

Since hammer is in free fall,

$$a = -g = -9.81 \text{ m/s}^2$$

$$v = \int a dt = -gt + v_0 \quad (v_0 = 0 \text{ since dropped from rest})$$

$$v = -9.81 t$$

$$x = \int v dt = -\frac{1}{2}gt^2 + x_0 \quad (x_0 = 3 \text{ m}, x_f = 0)$$

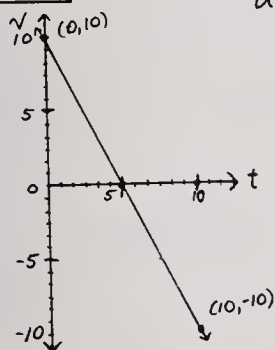
$$x = -4.91t^2 + 3 = 0 \text{ when hammer hits pile}$$

$$\text{Solving } x(t) \text{ for } t_f: t_f = 0.782 \text{ s}$$

$$v(0.782) = -9.81(0.782) \quad v = -7.67 \text{ m/s}$$

$$\text{or } v = 7.67 \text{ m/s} \downarrow$$

13.2



a.) Acceleration of particle:

$$v = \frac{(-10-10)t}{(10-0)} + 10$$

$$v = -2t + 10 \text{ [mm/s]}$$

$$a = \frac{dv}{dt} \Rightarrow a = -2 \text{ mm/s}^2$$

b.) Position of particle:

$$x = \int_{t_0}^t v dt + x_0 \quad (x_0 = 0)$$

$$x = \int_0^t (-2t + 10) dt$$

$$x = -t^2 + 10t \rightarrow \text{Solving for } t = 2, 5, 8, 10 \text{ s:}$$

$$x(2\text{s}) = 16 \text{ [mm]}$$

$$x(5\text{s}) = 25 \text{ [mm]}$$

$$x(8\text{s}) = 16 \text{ [mm]}$$

$$x(10\text{s}) = 0 \text{ [mm]}$$

c.) Distance traveled:

$$\Delta s = |\Delta x_1| + |\Delta x_2| \quad \Delta s(2\text{s}) = 16 \text{ [mm]}$$

$$\Delta s(5\text{s}) = 25 \text{ [mm]}$$

$$\Delta s(8\text{s}) = 25 + |16 - 25| = 34 \text{ [mm]}$$

$$\Delta s(10\text{s}) = 25 + |0 - 25| = 50 \text{ [mm]}$$

13.3

$$v = 4 = \text{constant}, \quad x = 0 \text{ when } t = 0$$

$$a = \frac{dv}{dt} \Rightarrow a(t) = 0$$

$$v = \frac{dx}{dt} = 4 \Rightarrow x = 4t + C \quad = 0 \text{ for } t = 0$$

$$\therefore C = 0 \quad x = 4t$$

$$a = \frac{dv}{dt} \Rightarrow a(t) = 4 \quad (\text{constant})$$

$$v = \frac{dx}{dt} = 4t \Rightarrow x(t) = 2t^2 + C$$

$$\text{For } x = 1, t = 1 \therefore 1 = 2(1)^2 + C \Rightarrow C = -1$$

$$x(t) = 2t^2 - 1$$

13.5

$$v = \sin 2t, \quad x = 0 \text{ when } t = 0$$

$$a = \frac{dv}{dt} \Rightarrow a(t) = 2 \cos 2t \quad (a)$$

$$v = \frac{dx}{dt} = \sin 2t \Rightarrow x = -\frac{\cos 2t}{2} + C$$

$$x = 0 \text{ when } t = 0 \therefore C = \frac{1}{2}$$

$$x(t) = -\frac{\cos 2t}{2} + \frac{1}{2} \quad (b.)$$

$$\text{By Eq. (b.) } -x + \frac{1}{2} = \frac{\cos 2t}{2}$$

$$\text{or } t = (\frac{1}{2}) \cos^{-1}(-2x + 1) \quad (c.)$$

Substitution of Eq. (c.) into Eq. (a.) gives:

$$a(x) = 2 \cos(\frac{1}{2} \cos^{-1}(-2x + 1))$$

13.6

$$v = e^{-2t}, \quad x = 2 \text{ when } t = 0$$

$$a = \frac{dv}{dt} = -2e^{-2t} \quad (a.)$$

$$v = \frac{dx}{dt} = e^{-2t} \quad x = \int v dt \Rightarrow x = -\frac{1}{2}e^{-2t} + C$$

$$x = 2 \text{ when } t = 0 \therefore C = \frac{5}{2}$$

$$x(t) = \frac{1}{2}(5 - e^{-2t}) \quad (b.)$$

$$\text{Solving Eq. (b.) for } t: e^{-2t} = 5 - 2x$$

$$t = -\frac{1}{2} \ln(5 - 2x) \quad (c.)$$

Substitution of Eq. (c.) into Eq. (a.) gives:

$$a(x) = -2e^{-2[-\frac{1}{2} \ln(5 - 2x)]} \Rightarrow a(x) = 4x - 10$$

13.7

$$a.) \Delta s_s = 2\pi(4000 \text{ miles}) = 25132.74 \text{ miles}$$

$$= 132,700,873 \text{ ft}$$

The displacement of your feet relative to the earth would be zero.  $\Delta x = 0 \text{ ft}$

