

# Abstract Algebra (3rd Edition)

Chapter 2.5, Problem 6E Bookmark Show all steps:  Done

## Step-by-step solution

### Step 1 of 7

By using the given lattices we have to find the centralizers of every element in the following groups:  
 $D_8$   
 $Q_8$   
 $S_3$   
 $D_8$

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### Step 2 of 7

**Lemma 1:** Let us consider that,  $G$  is a group and that  $x \in G$  and  $z \in Z(G)$ .  
 Thus, we have  
 $C_G(xz) = C_G(x)$   
 Now we have to prove Lemma 1:  
**Proof:** If we consider that  $y \in C_G(x)$  then we have  $yxz = xyx = xzy$ , such that  $y \in C_G(xz)$ .  
**Now let us suppose that  $y \in C_G(xz)$ . Then we have  $yxz = xzy = xyx$ , such that by cancellation we get  $yx = xy$  and thus we have  $y \in C_G(x)$ .**

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### Step 3 of 7

**Lemma 2:** Let  $G$  be a group and let  $x, g \in G$ . Then  
 $C_G(gxg^{-1}) = g(C_G(x))g^{-1}$   
 Now we have to prove the Lemma 2.  
**Proof:** If we consider that  $y \in C_G(x)$  then we have  $y = gxg^{-1}$  such that  $z \in C_G(x)$ .  
 Thus, we have,  
 $(gxg^{-1})(gxg^{-1}) = gxg^{-1}$   
 $= gxg^{-1}$   
 $= (gxg^{-1})(gxg^{-1})$   
 such that  $y \in C_G(gxg^{-1})$ .  
**Consider that  $y \in C_G(gxg^{-1})$ . Thus, we have**  
 $yxg^{-1} = gxg^{-1}y$   
**Hence,**  
 $(g^{-1}yg)x = x(g^{-1}yg)$   
**Hence we get  $g^{-1}yg \in C_G(x)$ , and we have  $y \in gC_G(x)g^{-1}$ .**

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### Step 4 of 7

**Lemma 3:** Let  $G$  be a group and let  $g \in G, A \subseteq G$ . Then  
 $g(A)g^{-1} = (gAg^{-1})$   
 Now we have to prove the Lemma 3:  
**Proof:** If we consider that  $y \in g(A)g^{-1}$  then we have  $y = gxg^{-1}$  where  $x \in A$ . We know that  $x = a_1a_2 \dots a_n$  where for each  $i$ , either  $a_i$  or  $a_i^{-1}$  is in  $A$ .  
 Thus, we have  $x = a_1g^{-1}ga_1g^{-1}g^{-1}ga_2g^{-1}g^{-1}ga_n$ , such that  
 $gxg^{-1} = (ga_1g^{-1})(ga_2g^{-1}) \dots (ga_n)$   
**Therefore, we have**  
 $y \in (gAg^{-1})$

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### Step 5 of 7

**Lemma 4:** Let  $G$  be a group and let  $a, b \in G$ . If  $(a) = (b)$ , then  
 $C_G(a) = C_G(b)$   
 Now we have to prove Lemma 4:  
**Proof:** we know that  $b = a^k$  for some  $k$ , such that if  $xa = ax$ , then we have  
 $xb = bx$   
**Therefore,**  
 $C_G(a) \subseteq C_G(b)$

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### Step 6 of 7

Now consider  $G = D_8$ . We know that,  
 $Z(D_8) = \{1, r^2\}$

$x$	Reasoning	$C_G(x)$
1	$1 \in Z(D_8)$	$D_8$
$r$	$\langle r \rangle \leq C_G(r)$ , so $C_G(r)$ is either $\langle r \rangle$ or $D_8$ . But $sr \neq rs$ , so $s \notin C_G(r)$ , hence $C_G(r) = \langle r \rangle$ .	$\langle r \rangle$
$r^2$	$r^2 \in Z(D_8)$	$D_8$
$r^3$	$r^3 = r^{-1}$	$\langle r \rangle$
$s$	$\langle s \rangle \leq C_G(s)$ and $\langle r^2 \rangle \leq C_G(s)$ since $r^2 \in Z(G)$ , so $C_G(s)$ is either $\langle s, r^2 \rangle$ or $D_8$ . But $r \notin C_G(s)$ since $sr \neq rs$ .	$\langle s, r^2 \rangle$
$sr$	$\langle sr \rangle \leq C_G(sr)$ and $\langle r^2 \rangle \leq C_G(sr)$ since $r^2 \in Z(G)$ , so $C_G(sr)$ is either $\langle sr, r^2 \rangle$ or $D_8$ . But $rsr \neq srs$ , so $r \notin C_G(sr)$ .	$\langle sr, r^2 \rangle$
$sr^2$	$sr^2 = sr^{-1}$ , so $C_G(sr^2) = C_G(sr)$ by Lemma 1	$\langle sr, r^2 \rangle$
$sr^3$	$sr^3 = sr^{-2}$ , so $C_G(sr^3) = C_G(sr)$ by Lemma 1	$\langle sr, r^2 \rangle$

