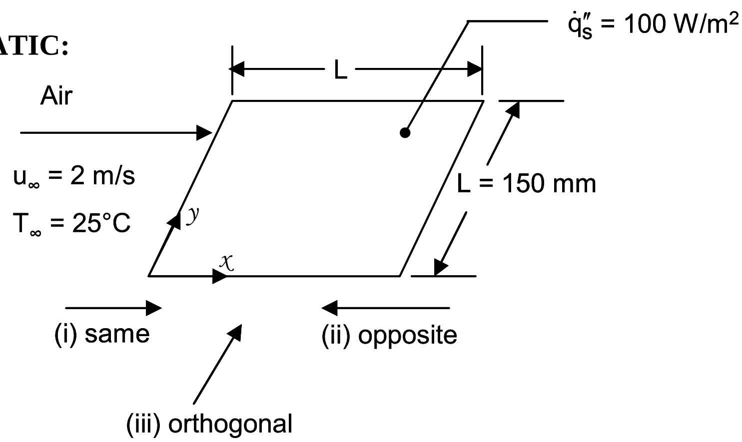
PROBLEM 7.16

KNOWN: Dimensions of and heat generation rate in thin membrane. Velocity and temperature of air flow parallel to membrane. Air streams above and below membrane are in same, opposite, or orthogonal directions.

FIND: (a) Minimum and maximum local membrane temperatures. Flow configuration that minimizes the membrane temperature. (b) Plot the surface temperature distribution for flow in the same and opposite directions. Find configuration that minimizes spatial temperature gradients.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Boundary layer assumptions hold, (3) Constant properties, (4) Solutions are bounded by constant surface temperature and constant heat flux cases for the opposite and orthogonal flow configurations.

PROPERTIES: Table A-4, Air ($T_f \approx 323$ K): $\nu = 18.20 \times 10^{-6}$ m^2/s, k = 0.0280 W/m·K, Pr = 0.704.

ANALYSIS:
(a) We begin by calculating the Reynolds number

$$Re_L = \frac{u_\infty L}{\nu} = \frac{2\text{ m/s} \times 0.15\text{ m}}{18.20 \times 10^{-6}\text{ m}^2/\text{s}} = 1.65 \times 10^4$$

Therefore, the flow is laminar.

(a) Top and bottom flows in same direction.
By symmetry, the heat flux from the membrane to the air is 50 W/m^2 everywhere for the top and bottom air flows. Since the heat flux is uniform, the local Nusselt number is given by Equation 7.37,

$$Nu_x = 0.453\, Re_x^{1/2}\, Pr^{1/3}$$

Thus $\quad h_x = c_q x^{-1/2}$
where

$$c_q = 0.453(u_\infty/\nu)^{1/2}\, Pr^{1/3}\, k$$
$$= 0.453(2\text{ m/s}/18.2 \times 10^{-6}\text{ m}^2/\text{s})^{1/2} \times (0.704)^{1/3} \times 0.028\text{ W/m}\cdot\text{K} = 3.74\text{ W/m}^{3/2}\cdot\text{K}$$

Then $\quad q''_s = h_x(T_s - T_\infty)$ and $\quad T_s - T_\infty = \dfrac{q''_s}{h_x} = \dfrac{q''_s}{c_q} x^{1/2} \qquad (1)$

Continued…

PROBLEM 7.16 (Cont.)

Clearly the minimum temperature occurs at x = 0 and is

$$T_{min} = T_\infty = 25°C \qquad <$$

The maximum temperature occurs at x = L and is

$$T_{max} = 25°C + 50\ W/m^2 \times (0.15\ m)^{1/2}/3.74\ W/m^{3/2}\cdot K = 30.2°C \qquad <$$

(ii) Top and bottom flows in opposite direction.
The heat flux entering each of the top and bottom flows will no longer be uniform. Near x = 0, where the top flow first encounters the plate, the heat transfer coefficient on the top surface is theoretically infinite, and all the generated heat will enter the top flow. The opposite situation will occur at x = L.

