Problem 1.1 Is each of these 1-D signals:

- Analog or digital?
- Continuous-time or discrete-time?

(a) Daily closes of the stock market
(b) Output from phonograph record pickup
(c) Output from compact disc pickup

Solution:

(a) Stock market closes are recorded only at the end of each day, but indices take on a continuous range of values. $\boxed{\text{Analog and discrete time.}}$

(b) Phonographs are entirely $\boxed{\text{Analog and continuous time.}}$

(c) CDs store music sampled at 44100 samples per s (discrete time) and quantized using 16 bits. So $\boxed{\text{Digital and discrete time.}}$

Problem 1.2 Is each of these 2-D signals:

- Analog or digital?
- Continuous-space or discrete-space?

(a) Image in a telescope eyepiece
(b) Image displayed on digital TV
(c) Image stored in a digital camera

Solution:

(a) The image seen in a telescope is $\boxed{\text{Analog and continuous space.}}$

(b) The image displayed on a digital TV is discrete space, since it is composed of pixels, but each pixel takes a continuous range of values.

So a digital TV image **at a given moment** is $\boxed{\text{Analog and discrete space.}}$

(c) The image **stored** in a digital camera consists of pixels (discrete space) which are quantized to a finite number of values. $\boxed{\text{Digital and discrete space.}}$

Problem 1.3 The following signals are 2-D in space and 1-D in time, so they are 3-D signals. Is each of these 3-D signals:

- Analog or digital?
- Continuous or discrete?

(a) The world as you see it
(b) A movie stored on film
(c) A movie stored on a DVD

Solution:

(a) The world you see is $\boxed{\text{Analog and continuous space and continuous time.}}$

(b) A movie on film is a sequence of images at 24 frames per second. Each image is continuous and analog (although film does has finite resolution). So a movie on film is

$$\boxed{\text{Analog and continuous in space and discrete in time.}}$$

(c) A movie on a DVD is a sequence of image at 30 frames per second. Each image is discrete space since it is composed of pixels, and each image pixel is quantized to a finite number of values.

So DVDs are entirely $\boxed{\text{Digital and discrete space and discrete time.}}$

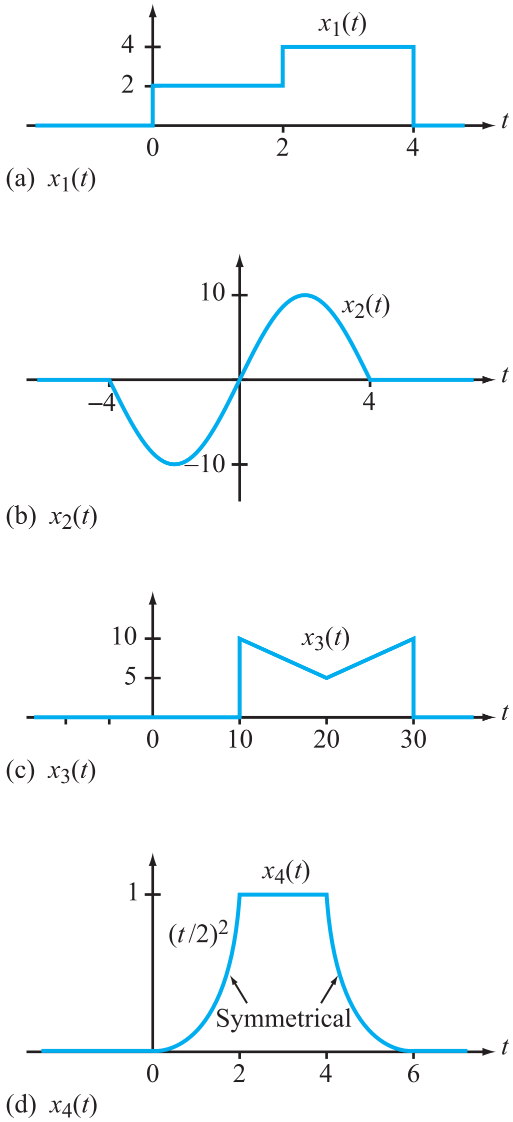


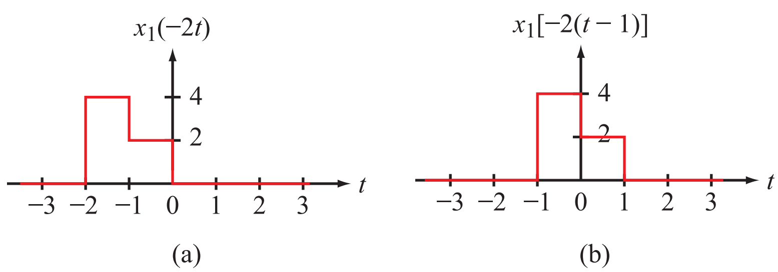
Figure P1.4: Waveforms for Problems 1.4 to 1.7.