$G = Q_8$ . We know that,  
 $Z(Q_8) = \{1, -1\}$

$x$	Reasoning	$C_G(x)$
1	$1 \in Z(G)$	$Q_8$
-1	$-1 \in Z(G)$	$Q_8$
$i$	$\langle i \rangle \leq C_G(i)$ so $C_G(i)$ is either $\langle i \rangle$ or $Q_8$ . But $j \notin C_G(i)$ since $ij = k \neq -k = ji$ .	$\langle i \rangle$
-i	$-i = i^{-1}$ , so $C_G(-i) = C_G(i)$	$\langle i \rangle$
$j$	$\langle j \rangle \leq C_G(j)$ so $C_G(j)$ is either $\langle j \rangle$ or $Q_8$ . But $k \notin C_G(j)$ since $jk = i \neq -i = kj$ .	$\langle j \rangle$
-j	$-j = j^{-1}$ , so $C_G(-j) = C_G(j)$ .	$\langle j \rangle$
$k$	$\langle k \rangle \leq C_G(k)$ , so $C_G(k)$ is either $\langle k \rangle$ or $Q_8$ . But $i \notin C_G(k)$ since $ki = j \neq -j = ik$ .	$\langle k \rangle$
-k	$-k = k^{-1}$ , so $C_G(-k) = C_G(k)$	$\langle k \rangle$

Now consider  $S_3$ :

$x$	Reasoning	$C_G(x)$
1		$S_3$
(1 2)	$\langle (1 2) \rangle \leq C_G((1 2))$ , so $C_G((1 2))$ is either $S_3$ or $\langle (1 2) \rangle$ . Thus we have $S_3$ which does not commute with $(1 2) G$ .	$\langle (1 2) \rangle$
(1 3)	We know that $(1 3) = (2 3)(1 2)(2 3)$ , so we can apply Lemmas 2 and 3.	$\langle (1 3) \rangle$
(2 3)	We have $(2 3) = (1 3)(1 2)(1 3)$ , so we can apply Lemmas 2 and 3.	$\langle (2 3) \rangle$
(1 2 3)	$\langle (1 2 3) \rangle \leq C_G((1 2 3))$ , so $C_G((1 2 3))$ is either $\langle (1 2 3) \rangle$ or $G$ . But $(1 2)$ does not commute with $(1 2 3)$ .	$\langle (1 2 3) \rangle$
(1 3 2)	$(1 3 2) = (1 2 3)^{-1}$	$\langle (1 2 3) \rangle$

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### Step 7 of 7

Now consider  $D_{16}$ :

$x$	Reasoning	$C_G(x)$
1		$D_{16}$
$r$	$\langle r \rangle \leq D_{16}$ , so $C_G(r)$ is either $D_{16}$ or $\langle r \rangle$ . But $rs \neq sr$ .	$\langle r \rangle$
$r^2$	$\langle r^2 \rangle \leq C_G(r^2)$ , so $C_G(r^2)$ is either $\langle r^2 \rangle, \langle s, r^2 \rangle, \langle r, sr, r^2 \rangle$ or $D_{16}$ . We know that $sr^2 \neq r^2s$ , and $rsr^2 = r^2r$ , and $sr^2 \neq r^2sr$ .	$\langle r^2 \rangle$
$r^3$	$r^3 = r^{-1}$	$\langle r \rangle$
$r^4$	$r^4 \in Z(G)$	$D_{16}$
$r^5$	$r^5 = r^{-1}$	$\langle r \rangle$
$r^6$	$r^6 = r^2r^4$	$\langle r \rangle$
$r^7$	$r^7 = r^{-1}$ .	$\langle r \rangle$
$s$	$\langle s \rangle \leq C_G(s)$ , so $C_G(s)$ is either $\langle s \rangle, \langle s, r^4 \rangle, \langle s, r^2 \rangle$ or $D_{16}$ . Now $\langle r^2 \rangle \leq C_G(s)$ , and $r^2s \neq sr^2$ .	$\langle s, r^4 \rangle$
$sr$	$\langle sr \rangle \leq C_G(sr)$ , so $C_G(sr)$ is either $\langle sr \rangle, \langle sr, r^4 \rangle, \langle sr, r^2 \rangle$ or $D_{16}$ . We know that $r^2sr = sr^2r^4$ and $sr^2r^2 \neq r^2sr$ .	$\langle sr, r^4 \rangle$
$sr^2$	$\langle sr^2 \rangle \leq C_G(sr^2)$ , so $C_G(sr^2)$ is either $\langle sr^2 \rangle, \langle sr^2, r^4 \rangle, \langle sr^2, r^2 \rangle$ or $D_{16}$ . We know that $r^2sr^2 = sr^2r^4$ and $r^2sr^2 \neq sr^2r^2$ .	$\langle sr^2, r^4 \rangle$
$sr^3$	$\langle sr^3 \rangle \leq C_G(sr^3)$ and $\langle r^2 \rangle \leq C_G(sr^3)$ . Hence we have $r^2sr^3 \neq sr^3r^2$ .	$\langle sr^3, r^4 \rangle$
$sr^4$	Lemma 1	$\langle s, r^4 \rangle$
$sr^5$	Lemma 1	$\langle sr, r^4 \rangle$
$sr^6$	Lemma 1	$\langle sr^2, r^4 \rangle$
$sr^7$	Lemma 1	$\langle sr^3, r^4 \rangle$

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