125 |
| *Nevada* | 40 | 40 | 100 | - | 200 | - | 125 | - | 150 |
| *Montana* | 25 | - | 175 | - | 175 | - | 125 | 25 | 125 |
| *Wyoming* | 50 | - | 150 | - | 175 | 50 | 100 | - | 150 |
| *Utah* | 30 | - | 150 | - | 150 | 30 | 100 | - | 200 |
| *Arizona* | 50 | 50 | 75 | - | 200 | - | 100 | - | 250 |
| *Colorado* | 65 | - | 150 | - | 125 | 65 | 25 | - | 250 |
| *New Mexico* | 40 | - | 125 | - | 125 | 40 | 75 | - | 300 |
| *North Dakota* | 30 | - | 300 | - | 200 | 30 | 150 | - | 200 |
| *South Dakota* | 20 | 0 | 300 | - | 175 | 20 | 125 | - | 200 |
| *Nebraska* | 30 | - | 250 | 30 | 100 | - | 125 | - | 250 |
| *Kansas* | 40 | - | 250 | 25 | 75 | 15 | 75 | - | 300 |
| *Oklahoma* | 55 | - | 250 | 55 | 25 | - | 125 | - | 300 |
| # of trips | 675 |  | 190 | - | 110 | - | 250 | - | 125 |
| # of Consultants |  |  | 8 |  | 5 |  | 10 |  | 5 |
| Fixed Cost of office |  |  | 165,428 |  | 131,230 |  | 140,000 |  | 145,000 |
| Cost of Trips |  |  | 15,250 |  | 6,250 |  | 20,750 |  | 9,875 |
| Total Office Cost |  |  | 180,678 |  | 137,480 |  | 160,750 |  | 154,875 |

The number of consultants is calculated based on the constraint of 25 trips per consultant. As trips to Kansas cost the same from Tulsa or Denver there are many other solutions possible by distributing the trips to Kansas between these two offices.

### (b)

If at most 10 consultants are allowed at each home office, then we need to add one more constraint i.e. the total number of trips from an office may not exceed 250. Or in terms of the optimization model, Ki, for all *i*, should have a value of 250. We can revise constraint (5.2) with this Ki value and resolve the model. The new model will answer (b).

However in this specific case, it is clear that only the Denver office violates this new condition. As trips to Kansas can be offloaded from Denver to Tulsa without any incremental cost, that is a good solution and still optimal.

Hence we just allocate 5 of the Denver-Kansas trips to Tulsa. This reduces the number of consultants at Denver to 10 while maintaining 5 consultants at Tulsa.

### (c)

Just like the situation in (b), though in general we need a new constraint to model the new requirement, it is not necessary in this specific case. We note that in the optimal solution of (b), each state is uniquely served by an office except for Kansas where the load is divided between Denver and Tulsa. The cost to serve Kansas is the same from either office. Hence we can meet the new constraint by making Tulsa fully responsible for Kansas. This brings the trips out of Tulsa to 125 and those out of Denver to 235. Again the number of consultants remains at 5 and 10 in Tulsa and Denver, respectively.

## 2.

DryIce Inc. faces the tradeoff between fixed cost (that is lower per item in a larger plant) versus the cost of shipping and manufacturing. The typical scenarios that need to be considered are either having regional manufacturing if the shipping costs are significant or have a centralized facility if the fixed costs show significant economies to scale.

We keep the units shipped from each plant to every region as variable and choose the fixed cost based on the emerging production quantities in each plant location. The total system cost is then minimized with the following constraints:

1. All shipment numbers need to be positive integers.
2. The maximum production capacity is 400,000
3. All shipments to a region should add up to the requirement for 2006 .

