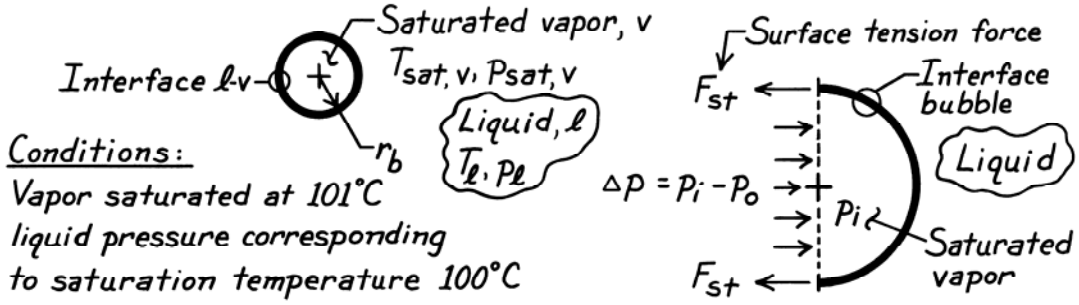
PROBLEM 10.3

KNOWN: Spherical bubble of pure saturated vapor in mechanical and thermal equilibrium with its liquid.

FIND: (a) Expression for the bubble radius, (b) Bubble vapor and liquid states on a p-v diagram; how changes in these conditions cause bubble to collapse or grow, and (c) Bubble size for specified conditions.

SCHEMATIC:



ASSUMPTIONS: (1) Liquid-vapor medium, (2) Thermal and mechanical equilibrium.

PROPERTIES: *Table A-6*, Water (T_{sat} = 101°C = 374.15K): p_{sat} = 1.0502 bar; (T_{sat} = 100°C = 373.15K): p_{sat} = 1.0133 bar, $\sigma = 58.9 \times 10^{-3}$ N/m.

ANALYSIS: (a) For mechanical equilibrium, the difference in pressure between the vapor inside the bubble and the liquid outside the bubble will be offset by the surface tension of the liquid-vapor interface. The force balance follows from the free-body diagram shown above (right),

$$F_{st} = \left(\pi r_b^2\right)\Delta p = \left(p_i - p_o\right)\left(\pi r_b^2\right)$$

$$\left(2\pi r_b\right)\sigma = \left(\pi r_b^2\right)\left(p_i - p_o\right)$$

$$r_b = 2\sigma / \left(p_i - p_o\right) \tag{1}$$

Thermal equilibrium requires that the temperatures of the vapor and liquid be equal. Since the vapor inside the bubble is saturated, $p_i = p_{sat,v}(T)$. Since $p_o < p_i$, it follows that the liquid outside the bubble must be superheated; hence, $p_o = p_\ell(T)$, the pressure of superheated liquid at T. Hence, we can write,

$$r_b = 2\sigma / \left(p_{sat,v} - p_\ell\right) \tag{2}$$ <

(b) The vapor [1] and liquid [2] states are represented on the following p-v diagram. Thermal equilibrium requires both the vapor and liquid to be at the same temperature [3]. But mechanical equilibrium requires that the outside liquid pressure be less than the inside vapor pressure [4]. Hence the liquid must be in a superheated state. That is, its saturation temperature, $T_{sat}(p_o)$ [5] is less than $T_{sat}(p_i)$; $T_\ell = T_{sat}(p_o)$ and $p_o = p_\ell$.

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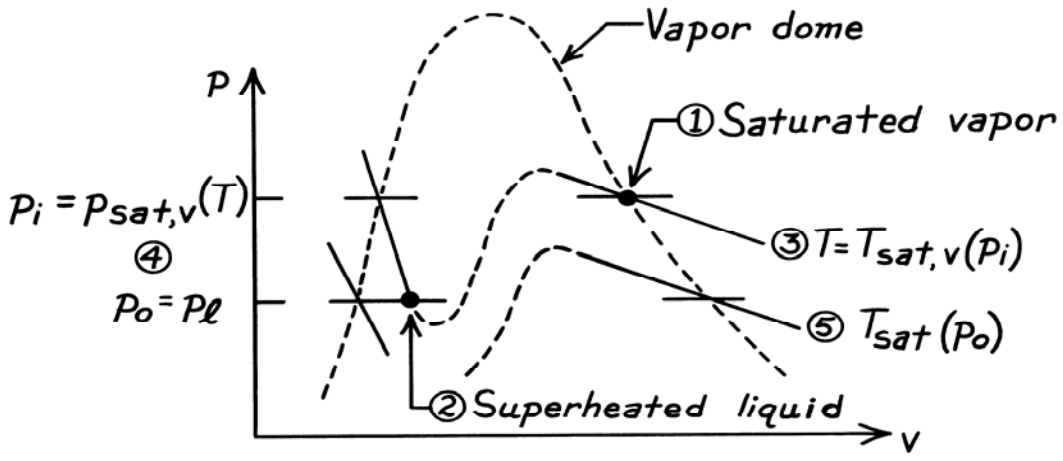
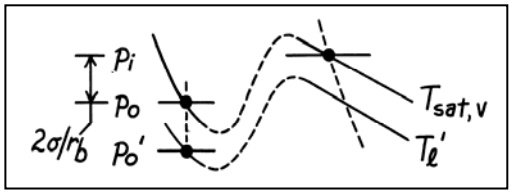


The equilibrium condition for the bubble is unstable. Consider situations for which the pressure of the surrounding liquid is greater or less than the equilibrium value. These situations are presented on portions of the p-v diagram



When $p_o' < p_o$, $T_\ell' < T_{sat,v}$ and heat must be transferred out of the bubble and vapor condenses. Hence, the bubble collapses.

A similar argument for the condition $p_o' > p_o$ leads to $T_\ell' > T_{sat,v}$ and heat is transferred into the bubble causing evaporation with the formation of vapor. Hence, the bubble begins to grow.

(c) Consider the specific conditions

$$T_{sat,v} = 101°C \qquad \text{and} \qquad T_\ell = T_{sat}(p_o) = 100°C$$

and calculate the radius of the bubble using the appropriate properties in Eq. (2).

$$r_b = 2 \times 58.9 \times 10^{-3} \frac{N}{m} / (1.0502 - 1.0133)\,\text{bar} \times \left(10^5 \frac{N}{m^2} / \text{bar}\right)$$

$$r_b = 0.032\,\text{mm}. \qquad <$$

Note the small bubble size. This implies that nucleation sites of the same magnitude formed by pits and crevices are important in promoting the boiling process.