Chapter 2

Signal and Linear System Analysis

2.1 Problem Solutions

Problem 2.1

a. For the single-sided spectra, write the signal as

$$\begin{aligned} x_1(t) &= 10\cos(4\pi t + \pi/8) + 6\sin(8\pi t + 3\pi/4) \\ &= 10\cos(4\pi t + \pi/8) + 6\cos(8\pi t + 3\pi/4 - \pi/2) \\ &= 10\cos(4\pi t + \pi/8) + 6\cos(8\pi t + \pi/4) \\ &= \text{Re}\left[10e^{j(4\pi t+\pi/8)} + 6e^{j(8\pi t+\pi/4)}\right] \end{aligned}$$

For the double-sided spectra, write the signal in terms of complex exponentials using Euler's theorem:

$$\begin{aligned} x_1(t) &= 5\exp[j(4\pi t + \pi/8)] + 5\exp[-j(4\pi t + \pi/8)] \\ &\quad +3\exp[j(8\pi t + 3\pi/4)] + 3\exp[-j(8\pi t + 3\pi/4)] \end{aligned}$$

The spectra are plotted in Fig. 2.1.

b. Write the given signal as

$$x_2(t) = \text{Re}\left[8e^{j(2\pi t+\pi/3)} + 4e^{j(6\pi t+\pi/4)}\right]$$

to plot the single-sided spectra. For the double-side spectra, write it as

$$x_2(t) = 4e^{j(2\pi t+\pi/3)} + 4e^{-j(2\pi t+\pi/3)} + 2e^{j(6\pi t+\pi/4)} + 2e^{-j(6\pi t+\pi/4)}$$

The spectra are plotted in Fig. 2.2.

c. Change the sines to cosines by subtracting $\pi/2$ from their arguments to get

$$\begin{aligned} x_3(t) &= 2\cos(4\pi t + \pi/8 - \pi/2) + 12\cos(10\pi t - \pi/2) \\ &= 2\cos(4\pi t - 3\pi/8) + 12\cos(10\pi t - \pi/2) \\ &= \text{Re}\left[2e^{j(4\pi t - 3\pi/8)} + 12e^{j(10\pi t - \pi/2)}\right] \\ &= e^{j(4\pi t - 3\pi/8)} + e^{-j(4\pi t - 3\pi/8)} + 6e^{j(10\pi t - \pi/2)} + 6e^{-j(10\pi t - \pi/2)} \end{aligned}$$

Spectral plots are given in Fig. 2.3.

d. Use a trig identity to write

$$3\sin(18\pi t + \pi/2) = 3\cos(18\pi t)$$

and get

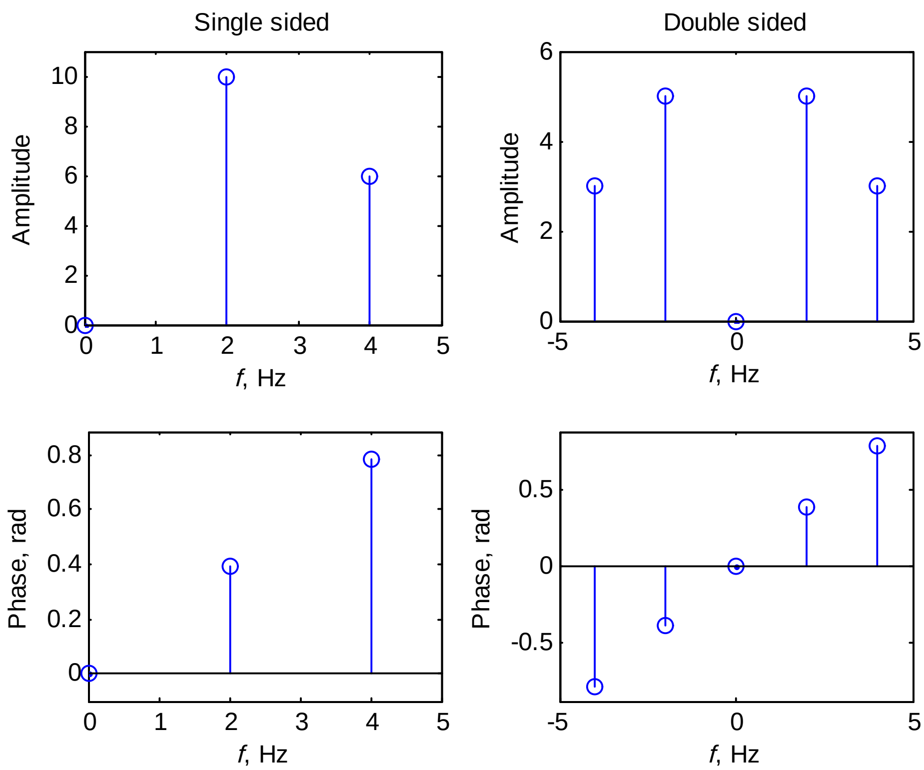
$$\begin{aligned} x_4(t) &= 2\cos(7\pi t + \pi/4) + 3\cos(18\pi t) \\ &= \text{Re}\left[2e^{j(7\pi t + \pi/4)} + 3e^{j18\pi t}\right] \\ &= e^{j(7\pi t + \pi/4)} + e^{-j(7\pi t + \pi/4)} + 1.5e^{j18\pi t} + 1.5e^{-j18\pi t} \end{aligned}$$