*Optimization model:*

|  |
| --- |
| *n = 4: potential sites.*  *m = 4: number of regional markets.*  *Dj = Annual units needed of regional market j*  *Ki = maximum possible capacity of potential sites.*  *Each Ki is assigned value 400000. If actually needed*  *capacity is less than or equal to 200000, we choose fixed cost accordingly.*  *fi = Annualized fixed cost of setting up a potential site.*  *cij = Cost of producing and shipping an air conditioner from site i to regional market j*  *yi = 1 if site i is open, 0 otherwise*  *xij = Number of air conditioners from site i to regional market j.*  *It should be integral and non-negative* |

|  |  |  |
| --- | --- | --- |
| SYMBOL | INPUT | CELL |
| *Dj* | *requirement at market j* | K10:K13 |
| *cij* | *Variable cost from plant i to market j* | C10:C13,E10:E13  G10:G13,I10:I13 |
| *fi* | *fixed cost of setting up plant i* | C7:C8,E7:E8  G7:G8,I7:I8 |
| *xij* | *number of consultants from office i to state j.* | D10:D13,F10:F13  H10:H13,J10:J13 |
| *obj.* | *objective function* | K21 |
| *5.1* | *demand constraints* | L10:L13 |

(Sheet DryIce in workbook exercise5.2.xls)

We get the following results:

The optimal solution suggests setting up 4 regional plants with each serving the needs of its own region. New York, Atlanta, Chicago and San Diego should each have a 200,000 capacity plant with production levels of 110000, 180000, 120000, 100000, respectively.

## 3

### (a)

Sunchem can use the projections to build an optimization model as shown below. In this case, the shipments from each plant to every market are assumed to be variable and solved to find the minimum total cost. This is done by utilizing the following constraints:

* Each plant runs at least at half capacity.
* Sum of all shipments from the plant needs to be less than or equal to the capacity in that plant.
* All production volumes are non-negative.
* All calculations are performed at the exchange rates provided.

*Optimization model:*

|  |
| --- |
| *n = 5: five manufacturing plants*  *m = 5: number of regional markets.*  *Dj = Annual tons of ink needed for regional market j*  *Ki = Maximum possible capacity of manufacturing plants.*  *Especially for (a) lower limit for capacity is 50%\*Ki .*  *cij = Cost of shipping one ton of printing ink from plant i to regional market j*  *pi = Cost of producing one ton of printing ink at plant i*  *xij = Tons of printing ink shipped from site i to regional market j.*  *It should be integral and non-negative* |

|  |  |  |
| --- | --- | --- |
| SYMBOL | INPUT | CELL |
| *Dj* | *Annual demand at market j* | N4:N8 |
| *cij* | *shipping cost from plant i to regional market j* | D4:D8,F4:F8,H4:H8,  J4:J8, L4:L8 |
| *pi* | *production cost of at plant i* | D12,F12,H12,J12,L12 |
| *xij* | *printing ink shipped from site i to regional market j* | E4:E8,G4:G8,I4:I8,  K4:K8, M4:M8 |
| obj. | *objective function* | N18 |
| *5.1* | *demand constraints* | O4:O8 |
| *5.2* | *capacity constraints* | E10,G10,I10,K10,M10 |
| *5.3* | *50% capacity constraints* | E10,G10,I10,K10,M10 |

(Sheet capacity\_constraints in workbook exercise5.3.xls)

The optimal result is summarized in the following table:



This is clearly influenced by the production cost per ton and the local market demand. Low cost structure plants need to operate at capacity.

### (b)

If there are no limits on production we can perform the same exercise as in (a) but without the capacity constraints (5.2) and (5.3). This gives us the following results:



Clearly by having no restrictions on capacity SunChem can reduce costs by $557,590. The analysis shows that there are gains from shifting a significant portion of production to Brazil and having no production in Japan, US and India.

### (c)

From the scenario in (a) we see that two of the plants are producing at full capacity. And in (b), we see that it is more economical to produce higher volumes in Brazil. Once we add 10 tons/year to Brazil, the cost reduces to $7,795,510.



### (d)

It is clear that fluctuations in exchange rates will change the cost structure of each plant. If the cost at a plant becomes too high, there is merit in shifting some of the production to another plant. Similarly if a plant’s cost structure becomes more favorable, there is merit in shifting some of the production from other plants to this plant. Either of these scenarios requires that the plants have built in excess capacity. Sunchem should plan on making excess capacity available at its plants.

## 4

### (a)

Starting from the basic models in (a), we will build more advanced models in the subsequent parts of this question. Prior to merger, Sleekfon and Sturdyfon operate independently, and so we need to build separate models for each of them.