Problem 1.4 Given the waveform of $x_1(t)$ shown in Fig. P1.4(a), generate and plot the waveform of:

(a) $x_1(-2t)$

(b) $x_1[-2(t-1)]$

Solution:



(a) $x_1(-2t)$ is $x_1(t)$ compressed by 2 and reversed in time.

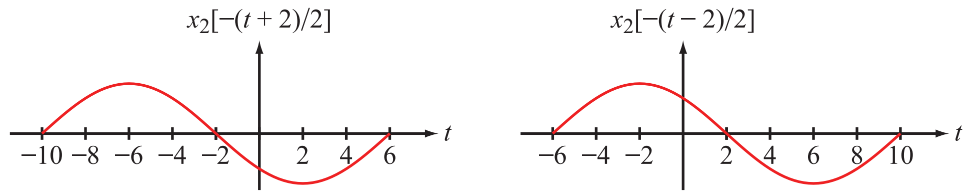
(b) $x_1(-2(t-1))$ is $x_1(t)$ compressed by 2 and reversed in time, then delayed by 1.

Problem 1.5 Given the waveform of $x_2(t)$ shown in Fig. P1.4(b), generate and plot the waveform of:

(a) $x_2[-(t+2)/2]$

(b) $x_2[-(t-2)/2]$

Solution:



(a) (b)

(a) $x_2[-(t+2)/2]$ is $x_2(t)$ expanded by 2 and reversed in time, then advanced by 2.

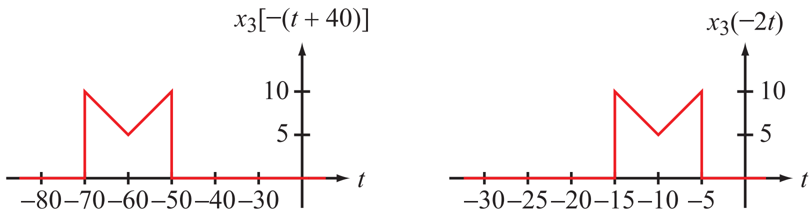
(b) $x_2[-(t-2)/2]$ is $x_2(t)$ expanded by 2 and reversed in time, then delayed by 2.

Problem 1.6 Given the waveform of $x_3(t)$ shown in Fig. P1.4(c), generate and plot the waveform of:

(a) $x_3[-(t+40)]$

(b) $x_3(-2t)$

Solution:



(a) (b)

(a) $x_3[-(t+40)]$ is $x_3(t)$ reversed in time, then advanced by 40.
(b) $x_3(-2t)$ is $x_3(t)$ compressed by 2 and reversed in time.

Problem 1.7 The waveform shown in Fig. P1.4(d) is given by:

$$x_4(t) = \begin{cases} 0 & \text{for } t \le 0, \\ \left(\frac{t}{2}\right)^2 & \text{for } 0 \le t \le 2 \text{ s}, \\ 1 & \text{for } 2 \le t \le 4 \text{ s}, \\ f(t) & \text{for } 4 \le t \le 6 \text{ s}, \\ 0 & \text{for } t \ge 6 \text{ s}. \end{cases}$$

(a) Obtain an expression for $f(t)$, the segment covering the time duration between 4 s and 6 s.

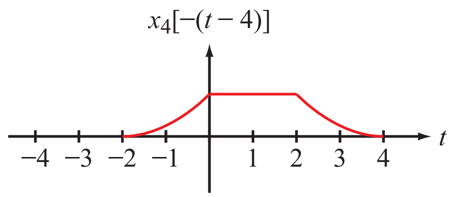
(b) Obtain an expression for $x_4[-(t-4)]$ and plot it.

