

# Applied Numerical Methods with MATLAB for Engineers and Scientists (4th Edition)

Chapter 24, Problem 29P

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Problem

Repeat Prob. 24.28, but for the following heat source:  $f(x) = 0.12x^3 - 2.4x^2 + 12x$ .

Step-by-step solution

Step 1 of 12

The insulated heated rod with uniform heat source is modelled as,  
$$\frac{d^2T}{dx^2} = -(0.12x^3 - 2.4x^2 + 12x)$$
The boundary conditions are:  
 $T(x=0) = 40^\circ\text{C}$   
 $T(x=10) = 200^\circ\text{C}$

Comment

Step 2 of 12

Convert the above boundary value problem into an initial value problem by defining the rate of change of temperature, that is,  $\frac{dT}{dx}$ .  
$$\frac{dT}{dx} = z$$
Let  $\frac{d^2T}{dx^2} = \frac{dz}{dx}$   
Therefore,  
$$\frac{dT}{dx} = z,$$
$$\frac{dz}{dx} = -(0.12x^3 - 2.4x^2 + 12x)$$
Let  $T(0) = 40\text{ K}$   
 $T(10) = 200\text{ K}$

Comment

Step 3 of 12

(a)  
Use the shooting method to develop a solution for the above initial-value problem.  
Write the following MATLAB code 'ShootingMethod2.m'.  
function ShootingMethod2  
% Inputs are L,h,Ta,Tb,T10,T10,z0  
% Outputs are y  
T0 = 40;  
T10 = 200;  
npts = 100;  
xspan = linspace(0,10,101);  
xlim([50 500])  
disp('-----')  
for i = 1:2  
z0(i) = input('nEnter a guess for dT/dx(0): ');  
disp('-----')  
y0 = [T0 z0(i)];  
[x,y] = ode45(@HeatTransfer,xspan,y0);  
T = y(:,1);  
Tlow(i) = y(length(y));  
plot(x,T)  
xlabel('x')  
ylabel('T')  
hold on  
fprintf('The final value of T is %5.2f\n', y(length(y)))  
end  
slope = (z0(2)-z0(1))/(Tlow(2)-Tlow(1));  
z1 = z0(1)+slope\*(T10-Tlow(1));  
[x,y] = ode45(@HeatTransfer,[0 10],[300,z1]);  
T = y(:,1);  
disp('-----')  
fprintf('The final value of T is %5.2f\n',y(length(y)))  
disp('-----')  
plot(x,T,'r')  
function dy = HeatTransfer(x,y)  
T = y(1,1);  
z = y(2,1);  
% Calculate state derivative vectory  
dTdx = z;  
dzdx = -(0.12\*x^3-2.4\*x^2+12\*x);  
% Save state derivative vector  
dy(1,1) = dTdx;  
dy(2,1) = dzdx;

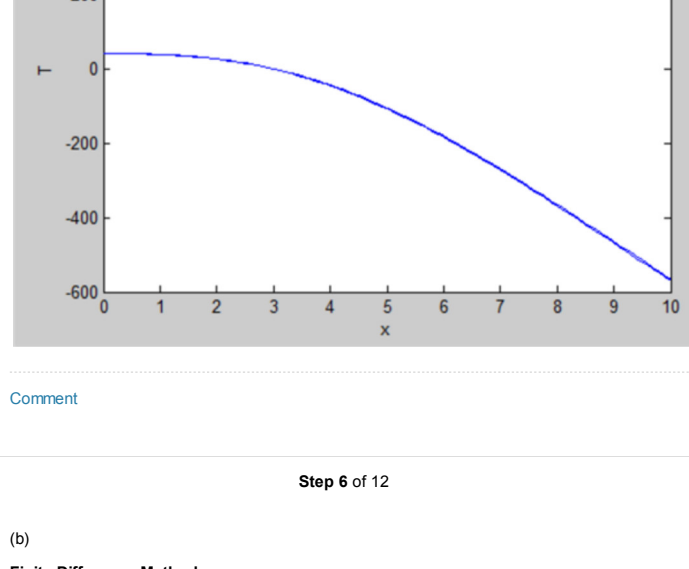
Comment

Step 4 of 12

Choose the below two guesses for the initial condition of z.  
Solve the ODEs with the 4th-order RK method using a step size of 0.125.  
The results are as follows:  
>> ShootingMethod2  
-----  
  
