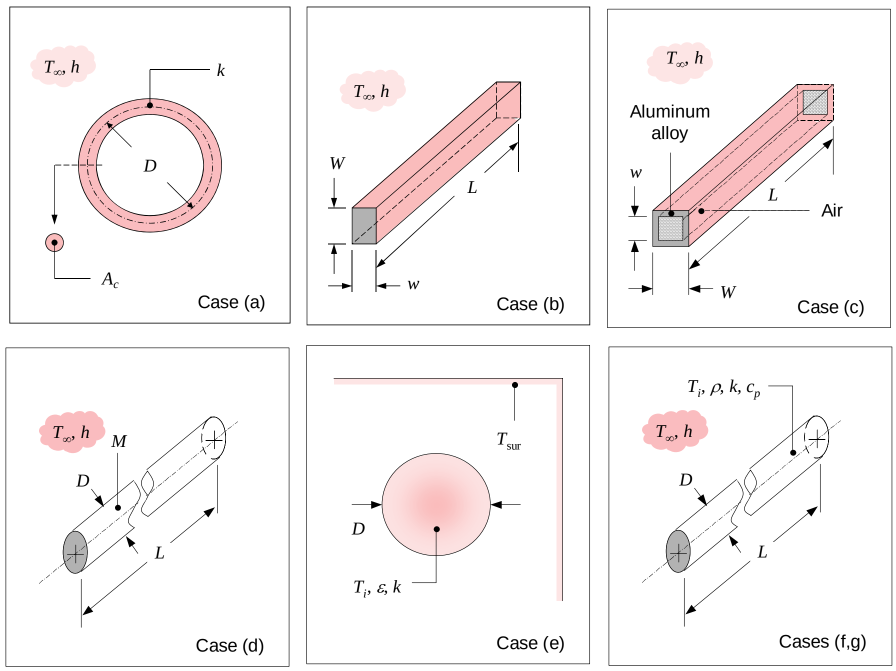
PROBLEM 5.5

KNOWN: Geometries of various objects. Material and/or properties. Cases (a) through (d): Convection heat transfer coefficient between object and surrounding fluid. Case (e): Emissivity of sphere, initial temperature, and temperature of surroundings. Cases (f) and (g): Initial temperature, spatially averaged temperature at a later time, and surrounding fluid temperature.

FIND: Characteristic length and Biot number. Validity of lumped capacitance approximation.

SCHEMATIC:



Case (a): $D = 65$ mm, $A_c = 7$ mm^2, $k = 2.3$ W/m·K, $h = 50$ W/m^2·K.
Case (b): W = 7 mm, w = 5 mm, L = 150 mm, h = 10 W/m^2·K, AISI 302 stainless steel.
Case (c): $w = 25$ mm, $W = 30$ mm, $h = 40$ W/m^2·K (L not specified), 2024 aluminum.
Case (d): $L = 300$ mm, $D = 13$ mm, $M = 0.328$ kg, $h = 30$ W/m^2·K. stainless steel.
Case (e): $D = 13$ mm, $k = 130$ W/m·K, $T_{sur} = 18$°C, $T_i = 100$°C, $\varepsilon = 0.75$.
Cases (f,g): $D = 20$ mm or 200 mm, $\rho = 2300$ kg/m^3, $c_p = 1750$ J/kg·K, $k = 16$ W/m·K, $T_\infty = 20$°C, $T_i =$ 200°C, $T = 100$°C at $t = 225$ s.

ASSUMPTIONS: (1) Constant properties, (2) In case (e), radiation is to large surroundings.

PROPERTIES: *Table A.1*, Stainless steel, AISI 302 ($T = 300$ K): $k = 15.1$ W/m·K. Aluminum 2024 ($T = 300$ K): $k = 177$ W/m·K.

ANALYSIS: Characteristic lengths can be calculated as $L_{c1} = V/A_s$, or they can be taken conservatively as the dimension corresponding to the maximum spatial temperature difference, L_{c2}. The former definition is more convenient for complex geometries. The lumped capacitance approximation is valid for $Bi = hL_c/k < 0.1$.

(a) The radius of the torus, r_o, can be found from $A_c = \pi r_o^2$. The characteristic lengths are

$$L_{c1} = \frac{V}{A_s} = \frac{A_c \pi D}{2\pi\sqrt{A_c/\pi} \times \pi D} = \frac{1}{2}\sqrt{\frac{A_c}{\pi}} = \frac{1}{2}\sqrt{\frac{7\text{ mm}^2}{\pi}} = 0.75\text{ mm} \qquad <$$

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L_{c2} = Maximum center to surface distance = radius $= \sqrt{A_c / \pi} = \sqrt{7 \text{ mm}^2 / \pi} = 1.49 \text{ mm}$ <

The corresponding Biot numbers are

$$Bi_1 = \frac{hL_{c1}}{k} = \frac{50 \text{ W/m}^2 \cdot \text{K} \times 0.00075 \text{ m}}{2.3 \text{ W/m} \cdot \text{K}} = 0.016$$ <

$$Bi_2 = \frac{hL_{c2}}{k} = \frac{50 \text{ W/m}^2 \cdot \text{K} \times 0.00149 \text{ m}}{2.3 \text{ W/m} \cdot \text{K}} = 0.032$$ <

