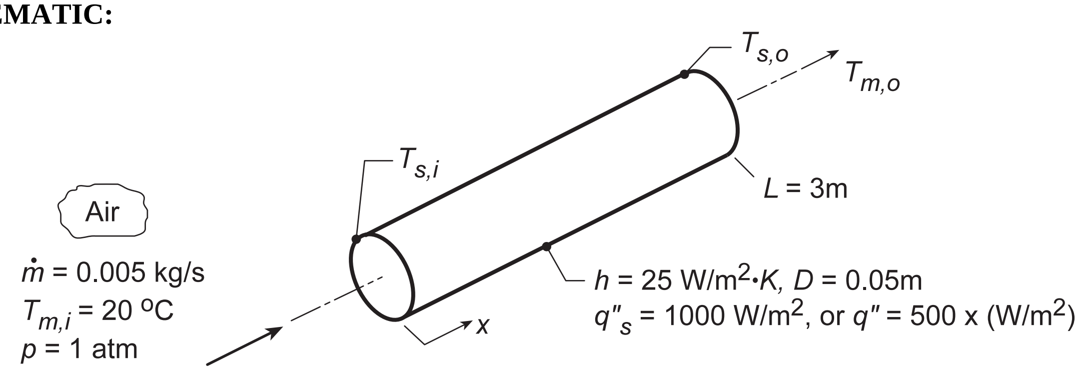
PROBLEM 8.18

KNOWN: Air inlet conditions and heat transfer coefficient for a circular tube of prescribed geometry. Surface heat flux.

FIND: (a) Tube heat transfer rate, q, air outlet temperature, $T_{m,o}$, and surface inlet and outlet temperatures, $T_{s,i}$ and $T_{s,o}$, for a uniform surface heat flux, q_s''. Air mean and surface temperature distributions. (b) Values of q, $T_{m,o}$, $T_{s,i}$ and $T_{s,o}$ for a linearly varying surface heat flux q_s'' = 500x (m). Air mean and surface temperature distributions, (c) For each type of heating process (a & b), compute and plot the mean fluid and surface temperatures, $T_m(x)$ and $T_s(x)$, respectively, as a function of distance; What is effect of four-fold increase in convection coefficient, and (d) For each type of heating process, heat fluxes required to achieve an outlet temperature of $T_{m,o}$ = 125°C; Plot temperatures.

SCHEMATIC:



ASSUMPTIONS: (1) Fully developed conditions in the tube, (2) Applicability of Eq. 8.34, (3) Heat transfer coefficient is the same for both heating conditions.

PROPERTIES: *Table A.4*, Air (for an *assumed* value of $T_{m,o}$ = 100°C, $\overline{T}_m$ = ($T_{m,i}$ + $T_{m,o}$)/2 = 60°C = 333 K): c_p = 1.008 kJ/kg·K.

ANALYSIS: (a) With constant heat flux, from Eq. 8.38,

$$q = q_s''(\pi DL) = 1000\,\mathrm{W/m^2}\,(\pi \times 0.05\,\mathrm{m} \times 3\,\mathrm{m}) = 471\,\mathrm{W}\,. \qquad (1)$$

From the overall energy balance, Eq. 8.34,

$$T_{m,o} = T_{m,i} + \frac{q}{\dot{m}c_p} = 20^\circ\mathrm{C} + \frac{471\,\mathrm{W}}{0.005\,\mathrm{kg/s} \times 1008\,\mathrm{J/kg \cdot K}} = 113.5^\circ\mathrm{C} \qquad (2) \quad <$$

From the convection rate equation, it follows that

$$T_{s,i} = T_{m,i} + \frac{q_s''}{h} = 20^\circ\mathrm{C} + \frac{1000\,\mathrm{W/m^2}}{25\,\mathrm{W/m^2 \cdot K}} = 60^\circ\mathrm{C} \qquad (3) \quad <$$

$$T_{s,o} = T_{m,o} + q_s''/h = 113.5^\circ\mathrm{C} + 40^\circ\mathrm{C} = 153.5^\circ\mathrm{C} \qquad <$$

From Eq. 8.39, (dT_m/dx) is a constant, as is (dT_s/dx) for constant h from Eq. 8.30. In the more realistic case for which h decreases with x in the entry region, (dT_m/dx) is still constant but (dT_s/dx) decreases with increasing x. See the plot below.

(b) From Eq. 8.37,

$$\frac{dT_m}{dx} = \frac{500x(\pi D)}{\dot{m}c_p} = \frac{500x\,\mathrm{W/m^2}\,(\pi \times 0.05\,\mathrm{m})}{0.005\,\mathrm{kg/s} \times 1008\,\mathrm{J/kg \cdot K}} = 15.6x\,\mathrm{K/m}\,. \qquad (4)$$

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Integrating from x = 0 to L it follows that

$$T_{m,o} = T_{m,i} + 15.6\int_0^3 x\,dx = 20^\circ C + 15.6 \left.\frac{x^2}{2}\right|_0^3 = 20^\circ C + 70.2^\circ C = 90.2^\circ C. \qquad (5) <$$

The heat rate is

$$q = \int q_s'' dA_s = 500(\pi \times 0.05\,m)\int_0^3 x\,dx = 78.5\left.\frac{x^2}{2}\right|_0^3 = 353\,W \qquad <$$

From Eq. 8.27 it then follows that

$$T_s = T_m + q_s''/h = T_{m,i} + 15.6\frac{x^2}{2} + \frac{500}{25}x = 20^\circ C + 7.8x^2 + 20x \qquad (6)$$

Hence, at the inlet (x = 0) and outlet (x = L),

$$T_{s,i} = T_{m,i} = 20^\circ C \qquad \text{and} \qquad T_{s,o} = 150.2^\circ C \qquad <$$

Note that (dT_s/dx) and (dT_m/dx) both increase linearly with x, but $(dT_s/dx) > (dT_m/dx)$.