$$b.) \Delta s_h = 2\pi[4000 \text{ mi}(\frac{5280 \text{ ft}}{\text{mi}}) + 6 \text{ ft}]$$

$$= 132,700,911.4 \text{ ft}$$

$$\text{Difference: } 132,700,911.4 - 132,700,873.7$$

$$= 37.7 \text{ ft}$$

$$\therefore \text{Your head travels } 37.7 \text{ ft more}$$

$$\text{Alternatively: } 2\pi(6 \text{ ft}) = 37.7 \text{ ft}$$

13.8  $d$  - total distance traveled = 2 mi  
 $t$  - total time over 2 mile stretch  
 $t = \frac{1 \text{ mile}}{30 \text{ mph}} + \frac{1 \text{ mile}}{V_{\text{reg}}}$

$V_{\text{reg}}$  = speed required to average 45 mph over 2 mile stretch

$$\frac{d}{t} = 45 \text{ mph} \Rightarrow \frac{2 \text{ mi}}{\frac{1 \text{ mi}}{30 \text{ mph}} + \frac{1 \text{ mi}}{V_{\text{reg}}}} = 45 \text{ mph}$$

$$V_{\text{reg}} = \frac{1 \text{ mi}}{\frac{2 \text{ mi}}{45 \text{ mph}} - \frac{1 \text{ mi}}{30 \text{ mph}}} \Rightarrow \underline{V_{\text{reg}} = 90 \text{ mph}}$$

13.9  $d$  - total distance traveled = 2 mi  
 $t$  - total time around 2 mile stretch  
 $t = \frac{1 \text{ mile}}{30 \text{ mph}} + \frac{1 \text{ mile}}{V_{\text{reg}}}$

$V_{\text{reg}}$  = speed required to average 60 mph over 2 mile stretch

$$\frac{d}{t} = 60 \text{ mph} \Rightarrow \frac{2 \text{ mi}}{\frac{1 \text{ mi}}{30 \text{ mph}} + \frac{1 \text{ mi}}{V_{\text{reg}}}} = 60 \text{ mph}$$

$$V_{\text{reg}} = \frac{1 \text{ mi}}{\frac{2 \text{ mi}}{60 \text{ mph}} - \frac{1 \text{ mi}}{30 \text{ mph}}} \Rightarrow \underline{V_{\text{reg}} = \infty}$$

(It is impossible to go fast enough to average 60 mph — there is no time left.)

13.10 a.)  $v_0 = \left(\frac{1}{6} \frac{\text{mi}}{\text{s}}\right) (5280 \frac{\text{ft}}{\text{mi}}) = 880 \text{ ft/s}$

After power is reduced,  $a = -g = -32.2 \text{ ft/s}^2$

$$\therefore v = v_0 + at = 880 - 32.2 t = 0 \text{ at highest pt.}$$

$$\therefore t = 880 / 32.2 \quad \underline{t = 27.4 \text{ s}}$$

b.) The height the plane rises above the point at which the power is reduced is:

$$x = v_0 t + \frac{1}{2} a t^2 = 880(27.35) - \frac{1}{2}(32.2)(27.35)^2$$

$$x = 12,025 \text{ ft} = \underline{2.28 \text{ mi}}$$

13.11  $v_y = \dot{y} \quad a_y = \ddot{y}$

$$L^2 = x^2 + y^2$$

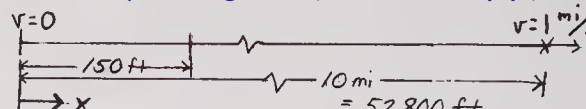
$$\text{Differentiating: } 0 = 2x\dot{x} + 2y\dot{y}$$

$$\dot{x} = -\frac{y\dot{y}}{x} \Rightarrow \underline{\underline{\dot{x} = \frac{v_y \sqrt{L^2 - x^2}}{x}}}$$

$$\ddot{x} = \frac{(-\dot{v}_y \sqrt{L^2 - x^2} - v_y^2 \frac{x}{\sqrt{L^2 - x^2}})}{x} - \frac{v_y^2}{x}$$

$$\ddot{x} = \frac{a_y \sqrt{L^2 - x^2}}{x} - \frac{v_y^2 (L^2 - x^2)}{x^3} - \frac{v_y^2}{x}$$

$$\underline{\underline{\ddot{x} = \frac{a_y x^2 \sqrt{L^2 - x^2} - v_y^2 L^2}{x^3}}}$$

13.12  $v=0$   $v=1 \text{ mi/s}$   


$$v = \int a dt = at + v_0 \quad \text{Since } v=0 \text{ when } t=0$$

$$\therefore t = v/a \quad (A)$$

$$x = \int v dt = \frac{1}{2} a t^2 + x_0 = \frac{1}{2} a t^2 \quad \text{since } x=0 \text{ when } t=0$$

$$\therefore x = \frac{1}{2} a t^2 \quad (B)$$

By Eqs (A) and (B):  $x = \frac{1}{2} a t^2 = \frac{1}{2} \frac{v^2}{a} \quad (C)$

$$\therefore a = \frac{1}{2} \frac{v^2}{x} = \frac{1}{2} \frac{[(1 \text{ mi/s})(5280 \text{ ft/mi})]^2}{(10 \text{ mi})(5280 \text{ ft/mi})} = 264 \text{ ft/s}^2$$