The lumped capacitance approximation is valid according to either definition. <

(b) For this complex shape, we will calculate only L_{c1}.

$$L_{c1} = \frac{V}{A_s} = \frac{WwL}{2(W+w)L + 2Ww} = \frac{7 \text{ mm} \times 5 \text{ mm} \times 150 \text{ mm}}{2(7 \text{ mm} + 5 \text{ mm}) \times 150 \text{ mm} + 14 \text{ mm} \times 5 \text{ mm}} = 1.43 \text{ mm}$$ <

Notice that the surface area of the ends has been included, and does have a small effect on the result – 1.43 mm versus 1.46 mm if the ends are neglected. The corresponding Biot number is

$$Bi_1 = \frac{hL_{c1}}{k} = \frac{10 \text{ W/m}^2 \cdot \text{K} \times 0.00143 \text{ m}}{15.1 \text{ W/m} \cdot \text{K}} = 0.00095$$ <

The lumped capacitance approximation is valid. <
Furthermore, since the Biot number is very small, the lumped capacitance approximation would certainly still be valid using a more conservative length estimate.

(c) Again, we will only calculate L_{c1}. There will be very little heat transfer to the stagnant air inside the tube, therefore in determining the surface area for convection heat transfer, A_s, only the outer surface area should be included. Thus,

$$L_{c1} = \frac{V}{A_s} = \frac{(W^2 - w^2)L}{4WL} = \frac{(30 \text{ mm})^2 - (25 \text{ mm})^2}{4 \times 30 \text{ mm}} = 2.29 \text{ mm}$$ <

The corresponding Biot number is

$$Bi_1 = \frac{hL_{c1}}{k} = \frac{40 \text{ W/m}^2 \cdot \text{K} \times 0.00229 \text{ m}}{177 \text{ W/m} \cdot \text{K}} = 5.18 \times 10^{-4}$$ <

The lumped capacitance approximation is valid. <
Furthermore, since the Biot number is very small, the lumped capacitance approximation would certainly still be valid using a more conservative length estimate.

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(d) We are not told which type of stainless steel this is, but we are told its mass, from which we can find its density:

$$\rho = \frac{M}{V} = \frac{M}{\pi D^2 L/4} = \frac{0.328\text{ kg}}{\pi(0.013\text{ m})^2 \times 0.3\text{ m}/4} = 8237\text{ kg/m}^3$$

This appears to be AISI 316 stainless steel, with a thermal conductivity of k = 13.4 W/m·K at T = 300 K.

The characteristic lengths are

$$L_{c1} = \frac{V}{A_s} = \frac{\pi D^2 L/4}{\pi DL + 2\pi D^2/4} = \frac{DL/4}{L + D/2} = \frac{13\text{ mm} \times 300\text{ mm}/4}{300\text{ mm} + 13\text{ mm}/2} = 3.18\text{ mm} \qquad <$$

$$L_{c2} = \text{Maximum center to surface distance} = D/2 = 6.5\text{ mm} \qquad <$$

Notice that the surface area of the ends has been included in L_{c1}, and does have a small effect on the result. The corresponding Biot numbers are

$$Bi_1 = \frac{hL_{c1}}{k} = \frac{15\text{ W/m}^2\cdot\text{K} \times 0.00318\text{ m}}{13.4\text{ W/m}\cdot\text{K}} = 0.0036 \qquad <$$

$$Bi_2 = \frac{hL_{c2}}{k} = \frac{15\text{ W/m}^2\cdot\text{K} \times 0.0065\text{ m}}{13.4\text{ W/m}\cdot\text{K}} = 0.0073 \qquad <$$

The lumped capacitance approximation is valid according to either definition. $<$

(e) The characteristic lengths are

$$L_{c1} = \frac{V}{A_s} = \frac{\pi D^3/6}{\pi D^2} = \frac{D}{6} = 2.17\text{ mm} \qquad <$$

$$L_{c2} = \text{Maximum center to surface distance} = D/2 = 6.5\text{ mm} \qquad <$$

Since heat transfer from the sphere is by radiation, we will calculate the effective radiation heat transfer coefficient (Equation 1.9):

$$\begin{aligned} h_r &= \varepsilon\sigma(T_s + T_{\text{sur}})(T_s^2 + T_{\text{sur}}^2) \\ &= 0.75 \times 5.67 \times 10^{-8}\text{ W/m}^2\cdot\text{K}^4[373\text{ K} + 291\text{ K}][(373\text{ K})^2 + (291\text{ K})^2] = 6.32\text{ W/m}^2\cdot\text{K} \end{aligned}$$

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The surface temperature has been taken as the initial value, to give the largest possible heat transfer coefficient. The Biot numbers are

$$Bi_1 = \frac{h_r L_{c1}}{k} = \frac{6.32\ \mathrm{W/m^2 \cdot K} \times 0.00217\ \mathrm{m}}{130\ \mathrm{W/m \cdot K}} = 1.05 \times 10^{-4}$$ <

$$Bi_2 = \frac{h_r L_{c2}}{k} = \frac{6.32\ \mathrm{W/m^2 \cdot K} \times 0.00632\ \mathrm{m}}{130\ \mathrm{W/m \cdot K}} = 3.16 \times 10^{-4}$$ <