*Optimization model for Sleekfon:*

|  |
| --- |
| *n = 3: Sleekfon production facilities.*  *m = 7: number of regional markets.*  *Dj = Annual market size of regional market j*  *Ki = maximum possible capacity of production facility i*  *cij = Variable cost of producing, transporting and duty from facility i to market j*  *fi = Annual fixed cost of facility i*  *xij = Number of units from facility i to regional market j.*  *It should be integral and non-negative.* |

Please note that we need to calculate the variable cost *cij* before we plug it into the optimization model. Variable cost *cij* is calculated as follows:

*cij = production cost per unit at facility i + transportation cost per unit from facility i to market j + duty\*(* *production cost per unit at facility i + transportation cost per unit from facility i to market j + fixed cost per unit of capacity)*

|  |  |  |
| --- | --- | --- |
| SYMBOL | INPUT | CELL |
| *Dj* | *Annual market size of regional market j* | B4:H4 |
| *Ki* | *maximum possible capacity of production facility i* | C12:C14 |
| *cij* | *Variable cost of producing, transporting and duty from facility i to market j* | B22:H28 |
| *fi* | *Annual fixed cost of facility i* | D12:D17 |
| *xij* | *Number of units from facility i to regional market j.* | C43:I45 |
| obj. | objective function | D48 |
| 5.1 | demand constraints | J43:J45 |
| 5.2 | capacity constraints | C46:I46 |

(Sheet sleekfon in workbook problem5.4)

The above model gives optimal result as in following table:



And we use the same model but with data from Sturdyfon to get following optimal production and distribution plan for Sturdyfon:



### (b)

Under conditions of no plant shutdowns, the previous model is still applicable. However, we need to increase the number of facilities to 6, i.e., 3 from Sleekfon and 3 from Sturdyfon. And the market demand at a region needs revised by combining the demands from the two companies. Decision maker has more facilities and greater market share in each region, and hence has more choices for production and distribution plans. The optimal result is summarized in the following table.



### (c)

This model is more advanced since it allows facilities to be scaled down or shutdown. Accordingly we need more variables to reflect this new complexity.

*Optimization model for Sleekfon:*

|  |
| --- |
| *n = 6: Sleekfon and Sturdyfon production facilities.*  *m = 7: number of regional markets.*  *Dj = Annual market size of regional market j, sum of the Sleekfon and Sturdyfon market share.*  *Ki =capacity of production facility i*  *Li =capacity of production facility if it is scaled back*  *cij = Variable cost of producing, transporting and duty from facility i to market j*  *fi = Annual fixed cost of facility i*  *gi = Annual fixed cost of facility i if it is scaled back*  *hi = Shutdown cost of facility i*  *xij = Number of units from facility i to regional market j.*  *It should be integral and non-negative.*  *yi  = Binary variable indicating whether to scale back facility i. yi = 1 means to scale it back, 0 otherwise.*  *Since two facilities, Sleekfon S America and Sturdyfon Rest of Asia, can not be scaled back, the index i*  *doesn’t include these two facilities.*  *zi = Binary variable indicating whether to shutdown facility i. zi =1 means to shutdown it, 0 otherwise.*  *(1-yi –zi) would be the binary variable indicating whether the facility is unaffected.* |

Please note that we need to calculate the variable cost *cij* before we plug it into the optimization model. Variable cost *cij* is calculated as following:

*cij = production cost per unit at facility i + transportation cost per unit from facility i to market j + duty\*(* *production cost per unit at facility i + transportation cost per unit from facility i to market j + fixed cost per unit of capacity)*

And we also need to prepare fixed cost data for the two new scenarios: shutdown and scale back. As explained in the problem description, fixed cost for a scaled back facility is 70% of the original one; and it costs 20% of the original annual fixed cost to shutdown it.

Above model gives optimal solution as summarized in the following table. The lowest cost possible in this model is $988.93, much lower than the result we got in (b) $1066.82. As shown in the result, the Sleekfon N.America facility is shutdown, and the market is mainly served by Sturdyfon N.America facility. The N.America market share is 22, and there are 40 in terms of production capacity, hence it is wise to shutdown one facility whichever is more expensive.



For questions (d) and (e), we need to change the duty to zero and run the optimization model again to get the result. We can achieve this by resetting B7:H7 to zeros in sheet merger (shutdown) in workbook problem5.4.xls.

## 5

### (a)

The model we developed in 4.d is applicable to this question. We only need to update the demand data accordingly. And the new demand structure yields a quite different optimal configuration of the network.