From this it is seen that the singe-sided amplitude spectrum consists of lines of amplitudes 2 and 3 at frequencies of 3.5 and 9 Hz, respectively, and the phase spectrum consists of a line of height $\pi/4$ at 3.5 Hz. The double-sided amplitude spectrum consists of lines of amplitudes 1, 1, 1.5, and 1.5 at frequencies of 3.5, -3.5, 9, and -9 Hz, respectively. The double-sided phase spectrum consists of lines of heights $\pi/4$ and $-\pi/4$ at frequencies 3.5 Hz and -3.5 Hz, respectively.

e. Use $\sin(2\pi t) = \cos(2\pi t - \pi/2)$ to write

$$\begin{aligned} x_5(t) &= 5\cos(2\pi t - \pi/2) + 4\cos(5\pi t + \pi/4) \\ &= \text{Re}\left[5e^{j(2\pi t - \pi/2)} + 4e^{j(5\pi t + \pi/4)}\right] \\ &= 2.5e^{j(2\pi t - \pi/2)} + 2.5e^{-j(2\pi t - \pi/2)} + 2e^{j(5\pi t + \pi/4)} + 2e^{-j(5\pi t + \pi/4)} \end{aligned}$$

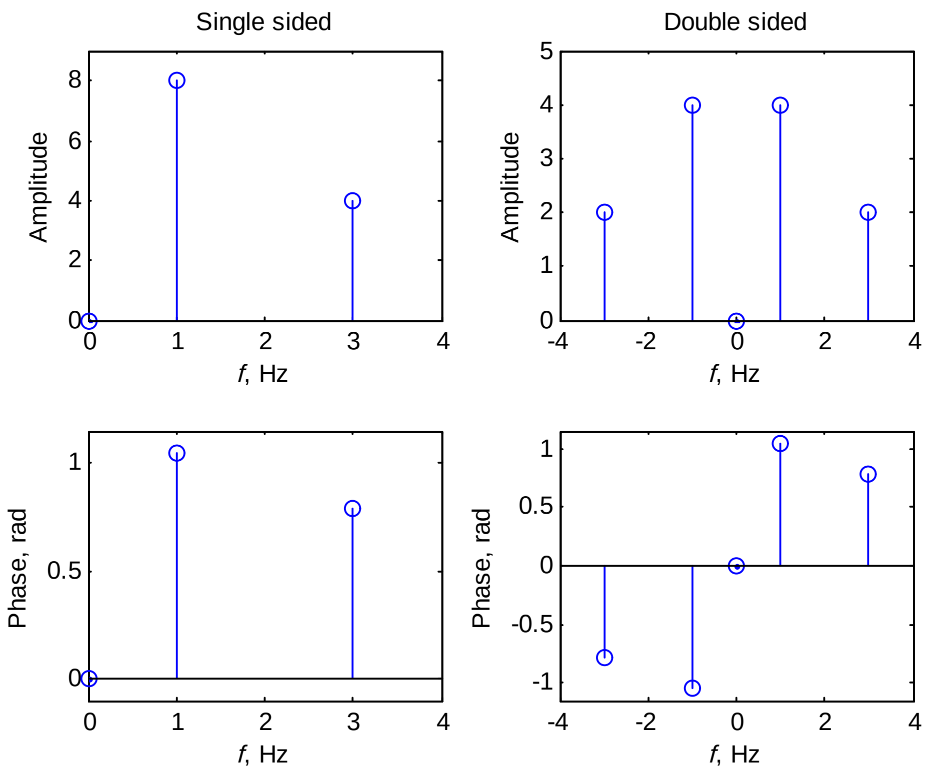
From this it is seen that the singe-sided amplitude spectrum consists of lines of amplitudes 5 and 4 at frequencies of 1 and 2.5 Hz, respectively, and the phase spectrum consists of lines of heights $-\pi/2$ and $\pi/4$ at 1 and 2.5 Hz, respectively. The double-sided amplitude spectrum consists of lines of amplitudes 2.5, 2.5, 2, and 2 at frequencies of 1, -1, 2.5, and -2.5 Hz, respectively. The double-sided phase spectrum consists of lines of heights $-\pi/2$, $\pi/2$, $\pi/4$, and $-\pi/4$ at frequencies of 1, -1, 2.5, and -2.5 Hz, respectively.