Substituting  $a = 264 \text{ ft/s}^2$  into (B):

$$150 \text{ ft} = \frac{1}{2} (264 \text{ ft/s}^2) t^2$$

$$\therefore \underline{t = 1.066 \text{ s}} = \text{time rocket is in contact with guides.}$$

13.13

a.)  $a = 32.2 \text{ ft/s}^2 \quad v_0 = 0$

$$v = \int a dt = at + v_0 \Rightarrow v = 32.2 t$$

$$\left(\frac{5280 \text{ ft}}{\text{mi}}\right) \left(\frac{186,000 \text{ mi}}{\text{s}}\right) = 32.2 \frac{\text{ft}}{\text{s}^2} (t)$$

$$t = 30,499,378.9 \text{ s} \Rightarrow \underline{t = 0.967 \text{ years}}$$

b.) At the end of 0.967 years, the distance traveled is:

$$S_1 = \frac{1}{2} a t^2 = \frac{1}{2} (32.2) (3.0499 \times 10^7)^2 \left(\frac{1}{5280}\right) = 2.836 \times 10^{12} \text{ mi}$$

Let  $S_2$  be distance remaining to be traveled

$$\therefore S_2 = 2.5 \times 10^{13} - S_1 = (2.5 - 0.2836) (10^{13}) \text{ mi} = 2.216 \times 10^{13} \text{ mi}$$

Let  $t_2$  = time to travel distance  $S_2$

$$\therefore t_2 = \frac{2.216 (10^{13})}{1.86 (10^5)} = 1.191 (10^8) \text{ s} = 3.774 \text{ years}$$

$$t_{\text{total}} = t_1 + t_2 = 0.967 + 3.774 = \underline{4.74 \text{ years}}$$

13.14

a.)  $v = 16t - t^2 \text{ mi/h} = \frac{1}{60} (16t - t^2) \text{ mi/min}$

$$\frac{dv}{dt} = \frac{1}{60} (16 - 2t) = 0 \text{ at } t = 8 \text{ min}$$

$$\therefore v_{\text{max}} = \frac{1}{60} [16(8) - (8)^2] = \frac{64}{60} \text{ mi/min}$$

Hence;

$$S = \frac{1}{60} \int_0^8 (16t - t^2) dt + \frac{64}{60} (12 - 8)$$

$$= \frac{1}{60} (8t^2 - \frac{1}{3} t^3) \Big|_0^8 + \frac{64(4)}{60} \quad \underline{S = 9.96 \text{ mi}}$$

b.)

$$v_{\text{ave}} (\text{mi/hr}) = \frac{9.956 \text{ mi} (60 \text{ min})}{12 \text{ min} (1 \text{ h.})} = \underline{49.78 \text{ mi/h}}$$

13.15  $a = \frac{dv}{dt} = \frac{k}{v}$

$\therefore \int k dt = \int v dv \rightarrow \frac{1}{2} v^2 = kt + C$

when  $t=0, v=2 \therefore C=2$

when  $t=3, v=4 \therefore k=2$

Hence,  $v^2 = 4(t+1)$

When  $t=8s, v^2=36 \rightarrow \underline{v=6 \text{ m/s}}$

13.16  $x = C_3 t^3 + C_2 t^2 + C_1$

a.)  $v = \frac{dx}{dt} = 3C_3 t^2 + 2C_2 t$

$a = \frac{dv}{dt} = 6C_3 t + 2C_2$

b.) Distance traveled from  $t=t_0$  to  $t=t_1$ :

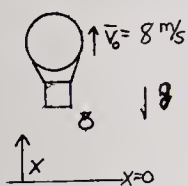
at  $t=t_0, x_0 = C_3(t_0)^3 + C_2(t_0)^2 + C_1$

at  $t=t_1, x_1 = C_3(t_1)^3 + C_2(t_1)^2 + C_1$

$\Delta S = \text{distance traveled} = x_1 - x_0$

$\Delta S = C_3(t_1^3 - t_0^3) + C_2(t_1^2 - t_0^2)$

13.17



a.)  $a = -g = -9.81 \text{ m/s}^2$

$v = \int a dt = -9.81 t + v_0$

$v = -9.81 t + 8$

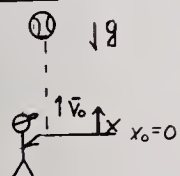
$x = \int v dt = -4.91 t^2 + 8t + x_0$

$x=0$  when it hits the ground

$0 = -4.91(10)^2 + 8(10) + x_0 \rightarrow \underline{x_0 = 411 \text{ m}}$

b.)  $v = -9.81(10) + 8 \quad v = -90.1 \text{ m/s} \text{ or } \underline{90.1 \text{ m/s} \downarrow}$