Solution:

(a) $f(t)$ is the segment $(\frac{t}{2})^2$ in the interval $0 \le t \le 2$, reversed in time and delayed by 6. So

$$f(t) = \left(\frac{-(t-6)}{2}\right)^2 = \left(\frac{6-t}{2}\right)^2.$$

(b) $x_4[-(t-4)]$ is $x_4(t)$ reversed in time, then delayed by 4.



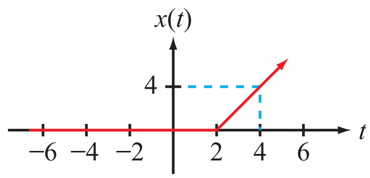
(b)

Problem 1.8 If

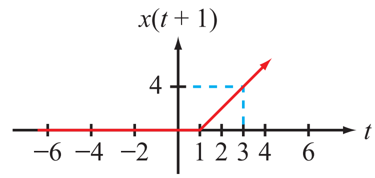
$$x(t) = \begin{cases} 0 & \text{for } t \leq 2 \\ (2t-4) & \text{for } t \geq 2, \end{cases}$$

plot $x(t)$, $x(t+1)$, $x\left(\frac{t+1}{2}\right)$, and $x\left[-\frac{(t+1)}{2}\right]$.

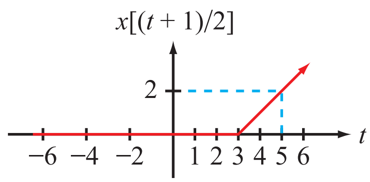
Solution:



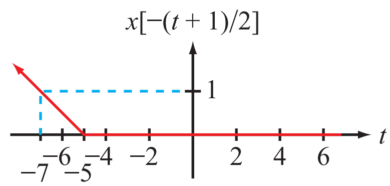
(a)



(b)



(c)



(d)

(a) $x(t) = 2(t-2)\,u(t-2) = 2r(t-2)$.

(b)

$$\begin{aligned} x(t+1) &= 2((t+1)-2)\,u(t+1-2) \\ &= 2(t-1)\,u(t-1) = 2r(t-1); \end{aligned}$$

(shift $x(t)$ left by 1).

(c)

$$\begin{aligned} x\left(\frac{t+1}{2}\right) &= 2\left(\frac{t+1}{2} - 2\right)\,u\left(\frac{t+1}{2} - 2\right) \\ &= (t-3)\,u\left(\frac{t}{2} - 1.5\right); \end{aligned}$$

slope is 1, instead of 2, and $u\left(\frac{t}{2} - 1.5\right)$ is zero for $t < 3$.

(d)

$$\begin{aligned} x\left(-\frac{t+1}{2}\right) &= 2\left(-\left(\frac{t+1}{2}\right) - 2\right)\,u\left(-\left(\frac{t+1}{2}\right) - 2\right) \\ &= (-t-5)\,u\left(-\frac{t}{2} - 2.5\right); \end{aligned}$$

slope is -1 and $u\left(-\frac{t}{2} - 2.5\right)$ is zero for $t > -5$.

Problem 1.9 Given $x(t) = 10(1 - e^{-|t|})$, plot $x(-t+1)$.

Solution: $x(-t+1) = x(-(t-1))$ is $x(t)$ reversed in time, then delayed by 1.

```
t=linspace(-5,5,1000);x=10-10*exp(-abs(1-t));plot(t,x)
```

