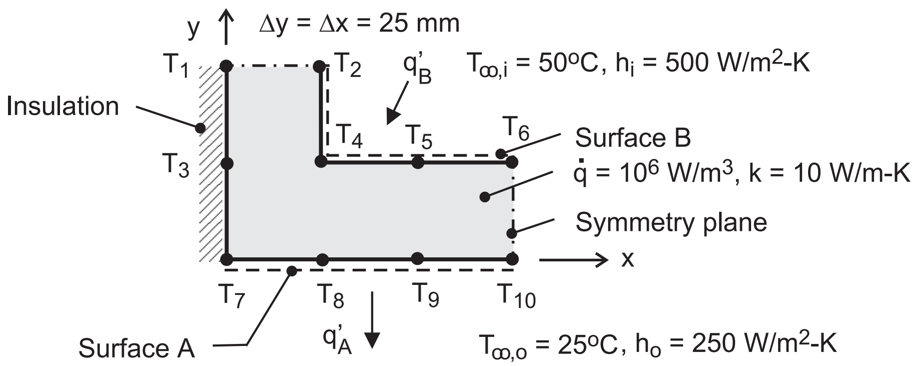
PROBLEM 4.57

KNOWN: Steady-state temperatures at selected nodal points of the symmetrical section of a flow channel with uniform internal volumetric generation of heat. Inner and outer surfaces of channel experience convection.

FIND: (a) Temperatures at nodes 1, 4, 7, and 9, (b) Heat rate per unit length (W/m) from the outer surface A to the adjacent fluid, (c) Heat rate per unit length (W/m) from the inner fluid to surface B, and (d) Verify that results are consistent with an overall energy balance.

SCHEMATIC:



$T_2 = 95.47°C$
$T_3 = 117.3°C$
$T_5 = 79.79°C$
$T_6 = 77.29°C$
$T_8 = 87.28°C$
$T_{10} = 77.65°C$

ASSUMPTIONS: (1) Steady-state, two-dimensional conduction, (2) Constant properties.

ANALYSIS: (a) The nodal finite-difference equations are obtained from energy balances on control volumes about the nodes shown in the schematics below.

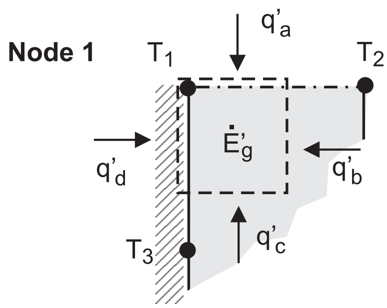
Node 1

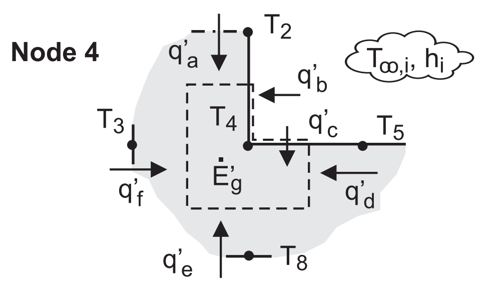
$$q'_a + q'_b + q'_c + q'_d + \dot{E}'_g = 0$$

$$0 + k(\Delta y/2)\frac{T_2 - T_1}{\Delta x} + k(\Delta x/2)\frac{T_3 - T_1}{\Delta y} + 0 + \dot{q}(\Delta x \cdot \Delta y/4) = 0$$

$$T_1 = (T_2 + T_3)/2 + \dot{q}\Delta x^2/4k$$

$$T_1 = (95.47 + 117.3)°C/2 + 10^6\ W/m^3(25\times25)\times10^{-6}\ m^2/(4\times10\ W/m\cdot K) = 122.0°C$$





Node 4

$$q'_a + q'_b + q'_c + q'_d + q'_e + q'_f + \dot{E}'_g = 0$$

$$k(\Delta x/2)\frac{T_2 - T_4}{\Delta y} + h_i(\Delta y/2)(T_{\infty,i} - T_4) + h_i(\Delta x/2)(T_\infty - T_4) +$$

Continued …

$$k(\Delta y/2)\frac{T_5 - T_4}{\Delta x} + k(\Delta x)\frac{T_8 - T_4}{\Delta y} + k(\Delta y)\frac{T_3 - T_4}{\Delta x} + \dot{q}(3\Delta x \cdot \Delta y/4) = 0$$

$$T_4 = \left[T_2 + 2T_3 + T_5 + 2T_8 + 2(h_i \Delta x/k)T_{\infty,i} + \left(3\dot{q}\Delta x^2/2k\right)\right] \Big/ \left[6 + 2(h_i \Delta x/k)\right]$$

$$T_4 = 94.50°C$$ <

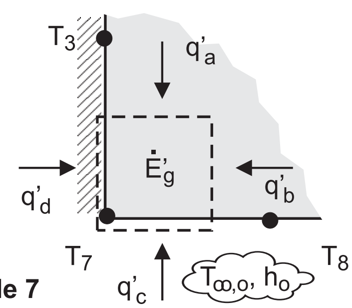
Node 7

$$q'_a + q'_b + q'_c + q'_d + \dot{E}'_g = 0$$

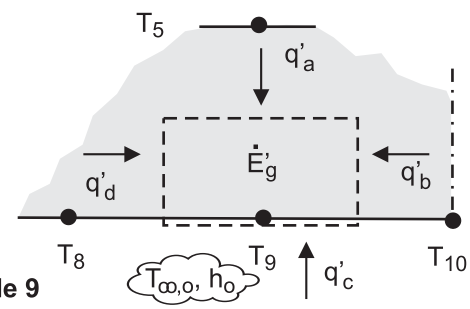
$$k(\Delta x/2)\frac{T_3 - T_7}{\Delta y} + k(\Delta y/2)\frac{T_8 - T_7}{\Delta x} + h_o(\Delta x/2)\left(T_{\infty,o} - T_7\right) + 0 + \dot{q}(\Delta x \cdot \Delta y/4) = 0$$

