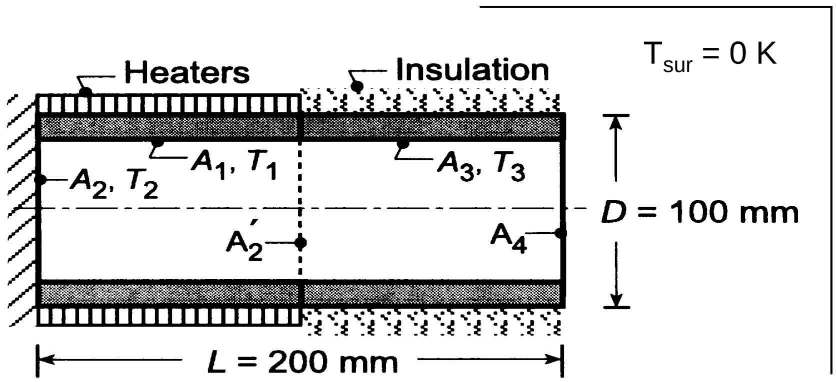
PROBLEM 13.26

KNOWN: Furnace constructed in three sections: insulated circular (2) and cylindrical (3) sections, as well as, an intermediate cylindrical section (1) with imbedded electrical resistance heaters. Cylindrical sections (1,3) are of equal length.

FIND: (a) Electrical power required to maintain the heated section at T_1 = 1000 K if all the surfaces are black, (b) Temperatures of the insulated sections, T_2 and T_3, and (c) Compute and plot q_1, T_2 and T_3 as functions of the length-to-diameter ratio, with $1 \le L/D \le 5$ and D = 100 mm.

SCHEMATIC:



ASSUMPTIONS: (1) All surfaces are black, (2) Areas (1, 2, 3) are isothermal. (3) Uniform surface radiosity and irradiation.

ANALYSIS: (a) To complete the enclosure representing the furnace, define the hypothetical surface A_4 as the opening at 0 K with unity emissivity. For each of the enclosure surfaces 1, 2, and 3, the energy balances following Eq. 13.13 are

$$q_1 = A_1F_{12}\left(E_{b1} - E_{b2}\right) + A_1F_{13}\left(E_{b1} - E_{b3}\right) + A_1F_{14}\left(E_{b1} - E_{b4}\right) \quad (1)$$

$$0 = A_2F_{21}\left(E_{b2} - E_{b1}\right) + A_2F_{23}\left(E_{b2} - E_{b3}\right) + A_2F_{24}\left(E_{b2} - E_{b4}\right) \quad (2)$$

$$0 = A_3F_{31}\left(E_{b3} - E_{b1}\right) + A_3F_{32}\left(E_{b3} - E_{b2}\right) + A_3F_{34}\left(E_{b3} - E_{b4}\right) \quad (3)$$

where the emissive powers are

$$E_{b1} = \sigma T_1^4 \qquad E_{b2} = \sigma T_2^4 \qquad E_{b3} = \sigma T_3^4 \qquad E_{b4} = 0 \quad (4-7)$$

For this four surface enclosure, there are N^2 = 16 view factors and N (N – 1)/2 = 4 × 3/2 = 6 must be directly determined (by inspection or formulas) and the remainder can be evaluated from the summation rule and reciprocity relation. By inspection,

$$F_{22} = 0 \qquad F_{44} = 0 \quad (8,9)$$

From the coaxial parallel disk relation, Table 13.2, find F_{24}

$$S = 1 + \frac{1 + R_4^2}{R_2^2} = 1 + \frac{1 + (0.250)^2}{(0.250)^2} = 18.00$$

$$R_2 = r_2 / L = 0.050\,\text{m} / 0.200\,\text{m} = 0.250 \qquad R_4 = r_4 / L = 0.250$$

$$F_{24} = 0.5\left\{S - \left[S^2 - 4\left(r_4 / r_2\right)^2\right]^{1/2}\right\}$$

$$F_{24} = 0.5\left\{18.00 - \left[18.00^2 - 4(1)^2\right]^{1/2}\right\} = 0.0557 \quad (10)$$

Consider the three-surface enclosure $1 - 2 - 2'$ and find F_{11} as beginning with the summation rule,