Enter a guess for dT/dx(0): -1  
-----  
The final value of T is -570.00  
  
Enter a guess for dT/dx(0): -0.5  
-----  
The final value of T is -565.00  
-----  
The final value of T is 460.00  
-----

Comment

Step 5 of 12

Use these values to derive the correct initial condition.  
$$z(0) = -1 + \frac{-0.5+1}{-565-(-570)}(200-(-570))$$
$$= 76$$
The resulting fit is displayed below:  


Comment

Step 6 of 12

(b)  
**Finite Difference Method:**  
The second derivative at node  $i$  represented by,  
$$\frac{d^2T}{dx^2} = \frac{T_{i-1} - 2T_i + T_{i+1}}{\Delta x^2}$$
 (centred finite difference)  
Substitute  $\frac{d^2T}{dx^2} = \frac{T_{i-1} - 2T_i + T_{i+1}}{(\Delta x)^2}$  into  $\frac{d^2T}{dx^2} = -(0.12x^3 - 2.4x^2 + 12x)$ , we find,  
$$\frac{T_{i+1} - 2T_i + T_{i-1}}{\Delta x^2} + 0.12x_i^3 - 2.4x_i^2 + 12x_i = 0$$
  
Substitute  $\Delta x = 2$  and collect the terms.  
$$T_{i+1} - 2T_i + 4(0.12x_i^3 - 2.4x_i^2 + 12x_i) + T_{i-1} = 0 \quad \dots (1)$$
or  
$$-T_{i+1} + 2T_i - T_{i-1} = 4(0.12x_i^3 - 2.4x_i^2 + 12x_i)$$

Comment

Step 7 of 12

This equation can be written for each node with the result  
$$\begin{bmatrix} 2 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & 2 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \end{bmatrix} = \begin{bmatrix} 101.44 \\ 69.12 \\ 46.08 \\ 215.36 \end{bmatrix}$$
  
This equation can be written for each node with the result  
>> A=[2 -1 0 0;-1 2 -1 0;0 -1 2 -1;0 0 -1 2];  
b=[101.44 69.12 46.08 215.36];  
x=A\b  
x =  
184.1280  
266.8160  
280.3840  
247.8720

Comment

Step 8 of 12

The first and last equations (from (1)) along with the boundary conditions are given by,  
$$T_2 - 2T_1 + 4(0.12x_1^3 - 2.4x_1^2 + 12x_1) + T_3 = 0$$
$$T_{n-1} - 2T_n + 4(0.12x_n^3 - 2.4x_n^2 + 12x_n) + T_n = 0$$
  
Here  $T_2$  and  $T_n$  are the boundary conditions.  
Differentiate  $f(T) = T_2 - 2T_1 + 4(0.12x_1^3 - 2.4x_1^2 + 12x_1) + T_3$  with respect to  $T_1$ .  
Thus,  $f'(T) = -2$

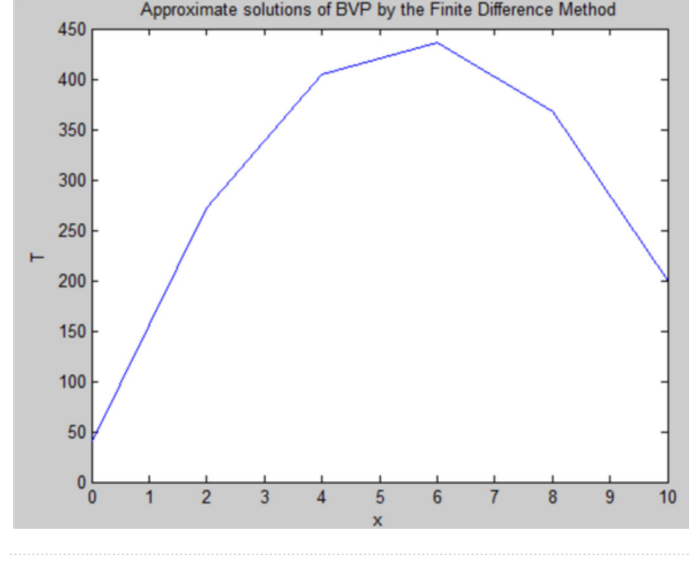
Comment

Step 9 of 12

Use the following MATLAB program, 'fda2.m' to approximate solutions for the linear BVP  
$$\frac{d^2T}{dx^2} = -(0.12x^3 - 2.4x^2 + 12x), T(0) = 40\text{ K}, T(10) = 200\text{ K}$$
  