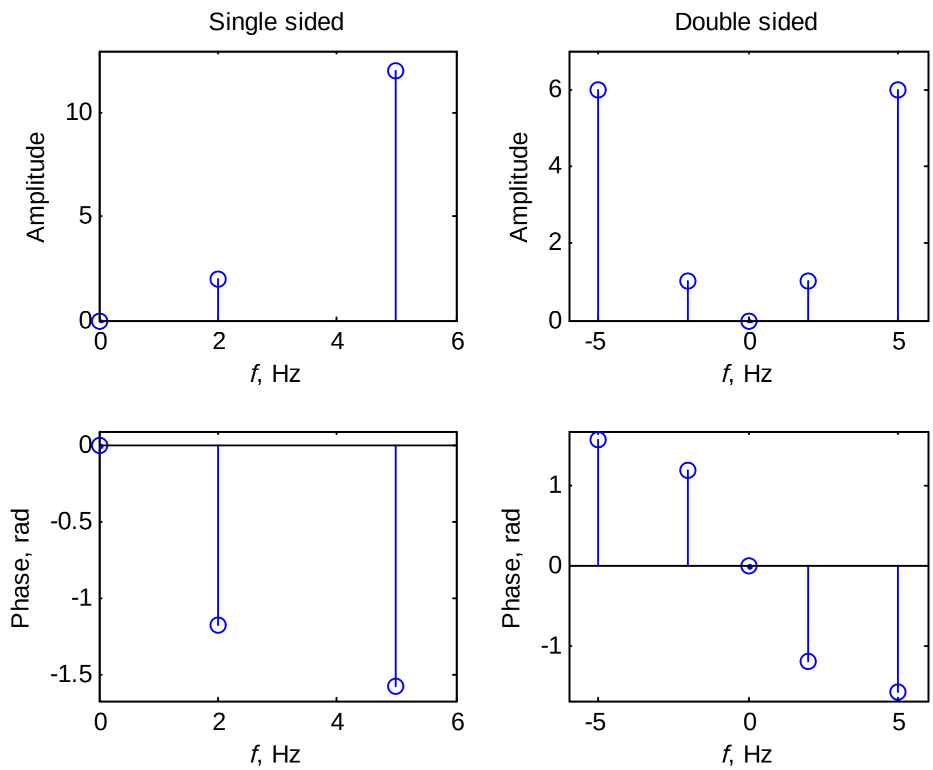


f. Use $\sin(10\pi t + \pi/6) = \cos(10\pi t + \pi/6 - \pi/2) = \cos(10\pi t - \pi/3)$ to write

$$\begin{aligned} x_6(t) &= 3\cos(4\pi t + \pi/8) + 4\cos(10\pi t - \pi/3) \\ &= \mathrm{Re}\left[3e^{j(4\pi t+\pi/8)} + 4e^{j(10\pi t-\pi/3)}\right] \\ &= 1.5e^{j(4\pi t+\pi/8)} + 1.5e^{-j(4\pi t+\pi/8)} + 2e^{j10\pi t-\pi/3)} + 2e^{-j(10\pi t-\pi/3)} \end{aligned}$$

From this it is seen that the singe-sided amplitude spectrum consists of lines of amplitudes 3 and 4 at frequencies of 2 and 5 Hz, respectively, and the phase spectrum consists of lines of heights $\pi/8$ and $-\pi/3$ at 2 and 5 Hz, respectively. The double-sided amplitude spectrum consists of lines of amplitudes 1.5, 1.5, 2, and 2 at frequencies of 2, -2, 5, and -5 Hz, respectively. The double-sided phase spectrum consists of lines of heights $\pi/8$, $-\pi/8$, $-\pi/3$, and $\pi/3$ at frequencies of 2, -2, 5, and -5 Hz, respectively.


Single sided
Double sided
Amplitude
Amplitude
f, Hz
f, Hz
Phase, rad
Phase, rad
f, Hz
f, Hz


Single sided
Double sided
Amplitude
Phase, rad
f, Hz
0
2
4
6
10
5
-5
-0.5
-1
-1.5
1

Problem 2.2

By noting the amplitudes and phases of the various frequency components from the plots, the result is

$$\begin{aligned} x(t) &= 4e^{j(8\pi t+\pi/2)} + 4e^{-j(8\pi t+\pi/2)} + 2e^{j(4\pi t-\pi/4)} + 2e^{-j(4\pi t-\pi/4)} \\ &= 8\cos(8\pi t+\pi/2) + 4\cos(4\pi t-\pi/4) \\ &= -8\sin(8\pi t) + 4\cos(4\pi t-\pi/4) \end{aligned}$$

Problem 2.3

a. Not periodic because $f_1 = 1/\pi$ Hz and $f_2 = 3$ Hz are not commensurable.

b. Periodic. To find the period, note that

$$\frac{6\pi}{2\pi} = 3 = n_1 f_0 \text{ and } \frac{30\pi}{2\pi} = 15 = n_2 f_0$$