The lumped capacitance approximation is valid according to either definition. <

(f) The characteristic lengths are

$$L_{c1} = \frac{V}{A_s} = \frac{\pi D^2 L / 4}{\pi D L} = \frac{D}{4} = 5\ \mathrm{mm}$$ <

$$L_{c2} = \text{Maximum center to surface distance} = D/2 = 10\ \mathrm{mm}$$ <

We are not told the convection heat transfer coefficient, but we do know the fluid temperature and the temperature of the rod initially and at t = 225 s. If we assume that the lumped capacitance approximation is valid, we can determine the heat transfer coefficient from Equation 5.5:

$$h = \frac{\rho V c}{A_s t} \ln\left(\frac{\theta_i}{\theta}\right) = \frac{\rho D c}{4t} \ln\left(\frac{T_i - T_\infty}{T - T_\infty}\right)$$
$$= \frac{2300\ \mathrm{kg/m^3} \times 0.020\ \mathrm{m} \times 1750\ \mathrm{J/kg \cdot K}}{4 \times 225\ \mathrm{s}} \ln\left(\frac{200°\mathrm{C} - 20°\mathrm{C}}{100°\mathrm{C} - 20°\mathrm{C}}\right) = 72.5\ \mathrm{W/m^2 \cdot K}$$

The resulting Biot numbers are:

$$Bi_1 = \frac{h L_{c1}}{k} = \frac{72.5\ \mathrm{W/m^2 \cdot K} \times 0.005\ \mathrm{m}}{16\ \mathrm{W/m \cdot K}} = 0.023$$ <

$$Bi_2 = \frac{h L_{c2}}{k} = \frac{72.5\ \mathrm{W/m^2 \cdot K} \times 0.01\ \mathrm{m}}{16\ \mathrm{W/m \cdot K}} = 0.045$$ <

The lumped capacitance approximation is valid according to either definition. <
This also means that it was appropriate to use the lumped capacitance approximation to calculate h.

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(g) With the diameter increased by a factor of ten, so are the characteristic lengths:

$$L_{c1} = \frac{V}{A_s} = \frac{\pi D^2 L/4}{\pi D L} = \frac{D}{4} = 50 \text{ mm}$$ <

$$L_{c2} = \text{Maximum center to surface distance} = D/2 = 100 \text{ mm}$$ <

Once again, we assume that the lumped capacitance approximation is valid to calculate the heat transfer coefficient according to

$$h = \frac{\rho V c}{A_s t} \ln\left(\frac{\theta_i}{\theta}\right) = \frac{\rho D c}{4t} \ln\left(\frac{T_i - T_\infty}{T - T_\infty}\right)$$
$$= \frac{2300 \text{ kg/m}^3 \times 0.20 \text{ m} \times 1750 \text{ J/kg}\cdot\text{K}}{4 \times 225 \text{ s}} \ln\left(\frac{200°\text{C} - 20°\text{C}}{100°\text{C} - 20°\text{C}}\right) = 725 \text{ W/m}^2\cdot\text{K}$$

The resulting Biot numbers are:

$$Bi_1 = \frac{hL_{c1}}{k} = \frac{725 \text{ W/m}^2\cdot\text{K} \times 0.05 \text{ m}}{16 \text{ W/m}\cdot\text{K}} = 2.3$$ <

$$Bi_2 = \frac{hL_{c2}}{k} = \frac{725 \text{ W/m}^2\cdot\text{K} \times 0.1 \text{ m}}{16 \text{ W/m}\cdot\text{K}} = 4.5$$ <

The lumped capacitance approximation is *not* valid according to either definition. <
This means that the calculated value of *h* is incorrect, therefore the above values of the Biot number are incorrect. However, we can still conclude that the *Bi* number is too large for lumped capacitance to be valid by the following reasoning. If the lumped capacitance approximation were valid, then the calculated *h* would be correct, and its value would be small enough to result in $Bi < 0.1$. Since the calculated Biot number does not satisfy the criterion to use the lumped capacitance approximation, the initial assumption that the lumped capacitance method is valid must have been false.

COMMENTS: (1) The determination of whether or not the lumped capacitance approximation can be used is, to some degree, dependent on how much precision is required in a given application. If the Biot number is close to 0.1 and good precision is required, the spatial variation of the temperature should be accounted for. If the geometry is simple, the analytical solutions presented in the text may be appropriate. For complex geometries, a numerical solution is often required, using the finite difference or finite element method. (2) In Case (d), the type of stainless steel is inferred from knowledge of its density. The variation of *k* among the four stainless steels listed in Table A.1 is on the order of 10%. If the object temperature varies significantly with time, the thermal conductivity may vary by more than 10% as a result. In that case, evaluating *k* at an appropriate average temperature is at least as important as distinguishing the type of stainless steel.