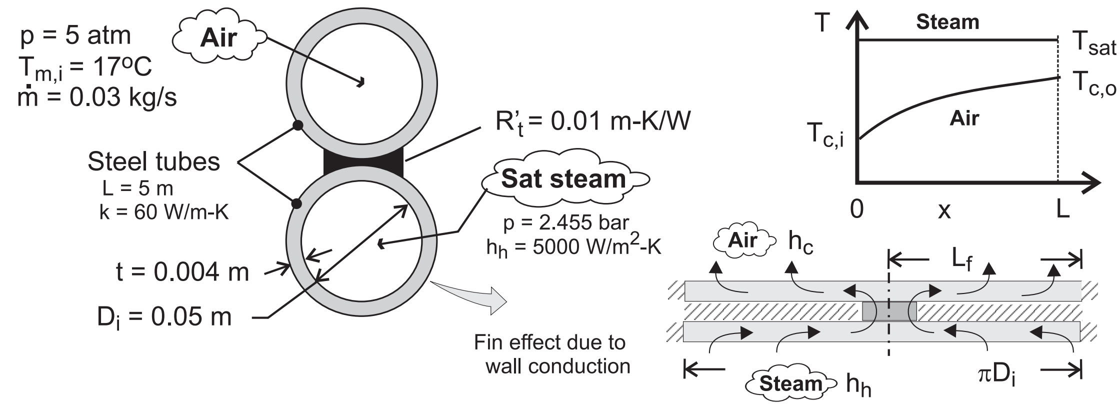
PROBLEM 11.30

KNOWN: Dimensions and thermal conductivity of twin-tube, counterflow heat exchanger. Contact resistance between tubes. Air inlet conditions for one tube and pressure of saturated steam in other tube.

FIND: Air outlet temperature and condensation rate.

SCHEMATIC:



ASSUMPTIONS: (1) Negligible heat exchange with surroundings, (2) Fully developed air flow, (3) Negligible fouling, (4) Constant properties.

PROPERTIES: *Table A-4*, air $\left(\overline{T}_c \approx 325\,K,\ p = 5\,atm\right)$: c_p = 1008 J/kg·K, $\mu = 196.4 \times 10^{-7}$ N·s/m^2, k = 0.0281 W/m·K, Pr = 0.703. *Table A-6*, sat. steam (p = 2.455 bar): $T_{h,i} = T_{h,o}$ = 400 K, h_{fg} = 2183 kJ/kg.

ANALYSIS: With $C_{max} \to \infty$, C_r = 0 and Eqs. 11.21 and 11.35a yield

$$\varepsilon = \frac{T_{c,o} - T_{c,i}}{T_{h,i} - T_{c,i}} = 1 - \exp(-NTU) \qquad (1)$$

From Eq. 11.1,

$$\frac{1}{UA} = \frac{1}{(\eta_o hA)_c} + \frac{R'_t}{L} + \frac{1}{(\eta_o hA)_h} \qquad (2)$$

With $Re_D = 4\dot{m}/\pi D_i \mu = 0.12\ kg/s/\pi(0.05m)196.4\times10^{-7}\,N\cdot s/m^2 = 38{,}900,$ the air flow is turbulent and the Dittus-Boelter correlation yields

$$h_c \approx h_{fD} = \left(\frac{k}{D_i}\right)0.023\ Re_D^{4/5}\,Pr^{0.4} = \left(\frac{0.0281\ W/m\cdot K}{0.05m}\right)0.023(38{,}900)^{4/5}(0.703)^{0.4} = 52.7\ W/m^2\cdot K$$

As shown on the inset, each tube wall may be modelled as two fins, each of length $L_f \approx \pi D_i/2$ = 0.0785m. The total surface area for heat transfer is $A_t = \pi D_i L$ = 0.785 m^2 = A_c, which is equivalent to the surface area of the fins. With $NA_f = A_t$ from Eq. 3.102, $\eta_o = \eta_f$. Because the outer surface of the tube is insulated, a wall thickness of 2t must be used in evaluating η_f. With m = $(2h/k \times 2t)^{1/2}$ = $(h/kt)^{1/2}$ = [52.7 W/m^2·K/(60 W/m·K × 0.004m)]$^{1/2}$ = 14.8 m^{-1}, $L_c = L_f$ for an adiabatic tip, and mL_f = 1.163, Eq. 3.92 yields

$$\eta_f = \frac{\tanh\ mL_f}{mL_f} = \frac{0.821}{1.163} = 0.706 = \eta_{o,c}$$

Continued …

Similarly, for the steam tube, $m = (h/kt)^{1/2} = [5{,}000\ W/m^2\cdot K/(60\ W/m\cdot K \times 0.004m)]^{1/2} = 144.3\ m^{-1}$ and $mL_f = 11.33$. Hence,

$$\eta_f = \frac{\tanh\ mL_f}{mL_f} = \frac{1.00}{11.33} = 0.088 = \eta_{o,h}$$

Substituting into Eq. (2),

$$UA = \left[\frac{1}{0.706 \times 52.7 \times 0.785} + \frac{0.01}{5} + \frac{1}{0.088 \times 5000 \times 0.785}\right]^{-1} \frac{W}{K} = 25.6 \frac{W}{K}$$

Hence, with $C_{min} = \left(\dot{m}\, c_p\right)_c = 0.03\ kg/s \times 1008\ J/kg \cdot K = 30.2\ W/K$, $NTU = UA/C_{min} = 0.847$ and $\varepsilon = 1 - \exp(-NTU) = 0.571$. From Eq. (1), the air outlet temperature is then

$$T_{c,o} = T_{c,i} + \varepsilon\left(T_{h,i} - T_{c,i}\right) = 17°C + 0.571(127 - 17)°C = 79.8°C \qquad <$$

The rate of heat transfer to the air is

$$q = \dot{m}\, c_p\left(T_{c,o} - T_{c,i}\right) = 0.03\,kg/s \times 1008\,J/kg \cdot K \times 62.8°C = 1900\,W$$

and the rate of condensation is

$$\dot{m}_{cond} = \frac{q}{h_{fg}} = \frac{1900\,W}{2.183 \times 10^6\ J/kg} = 8.70 \times 10^{-4}\ kg/s \qquad <$$

COMMENTS: (1) With $\overline{T}_c = 321.4\,K$, the initial estimate of 325K is reasonable and iteration on the property values is not necessary, (2) The major contribution to the total thermal resistance is due to air-side convection, (3) The foregoing results are independent of air pressure.