Therefore

$$\frac{15}{3} = \frac{n_2}{n_1}$$

Hence, take $n_1 = 1$, $n_2 = 5$, and $f_0 = 3$ Hz (we want the largest possible value for f_0 with n_1 and n_2 integer-valued).

c. Periodic. Using a similar procedure as used in (b), we find that $n_1 = 4$, $n_2 = 21$, and $f_0 = 0.5$ Hz.

d. Periodic. Using a similar procedure as used in (b), we find that $n_1 = 4$, $n_2 = 7$, $n_3 = 11$, and $f_0 = 0.5$ Hz.

e. Periodic. We find that $n_1 = 17$, $n_2 = 18$, and $f_0 = 0.5$ Hz.

f. Periodic. We find that $n_1 = 2$, $n_2 = 3$, and $f_0 = 0.5$ Hz.

g. Periodic. We find that $n_1 = 7$, $n_2 = 11$, and $f_0 = 0.5$ Hz.

h. Not periodic. The frequencies of the separate terms are incommensurable.

i. Periodic. We find that $n_1 = 19$, $n_2 = 21$, and $f_0 = 0.5$ Hz.

j. Periodic. We find that $n_1 = 6$, $n_2 = 7$, and $f_0 = 0.5$ Hz.

Problem 2.4

a. The single-sided amplitude spectrum consists of a single line of amplitude 5 at 6 Hz and the phase spectrum consists of a single line of height $-\pi/6$ rad at 6 Hz. The double-sided amplitude spectrum consists of lines of amplitude 2.5 at frequencies ± 6 Hz. The double -sided phase spectrum consists of a line of height $\pi/6$ at -6 Hz and a line of height $-\pi/6$ at 6 Hz.

b. Write the signal as

$$x_2(t) = 3\cos(12\pi t - \pi/2) + 4\cos(16\pi t)$$

From this it is seen that the single-sided amplitude spectrum consists of lines of heights 3 and 4 at frequencies 6 and 8 Hz, respectively, and the single-sided phase spectrum consists of a line of height $-\pi/2$ radians at frequency 6 Hz (the phase at 8 Hz is 0). The double-sided amplitude spectrum consists of lines of height 1.5 and 2 at frequencies of 6 and 8 Hz, respectively, and lines of height 1.5 and 2 at frequencies -6 and -8 Hz, respectively. The double-sided phase spectrum consists of a line of height $-\pi/2$ radians at frequency 6 Hz and a line of height $\pi/2$ radians at frequency -6 Hz.

c. Use the trig identity $\cos x \cos y = 0.5\cos(x+y) + 0.5\cos(x-y)$ to write

$$x_3(t) = 2\cos 20\pi t + 2\cos 4\pi t$$

From this we see that the single-sided amplitude spectrum consists of lines of height 2 at 2 and 10 Hz, and the single-sided phase spectrum is 0 at these frequencies. The double-sided amplitude spectrum consists of lines of height 1 at frequencies of -10, -2, 2, and 10 Hz. The double-sided phase spectrum is 0.

d. Use trig identies to get

$$\begin{aligned}
x_4(t) &= 4\sin(2\pi t)\left[1+\cos(10\pi t)\right] \\
&= 4\sin(2\pi t) - 2\sin(8\pi t + \pi) + 2\sin(12\pi t) \\
&= 4\cos(2\pi t - \pi/2) + 2\cos(8\pi t + \pi/2) + 2\cos(12\pi t - \pi/2) \\
&= \text{Re}\left[4e^{j(2\pi t-\pi/2)} + 2e^{j(8\pi t+\pi/2)} + 2e^{j(12\pi t-\pi/2)}\right] \\
&= 2e^{j(2\pi t-\pi/2)} + 2e^{-j(2\pi t-\pi/2)} + e^{j(8\pi t+\pi/2)} + e^{-j(8\pi t+\pi/2)} + e^{j(12\pi t-\pi/2)} + e^{-j(12\pi t-\pi/2)}
\end{aligned}$$

From this we see that the single-sided amplitude spectrum consists of lines of heights 4, 2, and 2 at frequencies 1, 4, and 6 Hz, respectively and the single-sided phase spectrum is $-\pi/2$ radians at 1 and 6 Hz and $\pi/2$ radians at 4 Hz. The double-sided amplitude spectrum consists of lines of height 2 at frequencies of 1 and -1 Hz and of height 1 at frequencies of 4, -4, 6, and -6 Hz. The double-sided phase spectrum is $\pi/2$ radians at -1, 4, and -6 Hz and $-\pi/2$ radians at 1, -4, and 6 Hz.

e. Clearly, from the form of the cosine sum, the single-sided amplitude spectrum has lines of heights 1 and 7 at frequencies of 3 and 15 Hz, respectively. The single-sided phase spectrum is zero. The double-sided amplitude spectrum has lines of heights 0.5, 0.5, 3.5, and 3.5 at frequencies of 3, -3, 15, and -15 Hz, respectfully. The double-sided phase spectrum is zero.

