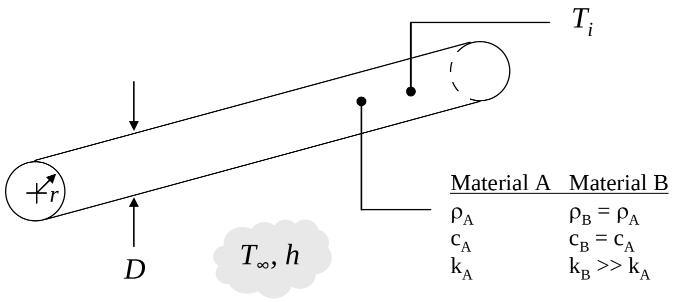
## PROBLEM 2.64

**KNOWN:** Long cylindrical rod with uniform initial temperature immersed in liquid at a lower temperature.

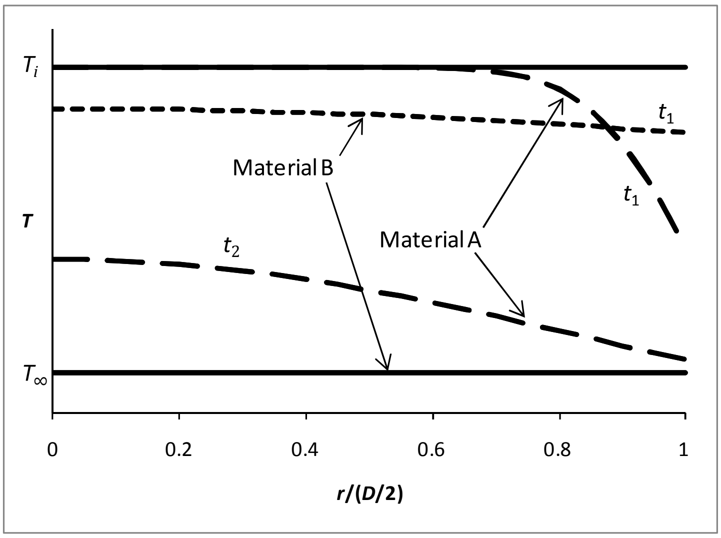
**FIND:** Sketch temperature distribution at initial time, steady state, and two intermediate times for two rods with different thermal conductivities. State boundary conditions at centerline and surface.

**SCHEMATIC**:




**ASSUMPTIONS**: (1) One-dimensional conduction in radial direction, (2) Constant properties, (3) Fluid temperature remains constant, (4) Convection heat transfer coefficient is constant.

**ANALYSIS:** Referring to the figure below, first consider Material A of moderate thermal conductivity. Initially, the rod temperature is uniform at $T_i$. When the rod is first exposed to the liquid, heat is transferred from the rod to the fluid due to convection, causing the surface temperature to decrease. The resulting temperature gradient in the rod causes heat to conduct radially outward, and the temperature further inside the rod decreases as well. Toward the beginning of this process, the temperature near the center of the rod is still very close to the initial temperature (see Material A, $t_1$). As time increases, the temperature everywhere in the rod decreases (see Material A, $t_2$). Eventually, at steady state, the rod temperature reaches the fluid temperature, $T_\infty$.

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The boundary condition at the rod surface expresses a balance between heat reaching the surface by conduction and heat leaving the surface by convection:

$$-k\frac{\partial T}{\partial r}\bigg|_{D/2} = h\left[T(D/2,t) - T_\infty\right] \qquad (1)$$ $<$

From this, it can be seen that the temperature gradient at the surface is negative and its magnitude decreases with time as the surface temperature approaches the fluid temperature. This is shown for the two intermediate times for Material A.

Next compare Material A to Material B having a very large thermal conductivity. At time $t$ = 0 when both rods have the same temperature $T_i$, it can be seen from the right hand side of Equation (1) that the heat flux is the same for both materials. Energy is being removed from both rods at the same rate. However, because of the large thermal conductivity of material B, its temperature gradient is smaller and its temperature tends to be nearly uniform, as shown in the figure for Material B, $t_1$. Its temperature is higher at the surface and lower in the center as compared to Material A. Because its surface temperature stays higher for longer, the heat flux leaving the rod is larger, and overall it cools faster. At time $t_2$, when Material A's surface temperature is close to $T_\infty$, but it is still warm in the center, Material B has already reached steady state.

The rod with the higher thermal conductivity reaches steady state sooner. $<$

The boundary condition at $r$ = 0 expresses radial symmetry:

$$\frac{\partial T}{\partial r}\bigg|_{0} = 0$$ $<$

The boundary condition at $r$ =$D$/2 was given in Equation (1).

**COMMENTS:** The problem of transient conduction in a cylinder will be solved in Chapter 5.