13.18



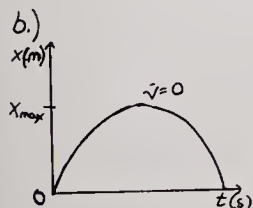
a.)  $a = -g = -9.81 \text{ m/s}^2$

$v = \int a dt = -9.81 t + v_0$

$x = \int v dt = -4.91 t^2 + v_0 t + x_0$

$0 = -4.91(4)^2 + v_0(4) \quad (x=0, t=4s)$

$\therefore \underline{v_0 = 19.64 \text{ m/s}}$



$v = -9.81 t + 19.64$

As shown on curve,  $x_{\max}$  occurs when  $\bar{v} = 0$ .

$\therefore 0 = -9.81 t_m + 19.64$

$t_m = 2.00 \text{ s}$

$x = -4.91 t^2 + 19.64 t$

$x_{\max} = -4.91(2)^2 + 19.64(2)$

$\underline{x_{\max} = 19.64 \text{ m}}$

a.) acceleration for  $t=4s$ :

$a = (4)^3 + 3(4) \rightarrow \underline{a = 76 \text{ ft/s}^2}$

velocity for  $t=4s$ :

$v = \int_0^t (t^3 + 3t) dt \rightarrow v = \frac{1}{4} t^4 + \frac{3}{2} t^2 + v_0$

Since  $v=0$  when  $t=0, v_0=0$

$v(4s) = \frac{1}{4} (4)^4 + \frac{3}{2} (4)^2 \rightarrow \underline{v = 88 \text{ ft/s}}$

displacement for  $t=4s$ :

$\Delta x = x(4) - x(0) = \int_0^4 v dt$

$= \int_0^4 (\frac{1}{4} t^4 + \frac{3}{2} t^2) dt = (\frac{1}{20} t^5 + \frac{1}{2} t^3) \Big|_0^4$

$\underline{\Delta x = 83.2 \text{ ft}}$

b.)  $v = \frac{1}{4} t^4 + \frac{3}{2} t^2$

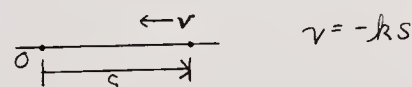
$v = 186,000 \text{ m/s} \left( 5280 \frac{\text{ft}}{\text{mi}} \right) = 9.8208 \times 10^8 \text{ ft/s}$

$9.8208 \times 10^8 = \frac{1}{4} t^4 + \frac{3}{2} t^2$

Using quadratic equation:

$\underline{t = 250.4 \text{ s}}$

13.20



a.)  $v = \frac{ds}{dt} = -ks \rightarrow \frac{1}{s} ds = -k dt$

$\int_{s_0}^s \frac{1}{s} ds = -\int_0^t k dt \rightarrow \ln \frac{s}{s_0} = -kt$

$\underline{s = s_0 e^{-kt}}$

b.)  $t=0, v=20 \text{ in/s}, s=10 \text{ in}$

Speed  $= |v| = ks$

$20 = k(10) \rightarrow \therefore k = 2 \text{ s}^{-1}$

$10 = s_0 e^{-2(0)} \rightarrow \therefore s_0 = 10 \text{ in}$

$s = 10 e^{-2t} \rightarrow \therefore t = -\frac{1}{2} \ln \frac{s}{10}$

$t(s=8 \text{ in}) = \underline{0.112 \text{ s}}$

$t(s=6 \text{ in}) = \underline{0.255 \text{ s}}$

$t(s=0 \text{ in}) = \underline{\infty}$

13.21

$v = t^2 + 20t \rightarrow t = \frac{-20 \pm \sqrt{20^2 - 4(1)(-v)}}{2(1)}$

Negative time does not apply:

$t(v=200) = 7.32 \text{ s} \quad t(v=600) = 16.46 \text{ s}$

$\Delta s = \int |v| dt = \int v dt$  since  $v$  is positive

$\Delta s = \int_{7.32}^{16.46} (t^2 + 20t) dt = \left[ \frac{1}{3} t^3 + 10t^2 \right] \Big|_{7.32}^{16.46}$

$\underline{\Delta s = 3530 \text{ ft}}$

13.22

a)  $a = -kv$   $v=v_0, x=0$  when  $t=0$

$$\int dt = \int \frac{dv}{a(v)} \Rightarrow t-0 = \int_{v_0}^v \frac{dv}{-kv}$$

$$t = -\frac{1}{k} \ln v \Big|_{v_0}^v = -\frac{1}{k} \ln \left( \frac{v}{v_0} \right) \quad (A)$$

$$v = v_0 e^{-kt}$$

$$\int dx = \int v(t) dt \Rightarrow x-0 = \int_0^t v_0 e^{-kt} dt$$

$$x = -\frac{v_0}{k} e^{-kt} \Big|_0^t = -\frac{v_0}{k} (e^{-kt} - 1)$$

$$x = \frac{v_0}{k} (1 - e^{-kt}) \quad (B)$$

b) By Eqn (A),  $t = -\frac{1}{k} \ln \left( \frac{v}{v_0} \right)$  and,

By Eqn (B),  $x = \frac{v_0}{k} (1 - e^{-kt})$ .