f. The single-sided amplitude spectrum has lines of heights 1 and 9 at frequencies of 2 and 10.5 Hz, respectively. The single-sided phase spectrum is $-\pi/2$ radians at 10.5 Hz and 0 otherwise. The double-sided amplitude spectrum has lines of heights 0.5, 0.5, 4.5, and 4.5 at frequencies of 2, -2, 10.5, and -10.5 Hz, respectfully. The double-sided phase spectrum is $\pi/2$ radians at -10.5 Hz and $-\pi/2$ radians at 10.5 Hz and 0 otherwise.

g. Convert the sine to a cosine by subtracting $\pi/2$ from its argument. It then follows that the single-sided amplitude spectrum is 2, 1, and 6 at frequencies of 2, 3, and 8.5 Hz and 0 otherwise. The single-sided phase spectrum is $-\pi/2$ radians at 8.5 Hz and 0 otherwise. The double-sided amplitude spectrum is 1, 1, 0.5, 0.5, 3, and 3 at frequencies of -2, 2, -3, 3 -8.5, and 8.5 Hz, respectively, and 0 otherwise. The double-sided phase spectrum is $\pi/2$ radians at a frequency of -8.5 Hz and $-\pi/2$ radians at a frequency of 8.5 Hz. It is 0 otherwise.

Problem 2.5

a. This function has area

$$\begin{aligned} \text{Area} &= \int_{-\infty}^{\infty} \epsilon^{-1} \left[\frac{\sin(\pi t/\epsilon)}{(\pi t/\epsilon)} \right]^2 dt \\ &= \int_{-\infty}^{\infty} \left[\frac{\sin(\pi u)}{(\pi u)} \right]^2 du = 1 \end{aligned}$$

where a tabulated integral has been used for $\text{sinc}^2 u$. A sketch shows that no matter how small ϵ is, the area is still 1. With $\epsilon \to 0$, the central lobe of the function becomes narrower and higher. Thus, in the limit, it approximates a delta function.

b. The area for the function is

$$\text{Area} = \int_{-\infty}^{\infty} \frac{1}{\epsilon} \exp(-t/\epsilon) u(t)\, dt = \int_{0}^{\infty} \exp(-u) du = 1$$

A sketch shows that no matter how small ϵ is, the area is still 1. With $\epsilon \to 0$, the function becomes narrower and higher. Thus, in the limit, it approximates a delta function.

c. Area $= \int_{-\epsilon}^{\epsilon} \frac{1}{\epsilon} (1 - |t|/\epsilon)\, dt = \int_{-1}^{1} \Lambda(t)\, dt = 1$. As $\epsilon \to 0$, the function becomes narrower and higher, so it approximates a delta function in the limit.

Problem 2.6

a. Make use of the formula $\delta(at) = \frac{1}{|a|}\delta(t)$ to write $\delta(2t-5) = \delta[2(t-5/2)] = \frac{1}{2}\delta\left(t - \frac{5}{2}\right)$ and use the sifting property of the δ-function to get

$$I_a = \frac{1}{2}\left(\frac{5}{2}\right)^2 + \frac{1}{2}\exp\left[-2\left(\frac{5}{2}\right)\right] = \frac{25}{8} + \frac{1}{2}\exp[-5] = 3.1284$$

b. Impulses at -10, -5, 0, 5, 10 are included in the integral. Use the sifting property after writing the expression as the sum of five integrals to get

$$I_b = (-10)^2 + 1 + (-5)^2 + 1 + 0^2 + 1 + 5^2 + 1 + 10^2 + 1 = 255$$

c. Matching coefficients of like derivatives of δ-functions on either side of the equation gives $A = 5$, $B = 10$, and $C = 3$.

d. Use $\delta(at) = \frac{1}{|a|}\delta(t)$ to write $\delta(4t+3) = \frac{1}{4}\delta\left(t + \frac{3}{4}\right)$. The integral then becomes $I = \frac{1}{4}\left[e^{-4\pi(-3/4)} + \tan\left(10\pi \times \left(-\frac{3}{4}\right)\right)\right] = \frac{1}{4}\left[e^{3\pi} + \tan(-7.5\pi)\right] = -9.277 \times 10^{13}$.

e. Use property 5 of the unit impulse function to get

$$\begin{aligned} I_e &= (-1)^2 \frac{d^2}{dt^2}\left[\cos 5\pi t + e^{-3t}\right]_{t=2} = \frac{d}{dt}\left[-5\pi \sin 5\pi t - 3e^{-3t}\right]_{t=2} \\ &= \left[-(5\pi)^2 \cos 5\pi t + 9e^{-3t}\right]_{t=2} = -(5\pi)^2 \cos 10\pi + 9e^{-6} = -246.73 \end{aligned}$$