function W=fda2(inter,bv,n)  
% Inputs: interval(inter), boundary values(bv), number of steps(n) are inputs  
% Command used: w=fda2( [0 10],[300 400],4)  
a=inter(1); b=inter(2); ya=bv(1); yb=bv(2);  
h=(b-a)/(n+1); % This gives the step size h  
W=zeros(n,1); % This initializes solution array w  
for i=1:20  
W=W+jac(W,inter,bv,n)/f(W,inter,bv,n);  
end  
plot([a+(1:n)\*h],[ya W' yb]); % This gives plot w with boundary data  
xlabel('x'),ylabel('T')  
title('Approximate solutions of BVP by the Finite Difference Method')  
%%%%%%%%%%%%%%  
function y=f(W,inter,bv,n)  
y=zeros(n,1); h=(inter(2)-inter(1))/(n+1);  
y(1)=bv(1)-2\*W(1)+100+W(2);  
y(n)=W(n)-2\*W(n)+100+bv(2);  
for i=2:n-1  
y(i)=W(i-1)-2\*W(i)+100+W(i+1);  
end  
%%%%%%%%%%%%%%  
function a=jac(W,inter,bv,n)  
a=zeros(n,n); h=(inter(2)-inter(1))/(n+1);  
for i=1:n  
a(i,i)=2;  
end  
for i=1:n-1  
a(i,i+1)=1;  
a(i+1,i)=1;  
end

Comment

Step 10 of 12

Output:  
>> w=fda2( [0 10],[40 200],4)  
  
w =  
  
272.0000  
404.0000  
436.0000  
368.0000  
Plot:  


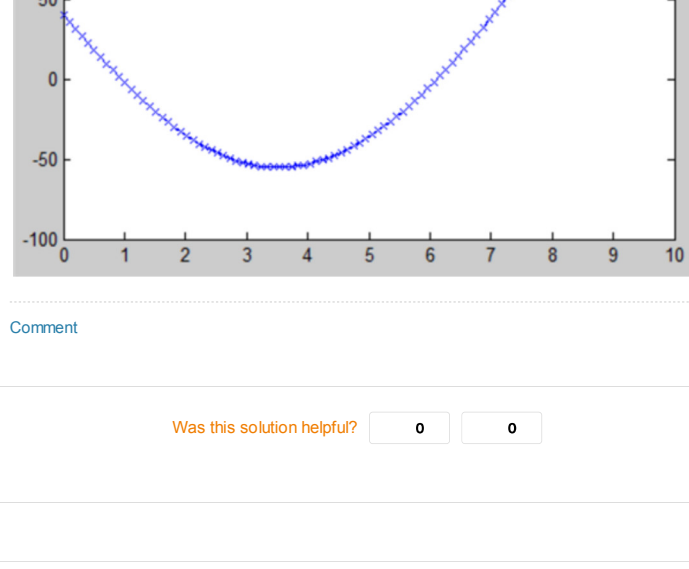
Comment

Step 11 of 12

(c)  
**Bvp4c:**  
Write the following SolveBVP.m file and then run:  
% Function to solve y'' + 25 = 0.  
function SolveBVP()  
solinit = bvpinit(linspace(0,10,100),[1;-1]); %initial guess 1,-1  
sol = bvp4c(@deriv,@bcs,solinit);  
x=linspace(0,10,100);  
y=deval(sol,x);  
plot(x,y(1,:),'b-x');  
function dydx = deriv(x,y)  
dydx(1) = Y(2);  
dydx(2) = 0.12\*x^3-2.4\*x^2+12\*x;  
% boundary conditions y(0)=40, y(10)=200.  
function res = bcs(ya,yb)  
res = [ ya(1)-40; yb(1)-200];

Comment

Step 12 of 12

Run the function to obtain the below graph:  


Comment

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