As shown in the table, Sturdyfon N.America is not shutdown in this optimal result. Instead, Sturdyfon EU facility is shutdown.

For questions (b), (c) and (d), we need to update Excel sheet data accordingly and rerun the optimization model.

## 6

### (a)

StayFresh faces a multi-period decision problem. If we treated each period separately, only two constraints are relevant, i.e., the demand and capacity constraints. Considering the multi-period nature of this problem, it must be noted that as the demand increases steadily, we need to add capacities eventually. However due to the discount factor, we want to increase capacities as late as possible. On the other hand, even when the total capacity at a certain period is greater than or equal to the total demand, we might want to increase capacity anyway. This is because a regional market might run short while the total supply is surplus, and it may be more expensive to ship from other regions than to increase local capacity. This complexity calls for an optimization model to find an optimal solution which can serve all demands, satisfy capacity constraints, adjust the regional imbalance, and take benefit of discount effect over periods.

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| --- |
|  |

|  |  |  |
| --- | --- | --- |
| SYMBOL | INPUT | CELL |
|  |  | B9:H9 |
|  |  | C12:C14 |
|  | Production and transportation cost from plant i to regional market j | B5:F8 |
|  |  | D12:D17 |
|  |  | C43:I45 |
|  |  | D48 |
|  |  | I5:Q8 |
|  |  | B22:E25  H22:K25  N22:Q25  T22:W25  Z22:AC25 |
| obj | objective function | C31 |
| 5.1 | capacity constraint | G22:G25  M22:M25  S22:S25  Y22:Y25  Ae22:Ae25 |
| 5.2 | demand constraint | B26:E26  H26:K26  N26:Q26  T26:W26  Z26:AC26 |

(Sheet StayFresh in workbook problem5.6.xls)

In the first year, original total capacity was 600,000 units, which was 60,000 units more than the total demand. However, a new plant in Kolkata is built in the optimal solution anyway, since it is cheaper to server the local market from Kolkata than to ship from other regions.

In the second year, no new capacity is added, since the plant location is reasonable and the total capacity still exceeds the demand.

In the third and fourth years, new capacity is added consecutively, which has lead to high surplus capacity. Note that this additional capacity is needed for the fifth year. While there is no reason to add capacity earlier than necessary, especially under the consideration of the discount factor, the solution is optimal in this particular model. Since the cost of fifth year will be added into the total cost six times, it is strategically correct to spend as little as possible in the fifth year. This explains why extra capacity is built into the network earlier than necessary.

For questions (b) and (c), we need to change data in the Excel sheet accordingly.

## 7

### (a)

Blue Computers has two plants in Kentucky and Pennsylvania, however both have high variable costs to serve the West regional market. On the other hand, West regional market has 2nd highest demand. Hence it is not hard to see that Blue Computers needs a new plant, which can serve the West regional market at a lower cost. From this point of view, California is a better choice than N.Carolina since California has a lower variable cost serving West regional market. However, N.Carolina has extra tax benefit. Even if a network of Kentucky, Pennsylvania, and California might yield higher before-tax profit than a network of Kentucky, Pennsylvania, and N.Carolina, the after-tax profit might be worse.

|  |
| --- |
| *n = 2 potential sites.*  *m = 4: number of regional markets.*  *Dj = Annual units needed of regional market j*  *Ki = maximum possible capacity of potential sites.*  *fi = Annualized fixed cost of setting up a potential site.*  *cij = Cost of producing and shipping a computer r from site i to regional market j*  *yi = 1 if site i is open, 0 otherwise*  *xij = Number of products from site i to regional market j.*  *It should be integral and non-negative* |

|  |  |  |
| --- | --- | --- |
| SYMBOL | INPUT | CELL |
| *Dj* | *Annual market size of regional market j* | B9:F9 |
| *Ki* | *maximum possible capacity of production facility i* | H5:H8 |
| *cij* | *Variable cost of producing, transporting and duty from facility i to market j* | B5:F8 |
| *fi* | *Annual fixed cost of facility i* | G5:G8 |
| *xij* | *Number of units from facility i to regional market j.* | B17:F20 |
| obj. | objective function | I21 |
| 5.1 | demand constraints | B21:F21 |
| 5.2 | capacity constraints | H17:H20 |
| 5.4 | see explanation in next paragraph |  |

(Sheet Blue in workbook problem5.7.xls)

Even though constraint (5.4) is simple in its mathematical notation, we can do better in practice. Since at most one site can be open, we can run the optimization three times for three scenarios respectively: none open, only California, or only N.Carolina. And we compare the three results and choose the best one. It is much faster to solve these three scenarios separately given that EXCEL solver cannot achieve a converging result with constraint (5.4). The result below shows the optimal solution when California is picked up.