Problem 2.7

(a), (c), and (e) are periodic. Their periods are 2 s (fundamental frequency of 0.5 Hz), 2 s, and 3 s, respectively. The waveform of part (c) is a periodic train of triangles, each 2 units wide, extending from -∞ to ∞ spaced by 2 s ((b) is similar except that it is zero for $t < -1$ thus making it aperiodic). Waveform (d) is aperiodic because the frequencies of its two components are incommensurable. The waveform of part (e) is a doubly-infinite train of square pulses, each of which is one unit high and one unit wide, centered at $\cdots$, -6, -3, 0, 3, 6, $\cdots$. Waveform (f) is identical to (e) for $t \geq -1/2$ but 0 for $t < -1/2$ thereby making it aperiodic.

Problem 2.8

a. The result is

$$x(t) = \cos(6\pi t) + 2\cos(10\pi t - \pi/2) = \operatorname{Re}\left(e^{j6\pi t}\right) + \operatorname{Re}\left(2e^{j(10\pi t - \pi/2)}\right) = \operatorname{Re}\left[e^{j6\pi t} + 2e^{j(10\pi t - \pi/2)}\right]$$

b. The result is

$$x(t) = e^{-j(10\pi t - \pi/2)} + \frac{1}{2}e^{-j6\pi t} + \frac{1}{2}e^{j6\pi t} + e^{j(10\pi t - \pi/2)}$$

c. The single-sided amplitude spectrum consists of lines of height 1 and 2 at frequencies of 3 and 5 Hz, respectively. The single-sided phase spectrum consists of a line of height $-\pi/2$ at frequency 5 Hz. The double-sided amplitude spectrum consists of lines of height 1, 1/2, 1/2, and 1 at frequencies of -5, -3, 3, and 5 Hz, respectively. The double-sided phase spectrum consists of lines of height $\pi/2$ and $-\pi/2$ at frequencies of -5 and 5 Hz, respectively.

Problem 2.9

a. Power. Since it is a periodic signal, we obtain

$$P_1 = \frac{1}{T_0}\int_0^{T_0} 4\cos^2(4\pi t + 2\pi/3)\, dt = \frac{1}{T_0}\int_0^{T_0} 2\left[1 + \cos(8\pi t + 4\pi/3)\right] dt = 2 \text{ W}$$

where $T_0 = 1/2$ s is the period. The cosine in the above integral integrates to zero because the interval of integratation is two periods.

b. Energy. The energy is

$$E_2 = \int_{-\infty}^{\infty} e^{-2\alpha t} u^2(t) dt = \int_0^{\infty} e^{-2\alpha t} dt = \frac{1}{2\alpha} \text{ J}$$

c. Energy. The energy is

$$E_3 = \int_{-\infty}^{\infty} e^{2\alpha t} u^2(-t) dt = \int_{-\infty}^{0} e^{2\alpha t} dt = \frac{1}{2\alpha} \text{ J}$$

d. Energy. The energy is

$$\begin{aligned} E_4 &= \lim_{T\to\infty} \int_{-T}^{T} \frac{dt}{(\alpha^2 + t^2)} = \lim_{T\to\infty} \frac{1}{\alpha^2} \int_{-T}^{T} \frac{dt}{\left(1 + (t/\alpha)^2\right)} \\ &= \lim_{T\to\infty} \frac{1}{\alpha} \tan^{-1} \left[\frac{t}{\alpha}\right]_{-T}^{T} = \lim_{T\to\infty} \frac{1}{\alpha} \left[\tan^{-1}(T/\alpha) - \tan^{-1}(-T/\alpha)\right] \\ &= \frac{1}{\alpha} \left[\frac{\pi}{2} - \left(-\frac{\pi}{2}\right)\right] = \frac{\pi}{\alpha} \text{ J} \end{aligned}$$

e. Energy. Since it is the sum of $x_2(t)$ and $x_3(t)$, its energy is the sum of the energies of these two signals, or $E_5 = 1/\alpha$ J.