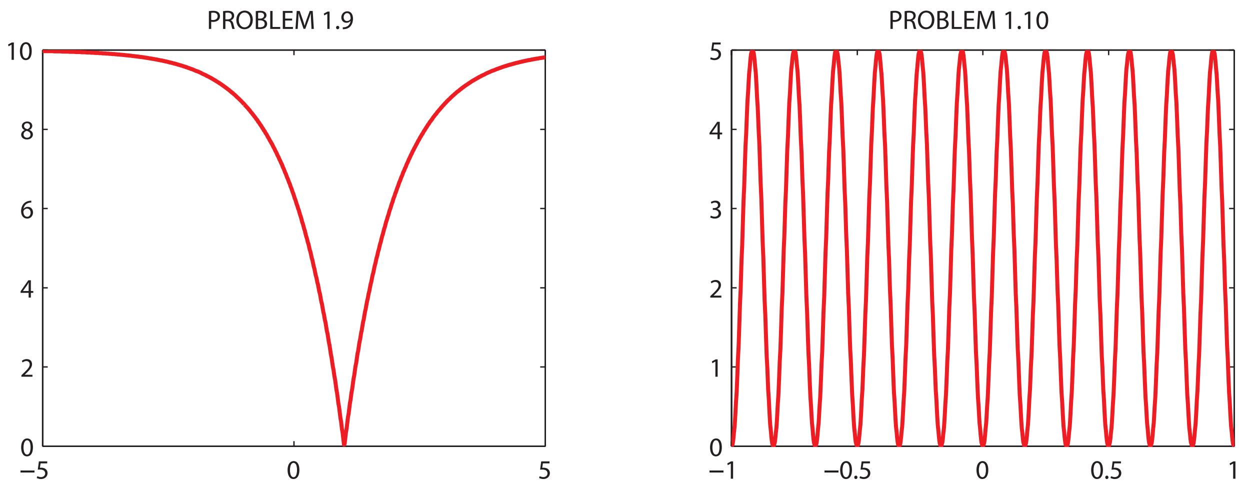


Figure P1.9: Waveforms for Problems 1.9 (left) and 1.10 (right).

Problem 1.10 Given $x(t) = 5\sin^2(6\pi t)$, plot $x(t-3)$ and $x(3-t)$.

Solution:

$$x(t) = 5\sin^2(6\pi t) = \frac{5}{2} - \frac{5}{2}\sin(12\pi t)$$

is unaltered by time shifts of ± 3. $x(t)$ is also an even function. So $x(t-3) = x(3-t) = x(t)$.

```
t=linspace(-1,1,1000);x=5*sin(6*pi*t).*sin(6*pi*t);plot(t,x)
```

Problem 1.11 Given the waveform of $x(t)$ shown in P1.11(b), generate and plot the waveform of:

(a) $x(2t+6)$

(b) $x(-2t+6)$

(c) $x(-2t-6)$

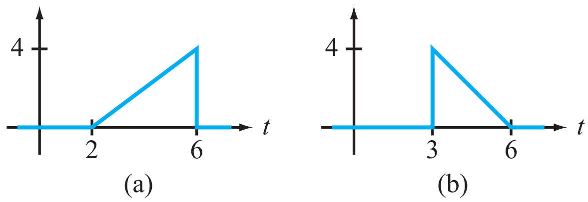
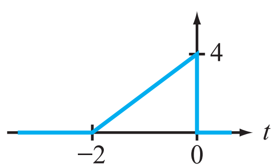


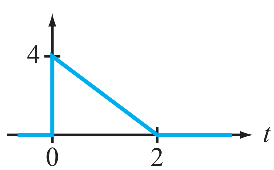
Figure P1.11: Waveforms for Problems 1.11 and 1.12.

Solution: Scale first, then shift, after writing each transformation in the form $x(a(t-b))$.

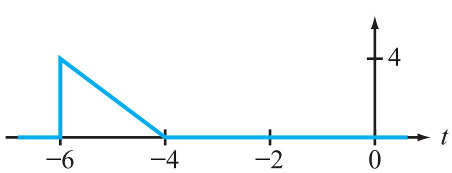
(a) $x(2t+6) = x(2(t+3))$



(b) $x(-2t+6) = x(-2(t-3))$



(c) $x(-2t-6) = x(-2(t+3))$



Problem 1.12 Given the waveform of $x(t)$ shown in P1.11(b), generate and plot the waveform of:

(a) $x(3t+6)$

(b) $x(-3t+6)$

(c) $x(-3t-6)$

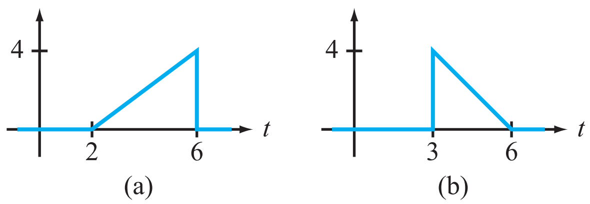
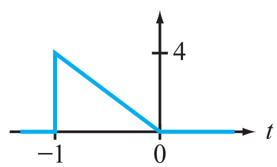


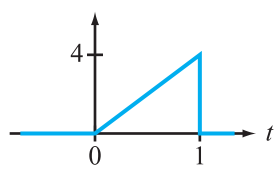
Figure P1.12: Waveforms for Problems 1.11 and 1.12.

Solution: Scale first, then shift, after writing each transformation in the form $x(a(t-b))$.