Combining (A) and (B) gives:

$$x = \frac{v_0}{k} (1 - e^{-k(\frac{1}{k} \ln \frac{v_0}{v})}) \quad \text{or} \quad \underline{x(v) = \frac{1}{k} (v_0 - v)}$$

13.23

$a = 4$ ,  $v = -6$ ,  $x = 0$  when  $t = 0$

$$v = \int a dt = \int 4 dt \Rightarrow v = 4t + v_0$$

$$v(t=0) = 0 + v_0 = -6 \Rightarrow v_0 = -6 \quad \underline{v = 4t - 6}$$

$$x = \int v dt = \int_0^t (4t - 6) dt$$

$$x = 2t^2 - 6t + x_0 \Rightarrow x(t=0) = 0 - 0 + x_0 = 0 \Rightarrow x_0 = 0$$

$$\underline{x = 2t^2 - 6t}$$

From  $v(t) \Rightarrow t = \frac{v+6}{4}$

Substitution of  $t$  into  $x(t)$  gives

$$x = 2 \left( \frac{v+6}{4} \right)^2 - 6 \left( \frac{v+6}{4} \right) = \frac{1}{8} v^2 - 4.5$$

$$\underline{v = \sqrt{8x + 36} = 2\sqrt{2x + 9}}$$

13.24

$a = 4t$ ;  $v = 2$ ,  $x = 1$  when  $t = 1$

$$v = \int_1^t a dt + v_1 \Rightarrow v = \int_1^t 4t dt + v_1$$

$$v = 2t^2 - 2 + v_1 \Rightarrow v(t=1) = 2 \Rightarrow v_1 = 2$$

$$\underline{v(t) = 2t^2} \quad (A)$$

$$x = \int_1^t v dt + x_1 \Rightarrow x = \int_1^t 2t^2 dt + x_1$$

$$x = \frac{2}{3} (t^3 - 1^3) + x_1 \Rightarrow x(t=1) = 1 = x_1 \Rightarrow x_1 = 1$$

$$\underline{x = \frac{2}{3} t^3 + \frac{1}{3}} \quad (B)$$

By Eqn (A),  $t = \sqrt{v/2} \quad (C)$

By Eqn (B) and (C),  $x = \frac{2}{3} \left( \frac{v}{2} \right)^{3/2} + \frac{1}{3}$

$$\underline{v = 2 \left( \frac{3}{2} x - \frac{1}{2} \right)^{2/3}}$$

13.25

$a = e^{-2t}$ ;  $v=0$ ,  $x=2$  when  $t=0$

$$v = \int a dt \Rightarrow v = \int e^{-2t} dt$$

$$v = -\frac{1}{2} e^{-2t} + v_0 \Rightarrow v(0) = 0 = -\frac{1}{2} (1) + v_0$$

$$\therefore v_0 = \frac{1}{2} \quad \therefore \underline{v(t) = \frac{1}{2} (1 - e^{-2t})} \quad (A)$$

$$x = \int v dt \Rightarrow x = \int \left( \frac{1}{2} - \frac{1}{2} e^{-2t} \right) dt$$

$$x = \frac{1}{2} t + \frac{1}{4} e^{-2t} + x_0 \Rightarrow x(0) = 2 = 0 + \frac{1}{4} + x_0$$

$$\therefore x_0 = 1.75 \quad \therefore \underline{x(t) = \frac{1}{2} t + \frac{1}{4} e^{-2t} + 7/4} \quad (B)$$

By Eqn (A);  $t = -\frac{1}{2} \ln (1 - 2v) \quad (C)$

By Eqns (B) and (C):

$$x = -\frac{1}{4} \ln (1 - 2v) + \frac{1}{4} e^{\ln(1-2v)} + 7/4$$

$$\therefore \underline{x(v) = -\frac{1}{4} [\ln (1 - 2v) - 8 + 2v]}$$

13.26

$a = \sin 2t$ ;  $v=0$ ,  $x=0$  when  $t=0$

$$v = \int a dt \Rightarrow v = \int \sin 2t dt$$

$$v = -\frac{1}{2} \cos 2t + v_0 \Rightarrow v(t=0) = -\frac{1}{2} (1) + v_0 = 0$$

$$\therefore v_0 = \frac{1}{2} \quad \therefore \underline{v(t) = \frac{1}{2} (1 - \cos 2t)}$$

$$\underline{v = \sin^2 t} \quad (A)$$

$$x = \int v dt \Rightarrow x = \int \left( \frac{1}{2} - \frac{1}{2} \cos 2t \right) dt$$

$$x = \frac{1}{2} t - \frac{1}{4} \sin 2t + x_0 \Rightarrow x(0) = 0 - 0 + x_0 = 0$$

$$\therefore x_0 = 0 \quad \therefore \underline{x = \frac{1}{2} t - \frac{1}{4} \sin 2t} \quad (B)$$

