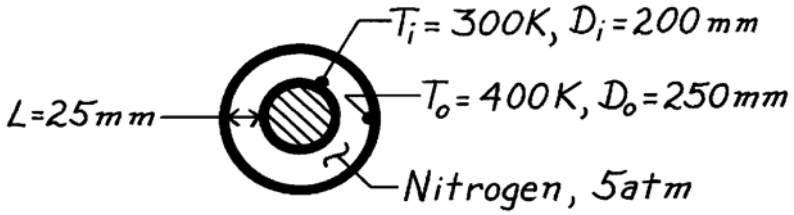
PROBLEM 9.105

KNOWN: Annulus formed by two concentric, horizontal tubes with prescribed diameters and surface temperatures is filled with nitrogen at 5 atm.

FIND: Convective heat transfer rate per unit length of the tubes.

SCHEMATIC:



ASSUMPTIONS: (1) Thermophysical properties k, μ, and Pr, are independent of pressure, (2) Density is proportional to pressure, (3) Perfect gas behavior.

PROPERTIES: *Table A-4*, Nitrogen $\left(\overline{T} = \left(T_i + T_o\right)/2 = 350K,\ 5\ atm\right)$: k = 0.0293 W/m·K, μ = 200×10^{-7} N·s/m^2, ρ(5 atm) = 5 ρ (1 atm) = 5 × 0.9625 kg/m^3 = 4.813 kg/m^3, Pr = 0.711, $\nu = \mu/\rho$ = 4.155×10^{-6} m^2/s, $\alpha = k/\rho c$ = 0.0293 W/m·K/(4.813 kg/m^3 × 1042 J/kg·K) = 5.842×10^{-6} m^2/s.

ANALYSIS: The length scale in Ra_c is given by Eq. 9.60,

$$L_c = \frac{2\left[\ln\left(r_o/r_i\right)\right]^{4/3}}{\left(r_i^{-3/5} + r_o^{-3/5}\right)^{5/3}} = \frac{2\left[\ln\left(125/100\right)\right]^{4/3}}{\left[(0.1\ m)^{-3/5} + (0.125\ m)^{-3/5}\right]^{5/3}} = 0.0095\ m$$

Then

$$Ra_c = \frac{g\beta(T_s - T_\infty)L_c^3}{\nu\alpha} = \frac{9.8\ m/s^2 \times (1/350\ K)\,(400 - 300)K\,(0.0095\ m)^3}{4.155 \times 10^{-6}\ m^2/s \times 5.842 \times 10^{-6}\ m^2/s} = 98{,}800$$

The effective thermal conductivity is found from Eq. 9.59,

$$\frac{k_{eff}}{k} = 0.386\left(\frac{Pr}{0.861 + Pr}\right)^{1/4} Ra_c^{1/4}$$

$$\frac{k_{eff}}{k} = 0.386\left(\frac{0.711}{0.861 + 0.711}\right)^{1/4} \left(98{,}800\right)^{1/4} = 5.61.$$

Hence, the heat rate, Eq. (1), becomes

$$q' = \frac{2\pi \times 5.61 \times 0.0293\ W/m \cdot K}{\ell n\left(125/100\right)}\left(400 - 300\right)K = 463\ W/m. \qquad <$$

COMMENTS: Note that the heat loss by convection is nearly six times that for conduction. Radiation transfer is likely to be important for this situation. The effect of nitrogen pressure is to decrease ν which in turn increases Ra_L; that is, free convection heat transfer will increase with increase in pressure.