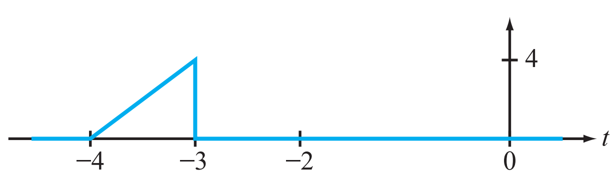
(a) $x(3t+6) = x(3(t+2))$



(b) $x(-3t+6) = x(-3(t-2))$



(c) $x(-3t-6) = x(-3(t+2))$



Problem 1.13 If $x(t) = 0$ unless $a \leq t \leq b$, and $y(t) = x(ct+d)$ unless $e \leq t \leq f$, compute e and f in terms of a, b, c, d. Assume $c > 0$ to make things easier for you.

Solution: Let $z(t) = x(ct)$. Then $x(t) = 0$ unless $a \leq t \leq b$ $\leftrightarrow$ $z(t) = 0$ unless $\frac{a}{c} \leq t \leq \frac{b}{c}$.

Then $y(t) = x(ct+d) = x(c(t+\frac{d}{c})) = z(t+\frac{d}{c})$ and $y(t) = 0$ unless

$$\boxed{\frac{a-d}{c} \leq t \leq \frac{b-d}{c}.}$$

Try applying this result to the preceding three problems.

Problem 1.14 If $x(t)$ is a musical note signal, what is $y(t) = x(4t)$? Consider sinusoidal $x(t)$.

Solution: $y(t)$ is $\boxed{x(t) \text{ raised by 2 octaves.}}$ For example, let $x(t) = \cos(2\pi 440t)$ (note A).

Then $y(t) = x(4t) = \cos(2\pi 440(4t)) = \cos(2\pi 1760t)$; (note A, but 2 octaves higher).

Problem 1.15 Give an example of a non-constant signal that has the property $x(t) = x(at)$ for all $a > 0$.

Solution: The step $\boxed{x(t) = u(t)}$ has $u(at) = u(t)$ for $a > 0$.

Problem 1.16 For each of the following functions, indicate if it exhibits even symmetry, odd symmetry, or neither one:

(a) $x_1(t) = 3t^2 + 4t^4$

(b) $x_2(t) = 3t^3$

Solution: A function has even symmetry if $x(-t) = x(t)$, and odd symmetry if $x(-t) = -x(t)$.

(a) $x_1(-t) = 3(-t)^2 + 4(-t)^4 = 3t^2 + 4t^4 = x_1(t)$. $\boxed{\text{Even.}}$

(b) $x_2(-t) = 3(-t)^3 = -3t^3 = -x_2(t)$. $\boxed{\text{Odd.}}$

Problem 1.17 For each of the following functions, indicate if it exhibits even symmetry, odd symmetry, or neither one:

(a) $x_1(t) = 4[\sin(3t) + \cos(3t)]$

(b) $x_2(t) = \dfrac{\sin(4t)}{4t}$

Solution: A function has even symmetry if $x(-t) = x(t)$, and odd symmetry if $x(-t) = -x(t)$.

(a) $x_1(t)$ is the sum of an odd function $4\sin(3t)$ and an even function $4\cos(3t)$, so it is neither even nor odd. $\boxed{\text{Neither.}}$

(b)

$$x_2(-t) = \frac{\sin(4(-t))}{4(-t)} = \frac{-\sin(4t)}{-4t} = \frac{\sin(4t)}{4t} = x_2(t). \qquad \boxed{\text{Even.}}$$

Problem 1.18 For each of the following functions, indicate if it exhibits even symmetry, odd symmetry, or neither one:

(a) $x_1(t) = 1 - e^{-2t}$.

(b) $x_2(t) = 1 - e^{-2t^2}$

Solution: A function has even symmetry if $x(-t) = x(t)$, and odd symmetry if $x(-t) = -x(t)$.