$$T_7 = \left[T_3 + T_8 + (h_o \Delta x/k)T_{\infty,o} + \dot{q}\Delta x^2/2k\right] \Big/ \left(2 + h_o \Delta x/k\right)$$

$$T_7 = 95.80°C$$ <



Node 7



Node 9

Node 9

$$q'_a + q'_b + q'_c + q'_d + \dot{E}'_g = 0$$

$$k(\Delta x)\frac{T_5 - T_9}{\Delta y} + k(\Delta y/2)\frac{T_{10} - T_9}{\Delta y} + h_o(\Delta x)\left(T_{\infty,o} - T_9\right)$$

$$+k(\Delta y/2)\frac{T_8 - T_9}{\Delta x} + \dot{q}(\Delta x \cdot \Delta y/2) = 0$$

$$T_9 = \left[T_5 + 0.5T_8 + 0.5T_{10} + (h_o \Delta x/k)T_{\infty,o} + \dot{q}\Delta x^2/2k\right] / \left(2 + h_o \Delta x/k\right)$$

$$T_9 = 79.67°C$$ <

(b) The heat rate per unit length from the outer surface A to the adjacent fluid, q'_A, is the sum of the convection heat rates from the outer surfaces of nodes 7, 8, 9 and 10.

$$q'_A = h_o\left[(\Delta x/2)\left(T_7 - T_{\infty,o}\right) + \Delta x\left(T_8 - T_{\infty,o}\right) + \Delta x\left(T_9 - T_{\infty,o}\right) + (\Delta x/2)\left(T_{10} - T_{\infty,o}\right)\right]$$

$$q'_A = 250\ \mathrm{W/m^2 \cdot K}\left[(25/2)(95.80 - 25) + 25(87.28 - 25)\right.$$

$$\left. +25(79.67 - 25) + (25/2)(77.65 - 25)\right] \times 10^{-3}\,\mathrm{m \cdot K}$$

Continued …

PROBLEM 4.57 (Cont.)

$$q'_A = 1117 \text{ W/m}$$ <

(c) The heat rate per unit length from the inner fluid to the surface B, q'_B, is the sum of the convection heat rates from the inner surfaces of nodes 2, 4, 5 and 6.

$$q'_B = h_i\left[(\Delta y/2)(T_{\infty,i} - T_2) + (\Delta y/2 + \Delta x/2)(T_{\infty,i} - T_4) + \Delta x(T_{\infty,i} - T_5) + (\Delta x/2)(T_{\infty,i} - T_6)\right]$$

$$q'_B = 500 \text{ W/m}^2\cdot\text{K}\left[(25/2)(50-95.47) + (25/2+25/2)(50-94.50)\right.$$

$$\left. +25(50-79.79) + (25/2)(50-77.29)\right]\times 10^{-3} \text{ m}\cdot\text{K}$$

$$q'_B = -1383 \text{ W/m}$$ <

(d) From an overall energy balance on the section, we see that our results are consistent since the conservation of energy requirement is satisfied.

$$\dot{E}'_{in} - \dot{E}'_{out} + \dot{E}'_{gen} = -q'_A + q'_B + \dot{E}'_{gen} = (-1117 - 1383 + 2500)\text{W/m} = 0$$

where $\dot{E}'_{gen} = \dot{q}\forall' = 10^6 \text{ W/m}^3[25\times 50 + 25\times 50]\times 10^{-6} \text{ m}^2 = 2500 \text{ W/m}$

COMMENTS: The nodal finite-difference equations for the four nodes can be obtained by using IHT Tool *Finite-Difference Equations* | *Two-Dimensional* | *Steady-state*. Options are provided to build the FDEs for interior, corner and surface nodal arrangements including convection and internal generation. The IHT code lines for the FDEs are shown below.

```
/* Node 1: interior node; e, w, n, s labeled 2, 2, 3, 3. */
0.0   = fd_2d_int(T1,T2,T2,T3,T3,k,qdot,deltax,deltay)

/* Node 4: internal corner node, e-n orientation; e, w, n, s labeled 5, 3, 2, 8. */
0.0   = fd_2d_ic_en(T4,T5,T3,T2,T8,k,qdot,deltax,deltay,Tinfi,hi,q″a4
q″a4 = 0                    // Applied heat flux, W/m^2; zero flux shown

/* Node 7: plane surface node, s-orientation; e, w, n labeled 8, 8, 3. */
0.0   = fd_2d_psur_s(T7,T8,T8,T3,k,qdot,deltax,deltay,Tinfo,ho,q″a7
q″a7=0                      // Applied heat flux, W/m^2; zero flux shown

/* Node 9: plane surface node, s-orientation; e, w, n labeled 10, 8, 5. */
0.0 = fd_2d_psur_s(T9, T10, T8, T5,k,qdot,deltax,deltay,Tinfo,ho,q″a9
q″a9 = 0                    // Applied heat flux, W/m^2; zero flux shown
```