We bound the solution by considering Nusselt number correlations for uniform surface temperature and uniform surface heat flux, Equations 7.21 and 7.37. In both cases, the heat transfer coefficient varies as $x^{-1/2}$, where x is the distance from the leading edge, thus for the top and bottom,

$$h_{x,t} = cx^{-1/2}, \quad h_{x,b} = c(L - x)^{-1/2}$$

And all of the generated heat is removed by the top and bottom flows:

$$\dot{q}'' = (h_{x,t} + h_{x,b})(T_s - T_\infty)$$

Thus $$T_s - T_\infty = \frac{\dot{q}''}{h_{x,t} + h_{x,b}} = \frac{\dot{q}''}{c\left[x^{-1/2} + (L - x)^{-1/2}\right]} \qquad (2)$$

The minimum temperature occurs at x = 0 or x = L, and is

$$T_{min} = T_\infty = 25°C \qquad <$$

The maximum temperature occurs where the denominator is minimum:

$$\frac{d}{dx}\left[x^{-1/2} + (L - x)^{-1/2}\right] = 0$$

$$-\frac{1}{2}x^{-3/2} + \frac{1}{2}(L - x)^{-3/2} = 0$$

$$x = L - x$$

$$x = L/2$$

At that location

$$T_{max} = T_\infty + \frac{\dot{q}''}{c2(L/2)^{-1/2}}$$

For uniform surface temperature,

$$c_T = 0.332(u_\infty/\nu)^{1/2}\ Pr^{1/3}\ k$$

$$= 0.332(2\ m/s/18.2 \times 10^{-6} m^2/s)^{1/2} \times (0.704)^{1/3} \times 0.0280\ W/m\cdot K = 2.74\ W/m^{3/2}\cdot K$$

And $$T_{max} = 25°C + \frac{100\ W/m^2}{2.74\ W/m^{3/2}\cdot K \times 2 \times (0.15\ m/2)^{-1/2}} = 30.0°C$$

Continued…

PROBLEM 7.16 (Cont.)

For uniform surface heat flux, we previously found $c_q = 3.74\ \mathrm{W/m^{3/2}\cdot K}$, thus, $T_{max} = 28.7°C$.

Therefore, for the opposite flow case, $28.7°C \le T_{max} \le 30.0°C$ <

(iii) <u>Top and bottom flows in orthogonal directions.</u>
Here the heat transfer coefficients are given by

$$h_{x,t} = cx^{-1/2}, \quad h_{y,b} = cy^{-1/2}$$

And $$\dot{q}'' = c(x^{-1/2} + y^{-1/2})(T_s - T_\infty)$$

So $$T_s - T_\infty = \frac{\dot{q}''}{c\left(x^{-1/2} + y^{-1/2}\right)}$$

The temperature will be minimum along x = 0 or y = 0, where

$$T_{min} = T_\infty = 25°C$$ <

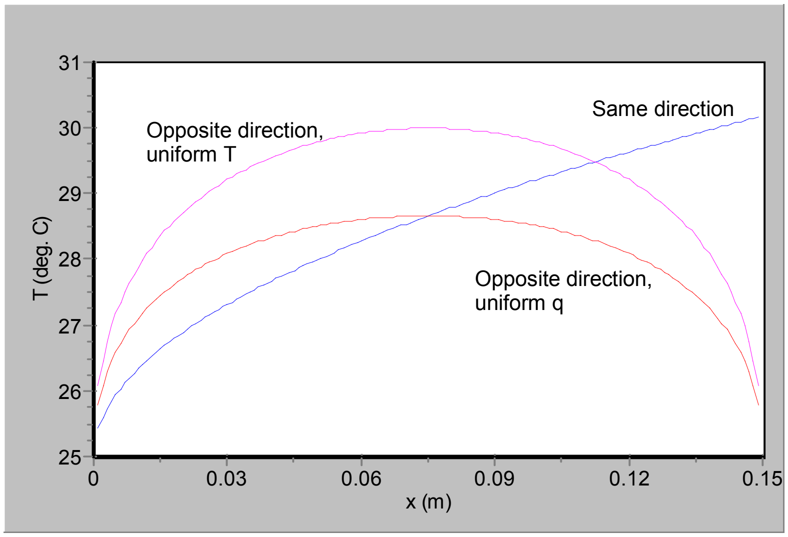
The temperature will be maximum along x = y = L, where

$$T_{max} = T_\infty + \frac{\dot{q}''}{c\,2\,L^{-1/2}}$$

The values of c are the same as previously, therefore we find $30.2°C \le T_{max} \le 32.1°C$ <

The surface temperature is minimized when the air streams are in opposite directions, because a small heat transfer coefficient on the top is paired with a large heat transfer coefficient on the bottom, and vice versa. <

(b) *IHT* was used to plot Equation (1) and (2) for $c = c_T$ or c_q. The result is shown below.



The spatial temperature gradients are somewhat less for the opposite flow case.

COMMENTS: To correctly treat the convective heat transfer would require a coupled numerical solution of the thermal energy equation for both boundary layers simultaneously.