(a) Clearly $x_1(-t) \neq \pm x_1(t)$ so it is neither even nor odd. $\boxed{\text{Neither.}}$

(b) $x_2(-t) = 1 - e^{-2(-t)^2} = 1 - e^{-2t^2} = x_2(t)$. $\boxed{\text{Even.}}$

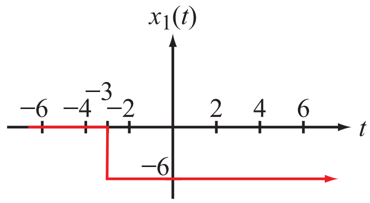
Problem 1.19 Generate plots for each of the following step-function waveforms over the time span from −5 s to +5 s:

(a) $x_1(t) = -6u(t+3)$

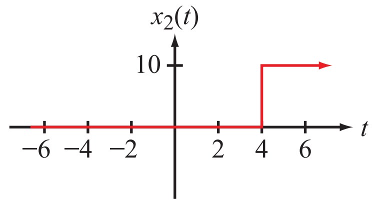
(b) $x_2(t) = 10u(t-4)$

(c) $x_3(t) = 4u(t+2) - 4u(t-2)$

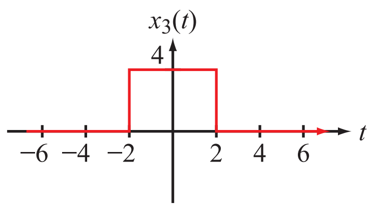
Solution:



(a)



(b)



(c)

(a) Advanced in time by 3 and multiplied by −6.
(b) Delayed in time by 4 and multiplied by 10.
(c) Up by 4 at $t = -2$, then down by 4 at $t = 2$.

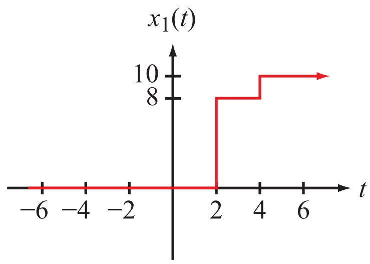
Problem 1.20 Generate plots for each of the following step-function waveforms over the time span from -5 s to $+5$ s:

(a) $x_1(t) = 8u(t-2) + 2u(t-4)$

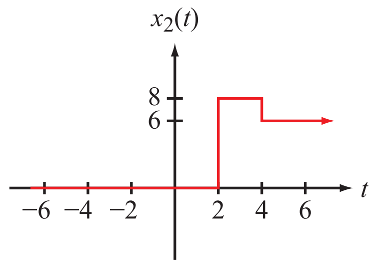
(b) $x_2(t) = 8u(t-2) - 2u(t-4)$

(c) $x_3(t) = -2u(t+2) + 2u(t+4)$

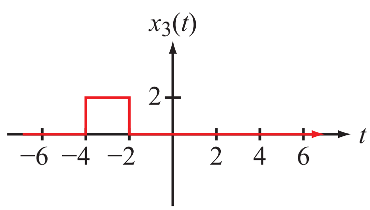
Solution:



(a)



(b)



(c)

(a) Up by 8 at $t = 2$, then up by 2 more at $t = 4$.
(b) Up by 8 at $t = 2$, then down by 2 at $t = 4$.
(c) Up by 2 at $t = -4$, then down by 2 at $t = -2$. Note $-4 < -2$.