f. Energy. The energy is

$$\begin{aligned} E_6 &= \lim_{T\to\infty} \int_{-T}^{T} \left[e^{-\alpha t} u(t) - e^{-\alpha(t-1)} u(t-1)\right]^2 dt \\ &= \lim_{T\to\infty} \int_{-T}^{T} \left[e^{-2\alpha t} u^2(t) - e^{-\alpha t} e^{-\alpha(t-1)} u(t) u(t-1) + e^{-2\alpha(t-1)} u^2(t-1)\right] dt \\ &= \lim_{T\to\infty} \left\{\int_0^T e^{-2\alpha t} dt - e^{-\alpha} \int_1^T e^{-2\alpha(t-1)} dt + \int_1^T e^{-2\alpha(t-1)} dt\right\} \\ &= \lim_{T\to\infty} \left\{\int_0^T e^{-2\alpha t} dt - e^{-\alpha} \int_0^{T-1} e^{-2\alpha t'} dt' + \int_0^{T-1} e^{-2\alpha t'} dt'\right\} \\ &= \lim_{T\to\infty} \left\{-\left.\frac{e^{-2\alpha t}}{2\alpha}\right|_0^T + e^{-\alpha} \left.\frac{e^{-2\alpha t'}}{2\alpha}\right|_0^{T-1} - \left.\frac{e^{-2\alpha t'}}{2\alpha}\right|_0^{T-1}\right\} \\ &= \frac{1}{2\alpha} - \frac{e^{-\alpha}}{2\alpha} + \frac{1}{2\alpha} = \frac{1}{\alpha}\left(1 - \frac{1}{2} e^{-\alpha}\right) \text{ J} \end{aligned}$$

Problem 2.10

a. Power. Since the signal is periodic with period $2\pi/\omega$, we have

$$P = \frac{\omega}{2\pi}\int_0^{2\pi/\omega} A^2 \left|\sin\left(\omega t+\theta\right)\right|^2 dt = \frac{\omega}{2\pi}\int_0^{2\pi/\omega} \frac{A^2}{2}\left\{1-\cos\left[2\left(\omega t+\theta\right)\right]\right\} dt = \frac{A^2}{2} \text{ W}$$

b. Neither. The energy calculation gives

$$E = \lim_{T\to\infty}\int_{-T}^{T} \frac{(A\tau)^2\, dt}{\sqrt{\tau+jt}\sqrt{\tau-jt}}dt = \lim_{T\to\infty}\int_{-T}^{T} \frac{(A\tau)^2\, dt}{\sqrt{\tau^2+t^2}}dt \to \infty$$

The power calculation gives

$$P = \lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T} \frac{(A\tau)^2\, dt}{\sqrt{\tau^2+t^2}}dt = \lim_{T\to\infty}\frac{(A\tau)^2}{2T}\ln\left(\frac{1+\sqrt{1+T^2/\tau^2}}{-1+\sqrt{1+T^2/\tau^2}}\right) = 0 \text{ W}$$

c. Energy:

$$E = \int_0^{\infty} A^2 t^2 \exp\left(-2t/\tau\right) dt = \frac{1}{8}A^2\tau\sqrt{\frac{\pi\tau}{2}} \text{ W (use a table of integrals)}$$

d. Energy: This is a "top hat" pulse which is height 2 for $|t| \leq \tau/2$, height 1 for $\tau/2 < |t| \leq \tau$, and 0 everywhere else. Making use of the even symmetry about $t = 0$, the energy is

$$E = 2\left(\int_0^{\tau/2} 2^2 dt + \int_{\tau/2}^{\tau} 1^2 dt\right) = 5\tau \text{ J}$$

e. Energy. The signal is a "house" two units wide and one unit up to the eves with a equilateral triangle for a roof. Because of symmetry, the energy calculation need be carried out for positive t and doubled. The calculation is

$$E = 2\int_0^1 (2-t)^2\, dt = -\frac{2}{3}(2-t)^3\Big|_0^1 = -\frac{2}{3} + \frac{2\times 8}{3} = \frac{14}{3} \text{ J}$$

f. Power. Since the two terms are harmonically related, we may add their respective powers and get

$$P = \frac{A^2}{2} + \frac{B^2}{2} \text{ W}$$

Problem 2.11

a. Using the fact that the power contained in a sinusoid is its amplitude squared divided by 2, we get

$$P = \frac{2^2}{2} = 2 \text{ W}$$

b. This is a periodic train of "box cars" 3 units high, 2 units wide, and occurring every 4 units (period of 4 seconds). The power calculation is

$$P = \frac{1}{4}\int_{-1}^{1} 3^2 dt = \frac{3^2 \times 2}{4} = 4.5 \text{ W}$$

c. This is a train of triangles 1 unit high, 4 s wide, and occuring every 6 s. Using the waveform period centered at 0, the power calculation is

$$P = \frac{1}{6}\int_{-2}^{2}\left(1 - \frac{t}{2}\right)^2 dt = -\frac{1}{6}\frac{2}{3}\left(1 - \frac{t}{2}\right)^3\Bigg|_0^2 = \frac{2}{9} \text{ W}$$

d. This is a train of "houses" each of which is 2 s wide, 1 unit high to the eves, with an isoceles triangle on top for the roof. They are separated by 4 s (the period). Using the even symmetry of each house, the power calculation is

$$P_d = \frac{2}{4}\int_0^1 (2-t)^2\, dt = -\frac{1}{2}\frac{(2-t)^3}{3}\Bigg|_0^1 = -\frac{1}{2}\left(\frac{1}{3} - \frac{2^3}{3}\right) = \frac{7}{6} \text{ W}$$