### (b)

We only need to change the objective function from minimize cost to maximize profit. On the Excel sheet, all we need to do is to set the target cell from I21 to L21, and change the direction of optimization from minimizing to maximizing. The following table shows the result. It is easy to see that lowest cost doesn’t mean maximum after tax profit.



## 8

### (a)

Starting from the basic models in (a), we will build more advanced models in the subsequent parts of this question. Prior to merger, Hot&Cold and CaldoFreddo operate independently, and we need to build separate models for each of them.

*Optimization model for Hot&Cold:*

|  |
| --- |
| *n = 3: Hot&Cold production facilities.*  *m = 4: number of regional markets.*  *Dj = Annual market size of regional market j*  *Ki = maximum possible capacity of production facility i*  *cij = Variable cost of producing, transporting and duty from facility i to market j*  *fi = Annual fixed cost of facility i*  *ti  =Tax rate at facility i*  *xij = Number of units from facility i to regional market j.*  *It should be integral and non-negative.* |

|  |
| --- |
| And replace above objective function to the following one to maximize after tax profit: |

|  |  |  |
| --- | --- | --- |
| SYMBOL | INPUT | CELL |
| *Dj* | *Annual market size of regional market j* | C8:F8 |
| *Ki* | *maximum possible capacity of production facility i* | G5:G7 |
| *cij* | *Variable cost of producing, transporting and duty from facility i to market j* | C5:F7 |
| *fi* | *Annual fixed cost of facility i* | H5:H7 |
| *xij* | *of units from facility i to regional market j.* | C20:F22 |
| obj. | objective function | H24 |
| 5.1 | demand constraints | C23:F23 |
| 5.2 | capacity constraints | G20:G22 |

(Sheet Hot&Cold in workbook problem5.8.xls)

The above model gives optimal result as in following table:



And we use the same model but with data from CaldoFreddo to get following optimal production and distribution plan for CaldoFreddo:



### (b)

If none of the plants is shut down, the previous model is still applicable. However, we need to update the number of facilities to 5, with 3 from Hot&cold and 2 from CaldoFreddo. And we need to update the market demand *Dj,* which should be the sum of market shares. Decision maker has more facilities and greater market share in each region, and hence has more choices for production and distribution plans. The optimal result is summarized in the following table.



### (c)

This model is more advanced since it allows facilities to be shutdown. Accordingly we need more variables to reflect this new complexity.

*Optimization model for Sleekfon:*

|  |
| --- |
| *n = 5: Hot&Cold and caldoFreddo production facilities.*  *m = 4: number of regional markets.*  *Dj = Annual market size of regional market j, sum of the : Hot&Cold and caldoFreddo market share.*  *Ki =capacity of production facility i*  *cij = Variable cost of producing, transporting and duty from facility i to market j*  *fi = Annual fixed cost of facility i*  *xij = Number of units from facility i to regional market j.*  *It should be integral and non-negative.*  *zi = Binary variable indicating whether to shutdown facility i. zi =1 means to shutdown it, 0 otherwise.* |

|  |  |  |
| --- | --- | --- |
| SYMBOL | INPUT | CELL |
| *Dj* | *Annual market size of regional market j* | C8:F8 |
| *Ki* | *maximum possible capacity of production facility i* | C12:C14 |
| *cij* | *Variable cost of producing, transporting and duty from facility i to market j* | C5:F7 |
| *fi* | *Annual fixed cost of facility i* | H5:H7 |
| *xij* | *Number of units from facility i to regional market j.* | C19:F23 |
| *Zi* | *open or shutdown facility i* | G19:G23 |
| obj. | objective function | I25 |
| 5.1 | demand constraints | C24:F24 |
| 5.2 | capacity constraints | H19:H23 |

(Sheet Merged in workbook problem5.8.xls)

It turned out that all sites are open so as to achieve best objective value. Following table shows the optimal configuration.

