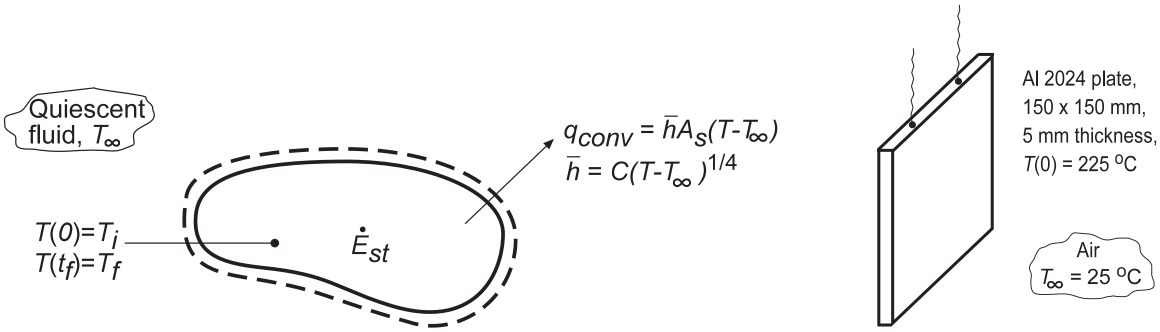
PROBLEM 9.12

KNOWN: Temperature dependence of free convection coefficient, $\bar{h} = C\Delta T^{1/4}$, for a solid suddenly submerged in a quiescent fluid.

FIND: (a) Expression for cooling time, t_f, (b) Considering a plate of prescribed geometry and thermal conditions, the time required to reach 80°C using the appropriate correlation from Problem 9.11 and (c) Plot the temperature-time history obtained from part (b) and compare with results using a constant $\bar{h}_o$ from an appropriate correlation based upon an average surface temperature $\bar{T} = (T_i + T_f)/2$.

SCHEMATIC:



ASSUMPTIONS: (1) Lumped capacitance approximation is valid, (2) Negligible radiation, (3) Constant properties.

PROPERTIES: *Table A.1*, Aluminum alloy 2024 $\left(\bar{T} = (T_i + T_f)/2 \approx 400\,K\right)$: ρ = 2770 kg/m^3, c_p = 925 J/kg·K, k = 186 W/m·K; *Table A.4*, Air ($\bar{T}_{film}$ = 362 K): $\nu = 2.221 \times 10^{-5}$ m^2/s, k = 0.03069 W/m·K, $\alpha = 3.187 \times 10^{-5}$ m^2/s, Pr = 0.6976, $\beta = 1/\bar{T}_{film}$.

ANALYSIS: (a) From Eq. 5.28,

$$\frac{\theta}{\theta_i} = \left[\frac{nCA_s\theta_i^n}{\rho Vc}t + 1\right]^{-1/n} \qquad (1)$$

where $\theta = T - T_\infty$ and n = 1/4. Solving for t_f, the time at which $T = T_f$,

$$t_f = \frac{4\rho Vc}{CA_s(T_i - T_\infty)^{1/4}}\left[\left(\frac{T_i - T_\infty}{T_f - T_\infty}\right)^{1/4} - 1\right] \qquad (2)$$ <

(b) Considering the aluminum plate, initially at T(0) = 225°C, and suddenly exposed to ambient air at $T_\infty = 25°C$, from Problem 9.11 the convection coefficient has the form

$$\bar{h}_i = 1.40\left(\frac{\Delta t}{L}\right)^{1/4} \qquad\qquad \bar{h}_i = C\Delta T^{1/4}$$

where C = $1.40/L^{1/4} = 1.40/(0.150)^{1/4}$ = 2. 2496 $W/m^2 \cdot K^{3/4}$. Using Eq. (2), find

Continued...