By Eqn (A),  $t = \sin^{-1} \sqrt{v} \quad (C)$

By Eqns (B) + (C),

$$\underline{x(v) = \frac{1}{2} \sin^{-1} \sqrt{v} - \frac{1}{4} \sin [2(\sin^{-1} \sqrt{v})]}$$

13.27

$a = 32 - 4v$ ;  $v=4$ ,  $x=0$  when  $t=0$

$$\int_0^t dt = \int_{v_0}^v \frac{dv}{a(v)} \Rightarrow \int_0^t dt = \int_4^v \frac{dv}{32 - 4v}$$

Using Integration tables:  $\int \frac{dx}{a+bx} = \frac{1}{b} \ln(a+bx)$

$$t = -\frac{1}{4} \ln (32 - 4v) \Big|_4^v$$

$$\therefore \underline{v(t) = 4(2 - e^{-4t})} \quad (A)$$

By Eqn (A):  $x = \int v dt = 4 \int (2 - e^{-4t}) dt$

or  $x = 8t + e^{-4t} + x_0$ ;  $x_0 = -1$  since  $x=0, t=0$

$$\therefore \underline{x = 8t + e^{-4t} - 1} \quad (B)$$

By Eqn (A):  $t = -\frac{1}{4} \ln \left( \frac{8-v}{4} \right) \quad (C)$

By Eqns (B) and (C)

$$x = 8 \left[ -\frac{1}{4} \ln \left( \frac{8-v}{4} \right) \right] + e^{\ln \left( \frac{8-v}{4} \right)} - 1$$

Simplifying,  $\underline{x(v) = 3.77 - 2 \ln (8-v) - \frac{v}{4}}$

13.28  $a = -\frac{4}{x^2}$ ;  $v = 4$ ,  $x = 2$  when  $t = 0$   $v > 0$

Find  $v(x)$ :

$$a = -\frac{4}{x^2} = \frac{dv}{dt} \frac{dx}{dx} = v \frac{dv}{dx} \quad \therefore \int v dv = -4 \int \frac{dx}{x^2}$$

$$\frac{1}{2} v^2 \Big|_4^v = \frac{4}{x} \Big|_2^x \quad \text{or} \quad \frac{v^2}{2} = \frac{4}{x} + 6$$

$$\therefore v = 2\sqrt{\frac{3x+2}{x}} \quad (A)$$

Find  $x(t)$ :

$$v = \frac{dx}{dt} = 2\sqrt{\frac{3x+2}{x}} \quad \therefore \int \frac{\sqrt{x} dx}{2\sqrt{3x+2}} = \int dt = t \quad (B)$$

By Integral tables: or computer:

$$\int \frac{\sqrt{x} dx}{\sqrt{3x+2}} = \int \frac{dx}{\sqrt{\frac{3}{x}+2}} = -2 \int \frac{y^{\frac{3}{2}} dy}{y^2 \sqrt{y+3}} \quad (C)$$

where  $y = 2/x$ .

By Integration tables or computer:

$$\begin{aligned} \int \frac{y^{\frac{3}{2}} dy}{y^2 \sqrt{y+3}} &= -\frac{\sqrt{y+3}}{3y} \Big|_{\frac{2}{x}}^{\frac{2}{x}} - \frac{1}{6} \int \frac{dy}{y \sqrt{y+3}} \\ &= -\frac{\sqrt{y+3}}{3y} \Big|_{\frac{2}{x}}^{\frac{2}{x}} - \frac{1}{6} \left[ \frac{1}{\sqrt{3}} \ln \frac{\sqrt{y+3}-\sqrt{3}}{\sqrt{y+3}+\sqrt{3}} \right]_{\frac{2}{x}}^{\frac{2}{x}} \quad (D) \end{aligned}$$

By Eqns (B), (C), and (D):

$$t = \frac{1}{6} \sqrt{2x+3x^2} + \frac{1}{10.392} \ln \left[ \frac{\sqrt{2x+3x^2}-\sqrt{3}x}{\sqrt{2x+3x^2}+\sqrt{3}x} \right] - 0.413$$

Find  $v(t)$ :

$$\text{By Eqn (A): } x = \frac{8}{v^2-12} \quad (F)$$

$$\text{Hence, } \sqrt{2x+3x^2} = \frac{4v}{v^2-12} \quad (G)$$

So, by substituting (F), (G), and  $x(t)$ :

$$t = \frac{2v}{3v^2-36} + \frac{1}{10.392} \ln \left( \frac{v-2\sqrt{3}}{v+2\sqrt{3}} \right) - 0.413$$

for  $v > 2\sqrt{3}$

13.29  $a = -8x$ ;  $v = 0$ ,  $x = 6$  when  $t = 0$

Find  $v(x)$ :

$$\frac{1}{2} (v(x)^2 - v(x_0)^2) = \int_{x_0}^x a(x) dx$$

$$\frac{1}{2} (v(x)^2 - (0)^2) = \int_6^x (-8x) dx$$

$$\frac{1}{2} v^2 = -4x^2 \Big|_6^x \rightarrow v^2 = 288 - 8x^2 \quad (A)$$

$$\therefore v = \sqrt{288 - 8x^2} \quad \text{or} \quad v = 2\sqrt{72 - 2x^2}$$

Find  $x(t)$ :

$$t - t_0 = \int_{x_0}^x \frac{dx}{v(x)} \rightarrow t = \int_6^x \frac{dx}{\sqrt{288 - 8x^2}} + t_0$$