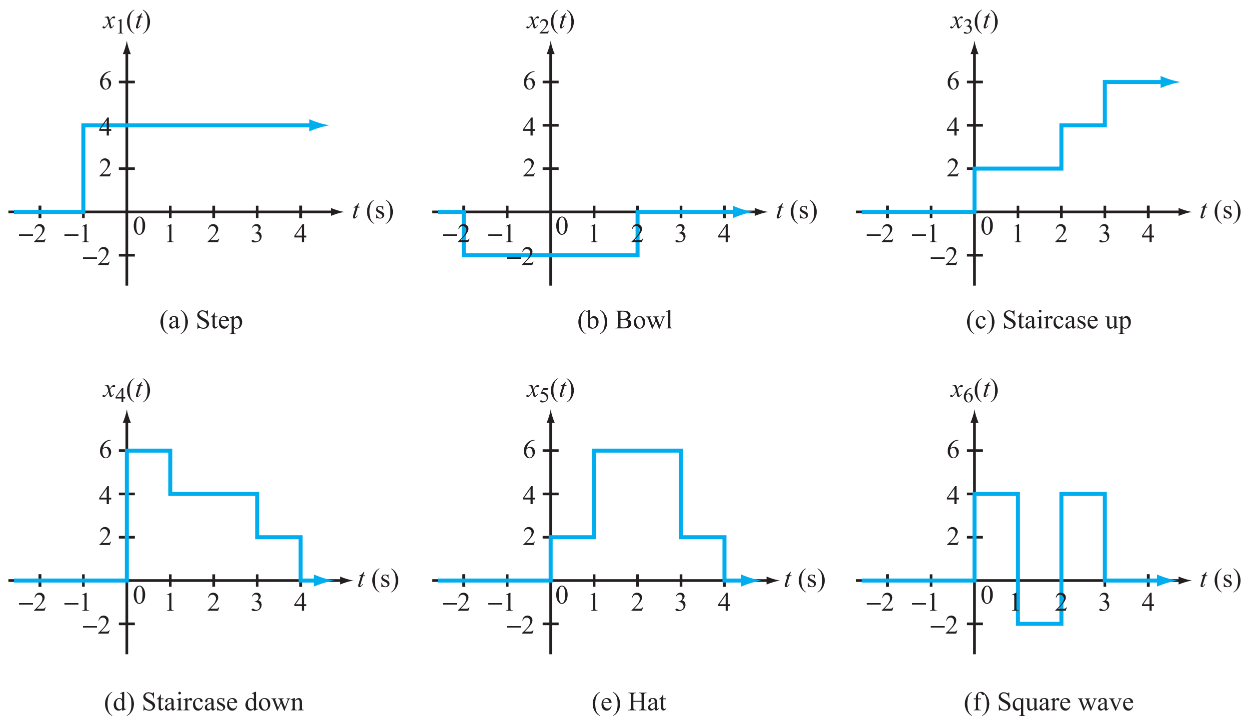


Figure P1.21: Waveforms for Problem 1.21.

Problem 1.21 Provide expressions in terms of step functions for the waveforms displayed in Fig. P1.21.

Solution:

(a) $\boxed{x_1(t) = 4u(t+1).}$ Advance in time by 1.

(b) $\boxed{x_2(t) = -2u(t+2) + 2u(t-2).}$ Step down at $t = -2$, then up at $t = 2$.

(c) $\boxed{x_3(t) = 2u(t) + 2u(t-2) + 2u(t-3).}$ Step up at $t = 0, 2, 3$ by 2 each time.

(d) $\boxed{x_4(t) = 6u(t) - 2u(t-1) - 2u(t-3) - 2u(t-4).}$

Step up by 6 at $t = 0$, then down by 2 each time at each of $t = 1, 3, 4$.

(e) $\boxed{x_5(t) = 2u(t) + 4u(t-1) - 4u(t-3) - 2u(t-4).}$

Step up by 2 at $t = 0$, up by 4 at $t = 1$, down by 4 at $t = 3$, then down by 2 at $t = 4$.

(f) $\boxed{x_6(t) = 4u(t) - 6u(t-1) + 6u(t-2) - 4u(t-3).}$

Step up by 4 at $t = 0$, down by 6 at $t = 1$, up by 6 at $t = 2$, then down by 5 at $t = 3$.

Problem 1.22 Generate plots for each of the following functions over the time span from −4 s to 4 s:

(a) $x_1(t) = 5r(t+2) - 5r(t)$

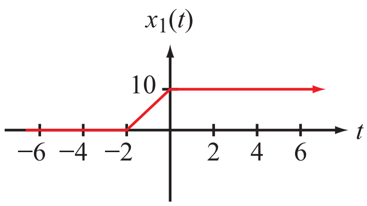
(b) $x_2(t) = 5r(t+2) - 5r(t) - 10u(t)$

(c) $x_3(t) = 10 - 5r(t+2) + 5r(t)$

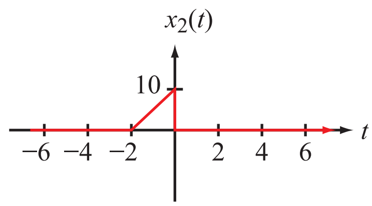
(d) $x_4(t) = 10\text{rect}\left(\frac{t+1}{2}\right) - 10\text{rect}\left(\frac{t-3}{2}\right)$

(e) $x_5(t) = 5\text{rect}\left(\frac{t-1}{2}\right) - 5\text{rect}\left(\frac{t-3}{2}\right)$

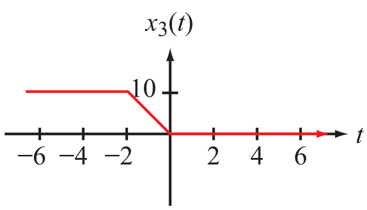
Solution:



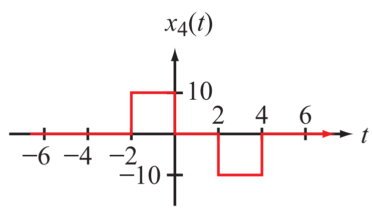
(a)



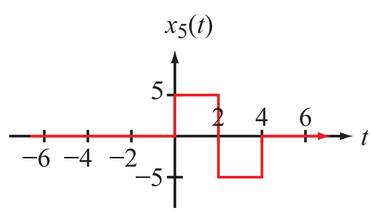
(b)



(c)



(d)



(e)

(a) Ramps up with slope 5 at $t = -2$, then levels off at $t = 0$.

(b) Ramps up with slope 5 at $t = -2$, then levels off and drops by 10 at $t = 0$. The final value is level at $5(t+2) - 5t - 10 = 0$.

(c) Starting at 10, ramps down with slope −5 at $t = -2$, then levels off at $t = 0$. The final value is level at $10 - 5(t+2) + 5t = 0$.

If $a > 0$, $\text{rect}\left(\frac{t-b}{2a}\right) = 1$ if $\left|\frac{t-b}{a}\right| < 1 \quad \leftrightarrow \quad (b-a) < t < (b+a)$, and 0 otherwise.

(d) $x_4(t)$ is two rectangular pulses on intervals $-2 < t < 0$ and $2 < t < 4$.

(e) $x_5(t)$ is two rectangular pulses on intervals $0 < t < 2$ and $2 < t < 4$.

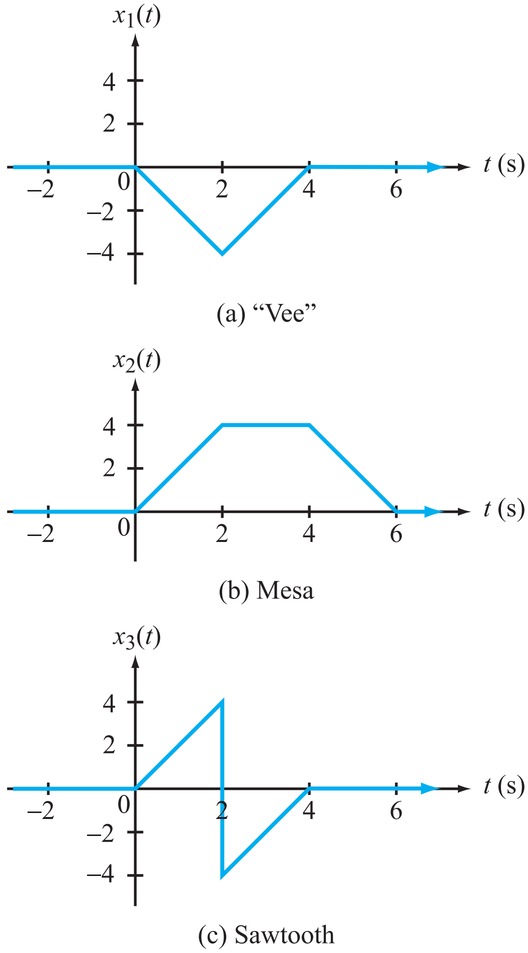


Figure P1.23: Waveforms for Problem 1.23.

Problem 1.23 Provide expressions for the waveforms displayed in Fig. P1.23 in terms of ramp and step functions.

Solution:

(a) $\boxed{x_1(t) = -2r(t) + 4r(t-2) - 2r(t-4).}$ $-2r(t)$ ramps down with slope -2.

$2r(t-2)$ would level off, so $4r(t-2)$ ramps up with slope 2. $-2r(t-4)$ levels off.

(b) $\boxed{x_2(t) = 2r(t) - 2r(t-2) - 2r(t-4) + 2r(t-6).}$

$2r(t)$ ramps up with slope 2.
$-2r(t-2)$ levels off, then $-2r(t-4)$ ramps down with slope -2. $2r(t-6)$ levels off.

(c) $\boxed{x_3(t) = 2r(t) - 8u(t-2) - 2r(t-4).}$

$2r(t)$ ramps up with slope 2.

$-8u(t-2)$ drops from $+4$ to -4, but $x_3(t)$ continues to ramp up. $-2r(t-4)$ levels off.