PROBLEM 9.12 (Cont.)

$$t_f = \frac{4\times 2770\,\mathrm{kg/m^3}\left(0.150^2\times 0.005\right)\mathrm{m^3}\times 925\,\mathrm{J/kg\cdot K}}{2.2496\,\mathrm{W/m^2\cdot K^{3/4}}\times 2\times\left(0.150\mathrm{m}\right)^2\left(225-25\right)^{1/4}\mathrm{K}^{1/4}}\left[\left(\frac{225-25}{80-25}\right)^{1/4}-1\right] = 1154\mathrm{s}$$

(c) For the vertical plate, Eq. 9.27 is an appropriate correlation. Evaluating properties at

$$\overline{T}_{film} = \left(\overline{T}_s + T_\infty\right)/2 = \left(\left(T_i + T_f\right)/2 + T_\infty\right)/2 = 362\ \mathrm{K}$$

where $\overline{T}_s = 426\mathrm{K}$, the average plate temperature, find

$$Ra_L = \frac{g\beta\left(\overline{T}_s - T_\infty\right)L^3}{\nu\alpha} = \frac{9.8\,\mathrm{m/s^2}\left(1/362\mathrm{K}\right)\left(426-298\right)\mathrm{K}\left(0.150\mathrm{m}\right)^3}{2.221\times 10^{-5}\,\mathrm{m^2/s}\times 3.187\times 10^{-5}\,\mathrm{m^2/s}} = 1.652\times 10^7$$

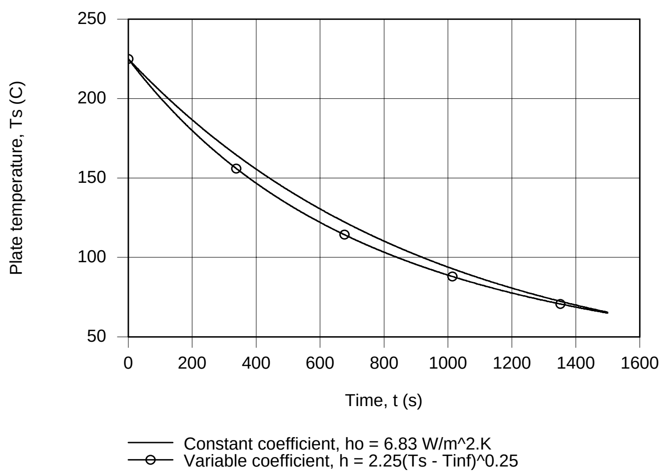
$$\overline{Nu}_L = 0.68 + \frac{0.670 Ra_L^{1/4}}{\left[1+\left(0.492/Pr\right)^{9/16}\right]^{4/9}} = 0.68 + \frac{0.670\left(1.652\times 10^7\right)^{1/4}}{\left[1+\left(0.492/0.6976\right)^{9/16}\right]^{4/9}} = 33.4$$

$$\overline{h}_o = \frac{k}{L}\overline{Nu}_L = \frac{0.03069\,\mathrm{W/m\cdot K}}{0.150\mathrm{m}}\times 33.4 = 6.83\,\mathrm{W/m^2\cdot K}$$

From Eq. 5.6, the temperature-time history with a constant convection coefficient is

$$T(t) = T_\infty + \left(T_i - T_\infty\right)\exp\left[-\left(\overline{h}_o A_s/\rho V c\right)t\right] \qquad (3)$$

where $A_s/V = 2L^2/\left(L\times L\times w\right) = 2/w = 400\mathrm{m}^{-1}$. The temperature-time histories for the $h = C\Delta T^{1/4}$ and $\overline{h}_o$ analyses are shown in plot below.



COMMENTS: (1) The times to reach $T(t_o) = 80°C$ were 1154 and 1212s for the variable and constant coefficient analysis, respectively, a difference of 5%. For convenience, it is reasonable to evaluate the convection coefficient as described in part (b).

(2) Note that $Ra_L < 10^9$ so indeed the expression selected from Problem 9.11 was the appropriate one.

(3) Recognize that if the emissivity of the plate were unity, the average linearized radiation coefficient using Eq. (1.9) is $\overline{h}_{rad} = 11.0\,\mathrm{W/m^2\cdot K}$ and radiative exchange becomes an important process.