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$$F_{11} = 1 - F_{12} - F_{12'} \qquad (11)$$

where, from symmetry, $F_{12} = F_{12'}$, and using reciprocity,

$$F_{12} = A_2 F_{21} / A_1 = \left(\pi D^2 / 4\right) F_{23} / \left(\pi DL / 2\right) = DF_{21} / 2L \qquad (12)$$

and from the summation rule on A_2

$$F_{21} = 1 - F_{22'} = 1 - 0.172 = 0.828\,, \qquad (13)$$

Using the coaxial parallel disk relation, Table 13.2, to find $F_{22'}$,

$$S = 1 + \frac{1 + R_{2'}^2}{R_2^2} = 1 + \frac{1 + 0.50^2}{0.50^2} = 6.000$$

$$R_2 = r_2 / L = 0.050\,\text{m} / \left(0.200 / 2\,\text{m}\right) = 0.500 \qquad R_{2'} = 0.500$$

$$F_{22'} = 0.5 \left\{ S - \left[S^2 - 4\left(r_{2'} / r_2\right)^2 \right]^{1/2} \right\}$$

$$F_{22'} = 0.5 \left\{ 6 - \left[6^2 - 4\left(1\right)^2 \right]^{1/2} \right\} = 0.1716$$

Evaluating F_{12} from Eq. (12), find

$$F_{12} = 0.100\ \text{m} \times 0.828 / 2 \times 0.200\ \text{m} = 0.2071$$

and evaluating F_{11} from Eq. (11), find

$$F_{11} = 1 - 2 \times F_{12} = 1 - 2 \times 0.207 = 0.586$$

From symmetry, recognize that $F_{33} = F_{11}$ and $F_{43} = F_{21}$. To this point we have directly determined six view factors (underlined in the matrix below) and the remaining F_{ij} can be evaluated from the summation rules and appropriate reciprocity relations. The view factors written in matrix form, $[F_{ij}]$ are.

$\underline{0.5858}$	$\underline{0.2071}$	0.1781	0.02896
$\underline{0.8284}$	$\underline{0}$	0.1158	$\underline{0.05573}$
0.1781	0.028960.5858	0.2071	
0.1158	0.055730.8284	$\underline{0}$	

Knowing all the required view factors, the energy balances and the emissive powers, Eqs. (4-6), can be solved simultaneously to obtain:

$$q_1 = 255\,\text{W} \qquad E_{b2} = 5.02 \times 10^4\,\text{W/m}^2 \qquad E_{b3} = 2.79 \times 10^4\,\text{W/m}^2$$ <

$$T_2 = 970\,\text{K} \qquad T_3 = 837.5\,\text{K}$$ <

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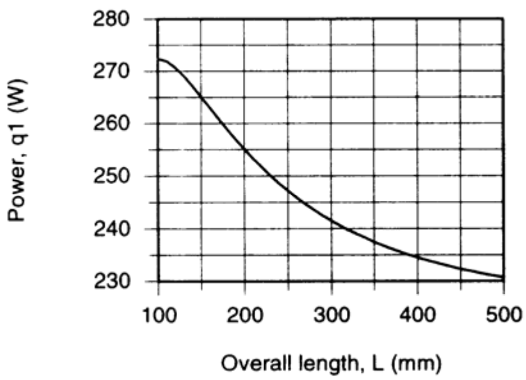
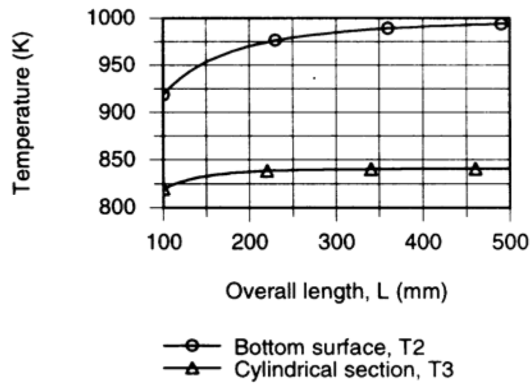
PROBLEM 13.26 (Cont.)

(b) Using the energy balances, Eqs. (1-3), along with the *IHT Radiation Tool, View Factors, Coaxial parallel disks*, a model was developed to calculate q_1, T_2, and T_3 as a function of length L for fixed diameter D = 100 m. The results are plotted below.





For fixed diameter, as the overall length increases, the power required to maintain the heated section at T_1 = 1000 K decreases. This follows since the furnace opening area is a smaller fraction of the enclosure surface area as L increases. As L increases, the bottom surface temperature T_2 increases as L increases and, in the limit, will approach that of the heated section, T_1 = 1000 K. As L increases, the temperature of the insulated cylindrical section, T_3, increases, but only slightly. The limiting value occurs when $E_{b3} = 0.5 \times E_{b1}$ for which $T_3 \rightarrow 840$ K. Why is that so?

COMMENT: If the electrical heating is uniformly distributed, the temperature of the heated section will not be uniform. In practice, a more detailed analysis involving more radiation surfaces might be warranted.