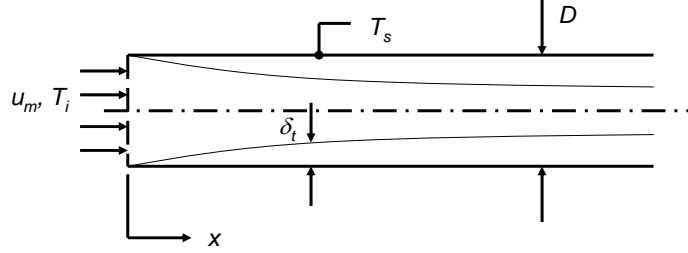


## PROBLEM 8.15

**KNOWN:** Laminar boundary layer development in a tube entrance.

**FIND:** (a) Expression for  $Nu_D$  in terms of  $Gz_D^{-1}$  and  $Pr$ . Plot of  $Nu_D$  versus  $Gz_D^{-1}$  for  $Pr = 0.7$ . (b) Expression for  $\overline{Nu}_D$  in terms of  $Gz_D^{-1}$  and  $Pr$ . Comparison to combined entrance length correlation in the limit of small  $x$ .

**SCHEMATIC:**



**ASSUMPTIONS:** (1) Constant properties. (2) Laminar conditions.

**ANALYSIS:** (a) From Equation 7.23, the Nusselt number based upon the streamwise coordinate  $x$  is

$$Nu_x = \frac{hx}{k} = 0.332 Re_x^{1/2} Pr^{1/3} \quad (1)$$

Multiplying both sides of Equation 1 by  $D/x$  and substituting  $Re_x = Re_D x/D$  yields

$$Nu_D = \frac{hD}{k} = 0.332 \left[ Re_x \left( \frac{x}{D} \right) \right]^{1/2} \left[ \frac{D}{x} \right] Pr^{1/3} = 0.332 \left[ Re_x \left( \frac{D}{x} \right) \right]^{1/2} Pr^{1/3} \quad (2)$$

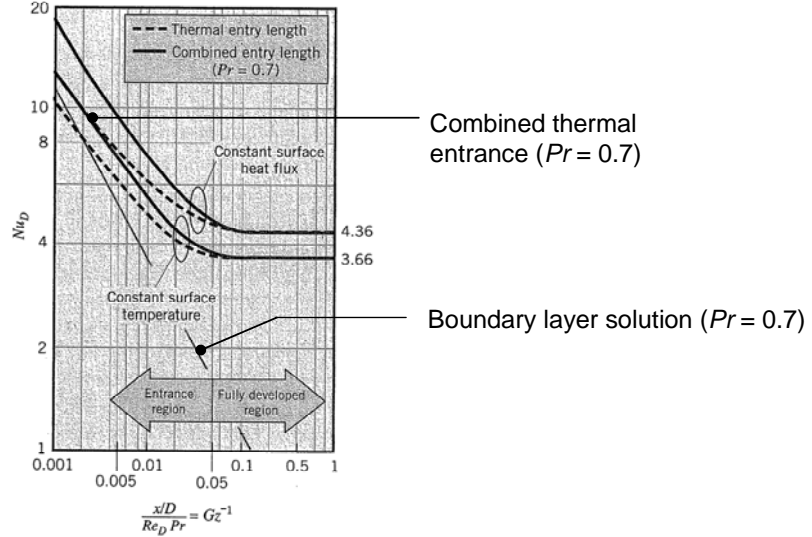
Substituting  $Gz_D^{-1} = (x/D)/(Re_D Pr)$  into Equation 2 and noting that  $Pr^{1/3} = Pr^{1/2} \cdot Pr^{-1/6}$  yields

$$Nu_D = 0.332 [Gz_D^{-1}]^{-1/2} Pr^{-1/6} <$$

The expression for the local Nusselt number,  $Nu_D$  with  $Pr = 0.7$  is plotted below.

Continued...

### PROBLEM 8.15 (Cont.)



(b) Equation 7.30 gives the following for the average Nusselt number:

$$\overline{Nu}_x = \frac{\bar{h}_x x}{k} = 0.664 Re_x^{1/2} Pr^{1/3}$$

Following the same steps as in part (a), this can be rewritten as

$$\overline{Nu}_D = 0.664 [Gz_D^{-1}]^{-1/2} Pr^{-1/6} \quad (3)$$

The average Nusselt number for the combined entrance length is given as

$$\overline{Nu}_D = \frac{\frac{3.66}{\tanh\left[2.264 Gz_D^{-1/3} + 1.7 Gz_D^{-2/3}\right]} + 0.0499 Gz_D \tanh(Gz_D^{-1})}{\tanh(2.432 Pr^{1/6} Gz_D^{-1/6})}$$

In the limit of small  $x$ ,  $Gz_D^{-1}$  is also small. Furthermore,  $Gz_D^{-2/3} \ll Gz_D^{-1/3}$ . Noting that  $\tanh(\varepsilon) \rightarrow \varepsilon$  as  $\varepsilon \rightarrow 0$ , we find

$$\begin{aligned} \overline{Nu}_D &= \frac{\frac{3.66}{\tanh\left[2.264 Gz_D^{-1/3} + 1.7 Gz_D^{-2/3}\right]} + 0.0499 Gz_D \tanh(Gz_D^{-1})}{\tanh(2.432 Pr^{1/6} Gz_D^{-1/6})} \\ &\rightarrow \frac{\frac{3.66}{2.264 Gz_D^{-1/3}} + 0.0499 Gz_D Gz_D^{-1}}{2.432 Pr^{1/6} Gz_D^{-1/6}} \rightarrow \frac{3.66}{2.264 \times 2.432 Gz_D^{-1/6} Gz_D^{-1/3} Pr^{1/6}} = 0.665 [Gz_D^{-1}]^{-1/2} Pr^{-1/6} \end{aligned}$$

This is in excellent agreement with Eq. (3).

Continued...

### PROBLEM 8.15 (Cont.)

**COMMENT:** The combined thermal entrance length solution and the boundary layer solution based upon the results of Chapter 7 exhibit asymptotic behavior at small inverse Graetz numbers. Small values of  $Gz_D^{-1}$  correspond to the locations where the boundary layer is very thin.