(c) The foregoing relations can be used to determine $T_m(x)$ and $T_s(x)$ for the two heating conditions:

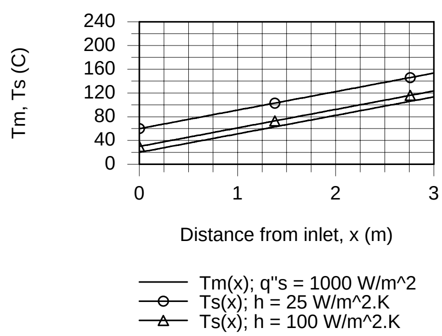
Uniform surface flux, q_s''; Eqs. (1-3),

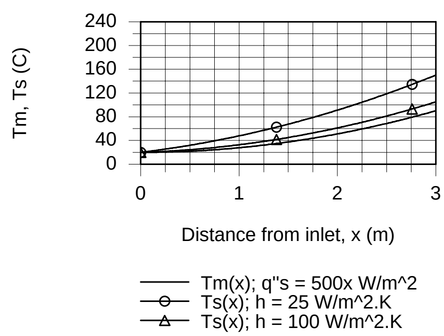
$$T_m(x) = T_{m,i} + q_s''\pi Dx/\dot{m}c_p \qquad T_s(x) = T_m(x) + q_s''/h \qquad (7,8)$$

Linear surface heat flux, $q_s'' = a_o x$, $a_o = 500\ W/m^3$; Eqs. (4-6),

$$T_m(x) = T_{m,i} + \left(a_o\pi D/2\dot{m}c_p\right)x^2 \qquad T_s(x) = T_m(x) + a_o x/h \qquad (9, 10)$$

Using Eqs. (7-10) in IHT, the mean fluid and surface temperatures as a function of distance are evaluated and plotted below. The calculations were repeated with the coefficient increased four-fold, $h = 4 \times 25 = 100\ W/m^2\cdot K$. As expected, the fluid temperature remained unchanged, but the surface temperatures decreased since the thermal resistance between the surface and fluid decreased.





(d) The foregoing set of equations, Eqs. (7-10), in the IHT model can be used to determine the required heat fluxes for the two heating conditions to achieve $T_{m,o} = 125°C$. The results with $h = 25\ W/m^2\cdot K$ are:

Uniform flux: $q_s'' = 1123\ W/m^2$ *Linear flux:* $q_s'' = 748.7x\ W/m^2$ <

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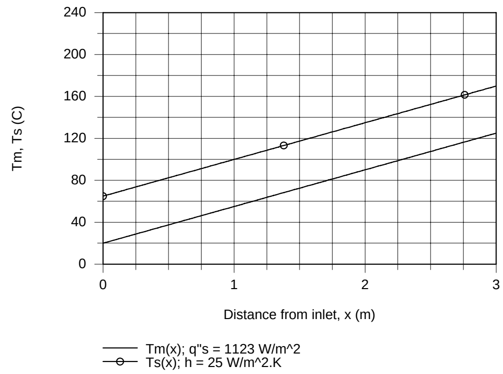
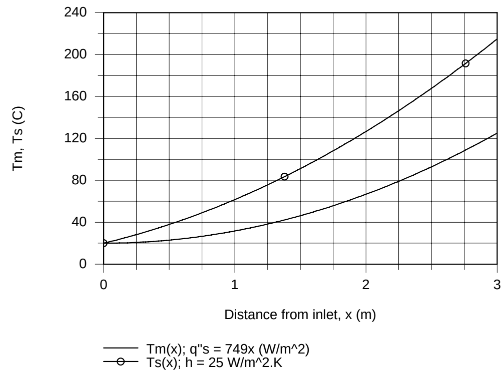
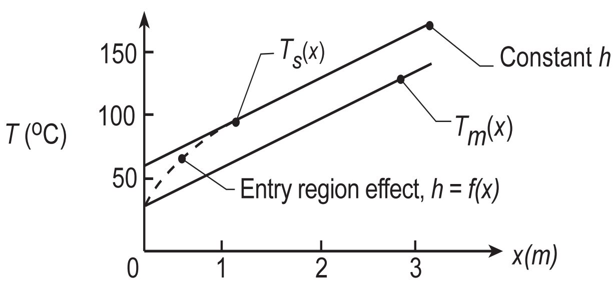
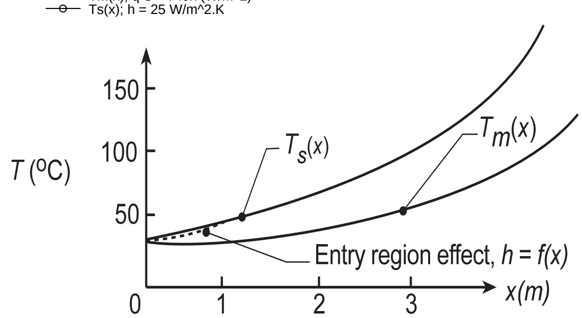
PROBLEM 8.18 (Cont.)

The temperature distributions resulting from these heat fluxes are plotted below. The heat rate for both heating processes is 529 W.









COMMENTS: Note that the assumed value for $T_{m,o}$ (100°C) in determining the specific heat of the air was reasonable.