By integration tables:

$$\int \frac{dx}{\sqrt{cx^2+bx+a}} = \frac{1}{\sqrt{-c}} \sin^{-1} \left( \frac{-2cx-b}{\sqrt{b^2-4ac}} \right) \quad \text{for } c < 0$$

Then,  $t = \frac{1}{\sqrt{8}} \sin^{-1} \left( \frac{x}{6} \right) - \frac{1}{\sqrt{8}} \sin^{-1} \left( \frac{x_0}{6} \right)$

$$t = \frac{1}{\sqrt{8}} \sin^{-1} \left( \frac{x}{6} \right) - \frac{\pi}{2\sqrt{8}}$$

$$t + \frac{\pi}{2\sqrt{8}} = \frac{1}{\sqrt{8}} \sin^{-1} \left( \frac{x}{6} \right) \rightarrow \sin(\sqrt{8}t + \frac{\pi}{2}) = \frac{x}{6}$$

$$\therefore x = \cos(\sqrt{8}t)$$

Find  $v(t)$ :

$$\text{From Eqn (A): } v^2 = 288 - 8x^2$$

$$\text{and } x = 6 \cos(\sqrt{8}t) \text{ so,}$$

$$\begin{aligned} v^2 &= 288 - 8(6 \cos(\sqrt{8}t))^2 \\ &= 288 - 288 \cos^2(\sqrt{8}t) = 288(1 - \cos^2 \sqrt{8}t) \end{aligned}$$

$$v = 288 \sin^2(\sqrt{8}t) \quad \therefore v = 12\sqrt{2} \sin(\sqrt{8}t)$$

13.30  $a = -kv$ ;  $v = v_0$ ,  $x = 0$  when  $t = 0$

Find  $v(t)$ :

$$a = -kv = \frac{dv}{dt} \rightarrow \int_0^t dt = -\frac{1}{k} \int_{v_0}^v \frac{dv}{v}$$

$$t = -\frac{1}{k} \ln(v) \Big|_{v_0}^v = -\frac{1}{k} \ln \frac{v}{v_0}$$

$$\therefore v = v_0 e^{-kt} \quad (A)$$

Find  $x(t)$ :

$$\frac{dx}{dt} = v \rightarrow \int_0^x dx = \int_0^t v dt = v_0 \int_0^t e^{-kt} dt$$

$$x = -\frac{v_0}{k} e^{-kt} \Big|_0^t \quad \text{or} \quad x = \frac{v_0}{k} (1 - e^{-kt}) \quad (B)$$

Find  $v(x)$ :

$$\text{By Eqn's (A) + (B): } x = \frac{v_0}{k} (1 - \frac{v}{v_0}) = \frac{1}{k} (v_0 - v)$$

$$v = v_0 - kx$$

13.31  $a = -kv^2$ ;  $v = v_0$ ,  $x = x_0$  when  $t = 0$

Find  $v(t)$ :

$$a = -kv^2 = \frac{dv}{dt} \rightarrow \int_0^t dt = -\frac{1}{k} \int_{v_0}^v \frac{dv}{v^2}$$

$$t = \frac{1}{kv} - \frac{1}{kv_0} \quad \therefore v = \frac{v_0}{1 + kv_0 t} \quad (A)$$

Find  $x(t)$ :

$$v = \frac{dx}{dt} \quad \text{or} \quad \int_{x_0}^x dx = \int_0^t v dt = \int_0^t \frac{v_0 dt}{1 + kv_0 t}$$

$$\therefore x = v_0 \int_0^t \frac{dt}{1 + kv_0 t} = \frac{1}{k} \ln(1 + kv_0 t) \quad (B)$$

By Eqn's (A) and (B):

$$x = \frac{1}{k} \ln \frac{v_0}{v}$$

13.32 **GIVEN:**  $Q [m^3/s] = \text{constant}$

$$\text{Volume of bubble} = \frac{4}{3} \pi r^3$$

**FIND**  $v(r)$  and  $a(r)$  for particle of soap film

**SOLUTION:** Time rate of change of Volume is:

$$Q = \frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt} = 4\pi r^2 v$$

where  $\frac{dr}{dt} = v$  is the velocity of a particle.

$$\text{Hence, } v = \frac{Q}{4\pi r^2}$$

The acceleration of a particle is given by the chain rule;

$$a = \frac{dv}{dt} = \frac{dv}{dr} \frac{dr}{dt} = v \frac{dv}{dr}$$

$$= \left( \frac{Q}{4\pi r^2} \right) \left( -\frac{2Q}{4\pi r^3} \right) \quad \text{or} \quad a = -\frac{Q^2}{8\pi^2 r^5}$$

13.33 **GIVEN** 

Position of part given by:

$$x = t^4 - 10t^2 + 24 \text{ [mm]}$$

**FIND** a)  $v$  and  $a$  of part

b)  $x$  as function of  $a$

**SOLUTION** a)  $v(t) = \frac{dx}{dt} = 4t^3 - 20t \text{ [mm/s]}$

$$a(t) = \frac{dv}{dt} = 12t^2 - 20 \text{ [mm/s}^2\text{]}$$

b)  $t(a) = \sqrt{\frac{a+20}{12}} \Rightarrow$  Sub into given  $x(t)$

$$\therefore x(a) = \left( \frac{a+20}{12} \right)^2 - 10 \left( \frac{a+20}{12} \right) + 24$$

13.34 **GIVEN** Body falls 16 ft in first second and falls 32 ft more each second thereafter.