Problem 2.12

a. The energy is

$$\begin{aligned} E &= \int_0^\infty \left|6e^{(-3+j4\pi)t}\right|^2 dt = 36\int_0^\infty e^{(-3+j4\pi)t}e^{(-3-j4\pi)t}dt \\ &= 36\int_0^\infty e^{-6t}dt = -36\,\frac{e^{-6t}}{6}\Bigg|_0^\infty = 6 \text{ J} \end{aligned}$$

The power is 0 W.

b. This signal is a "top hat" pulse which is 2 for $2 \le t \le 4$, 1 for $0 \le t < 2$ and $4 < t \le 6$, and 0 everywhere else. It is clearly an energy signal with energy

$$E = 2 \times 1^2 + 2 \times 2^2 + 2 \times 1^2 = 12 \text{ J}$$

Its power is 0 W.

c. This is a power signal with power

$$P = \lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T} 49e^{j6\pi t}e^{-j6\pi t}u(t)\,dt = \lim_{T\to\infty}\frac{49}{2T}\int_{0}^{T} dt = \frac{49}{2} = 24.5 \text{ W}$$

Its energy is infinite.

d. This is a periodic signal with power $P = \frac{2^2}{2} = 2$ W. Its energy is infinite.

e. This is neither an energy nor a power signal. Its energy is infinite and its power is

$$P = \lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T} t^2 dt = \lim_{T\to\infty}\frac{1}{2T}\left.\frac{t^3}{3}\right|_{-T}^{T} = \lim_{T\to\infty}\frac{1}{2T}\frac{2T^3}{3} \to \infty$$

f. This is neither an energy nor a power signal. Its energy is

$$E = \int_{1}^{\infty} t^{-1}dt = \ln(t)|_1^\infty \to \infty$$

and its power is

$$P = \lim_{T\to\infty}\frac{1}{2T}\int_{1}^{T} t^{-1}dt = \lim_{T\to\infty}\frac{1}{2T}\ln(t)|_1^\infty = 0$$

Problem 2.13

a. This is a cosine burst from $t = -6$ to $t = 6$ seconds. The energy is $E_1 = \int_{-6}^{6}\cos^2(6\pi t)\,dt = 2\int_0^6\left[\frac{1}{2}+\frac{1}{2}\cos(12\pi t)\right]dt = 6$ J

b. The energy is

$$\begin{aligned} E_2 &= \int_{-\infty}^{\infty}\left[e^{-|t|/3}\right]^2 dt = 2\int_0^\infty e^{-2t/3}dt \text{ (by even symmetry)} \\ &= -2\left.\frac{e^{-2t/3}}{2/3}\right|_0^\infty = 3 \text{ J} \end{aligned}$$

Since the result is finite, this is an energy signal.

c. The energy is

$$E_3 = \int_{-\infty}^{\infty}\{2[u(t) - u(t-8)]\}^2 dt = \int_0^8 4dt = 32 \text{ J}$$

Since the result is finite, this is an energy signal.

d. Note that

$$r(t) \triangleq \int_{-\infty}^{t} u(\lambda)\, d\lambda = \begin{cases} 0, & t < 0 \\ t, & t \geq 0 \end{cases}$$

which is called the unit ramp. Thus the given signal is a triangle between 0 and 20. The energy is

$$E_4 = \int_{-\infty}^{\infty} \left[r(t) - 2r(t-10) + r(t-20)\right]^2 dt = 2\int_0^{10} t^2 dt = \frac{2}{3}\, t^3\Big|_0^{10} = \frac{2000}{3} \text{ J}$$

where the last integral follows because the integrand is a symmetrical triangle about $t = 10$. Since the result is finite, this is an energy signal.

Problem 2.14

a. This is a cosine burst nonzero between 0 and 2 seconds. Its power is 0. Its energy is

$$E_1 = \int_0^2 \cos^2(10\pi t)\, dt = \frac{1}{2}\int_0^2 \left[1 + \cos(20\pi t)\right] dt = 1 \text{ J}$$

b. This is a periodic sequence of triangles of period 3 s. Its energy is infinite. Its power is

$$P_2 = \frac{2}{3}\int_0^2 (1 - t/2)^2\, dt = \frac{4}{9} \text{ J}$$

c. This is an energy signal. Its power is 0. Using evenness of the integrand, its energy is

$$\begin{aligned} E_3 &= 2\int_0^{\infty} e^{-2t}\cos^2(2\pi t)\, dt = \int_0^{\infty} e^{-2t} dt + \int_0^{\infty} e^{-2t}\cos(4\pi t)\, dt \\ &= \frac{1}{2} + \frac{2}{4 + 16\pi^2} \text{ J} \end{aligned}$$

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d. This is an energy signal. Its energy is

$$E_4 = 2\int_0^1 (2-t)^2\, dt = -\frac{2}{3}(2-t)^3\Big|_0^1 = \frac{14}{3} \text{ J}$$