**FIND** Show this is correct if  $g = 32 \text{ ft/s}^2$

**SOLUTION**

For the first second with  $x_0 = v_0 = 0$  for  $t=0$ , the body falls a distance:

$$x_1 = 16 \text{ ft} = \frac{1}{2} a t^2 = \frac{1}{2} a (1)^2 \quad \therefore a = 32 \text{ ft/s}^2$$

At end of first second,  $v_1 = at = 32(1) = 32 \text{ ft/s}$

Therefore, at the end of the next second, the body falls an additional distance:

$$x_2 = v_1 t_2 + \frac{1}{2} a t_2^2 = (32)(1) + \frac{1}{2} (32)(1) = 48 \text{ ft}$$

Since  $48 \text{ ft} = 32 \text{ ft} + 16 \text{ ft}$ , this verifies that in the second second, the body falls 32 ft more than it fell in the first second (16 ft).

At the end of the 2<sup>nd</sup> second,

$$v_2 = v_1 + at = 32 + 32(1) = 64 \text{ ft/s}$$

So, at the end of the third second, the body falls an additional distance:

$$x_3 = v_2 t + \frac{1}{2} a t^2 = (64)(1) + \frac{1}{2} (32)(1)^2 = 80 \text{ ft}$$

It can be seen that  $80 \text{ ft} = 48 \text{ ft} + 32 \text{ ft}$ .

Thus, in the third second, the body falls 32 ft more than it fell (48 ft) in the second second, and so on.

13.35 **GIVEN**  $a = 1 \text{ m/s}^2$ ,  $v_{\text{max}} = 100 \text{ km/hr}$

**FIND** a.) least time [s] required for car to travel 1 km if starting from rest

b.) sketch the velocity-time graph

**SOLUTION**  $v = 100 \text{ km/hr} = 27.78 \text{ m/s}$   $v_0 = 0$

a.) The minimum time to travel 1 km is attained if the car accelerates to 100 km/hr and then maintains that speed.

The time required to attain a speed of 100 km/hr is found by:  $v = at$ ;  $27.78 \text{ m/s} = (1)t$  or  $t_1 = 27.78 \text{ s}$

In this time, the car travels the distance

$$x = \frac{1}{2} a t^2 = \frac{1}{2} (1)(27.78)^2 = 385.8 \text{ m} = 0.386 \text{ km}.$$

In other words, it has not yet traveled 1 km.

To travel the remaining distance, the time required is found from the condition that:

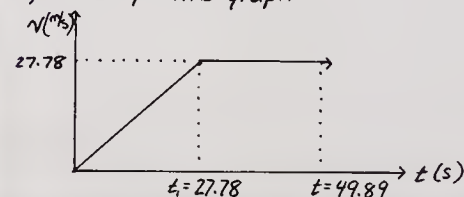
$$\Delta s = (1000 - 385.8) = v t_2 = 27.78 t_2$$

$$\text{or } t_2 = 22.11 \text{ s}$$

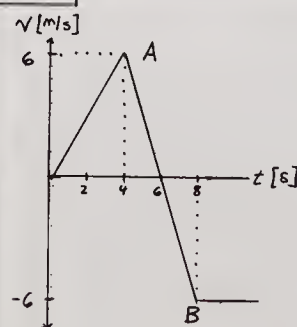
Therefore, total time is:

$$t = t_1 + t_2 = 27.78 + 22.11 = 49.89 \text{ s}$$

b.) Velocity-time graph:



13.36



**GIVEN** Machine part moves along straight line, has time-velocity graph shown.

**FIND** a.) displacement of part in intervals  $t = 0-4 \text{ s}$ ,  $t = 4-8 \text{ s}$ , and  $t = 0-8 \text{ s}$ .

b.) distance traveled by particle in same intervals

(Continued)

13.36 cont.

**SOLUTION** a) Displacement,  $t=0$  to  $4s$ :

$$\Delta X_{0-4} = \int_0^4 \frac{3}{2}t \, dt = \frac{3}{4}t^2 \Big|_0^4 \quad \underline{\Delta X_{0-4} = 12 \text{ m}}$$

Displacement,  $t=4$  to  $8s$ :

$$\Delta X_{4-8} = \int_4^8 (-3t + 18) \, dt = \left(-\frac{3}{2}t^2 + 18t\right) \Big|_4^8$$

$$\underline{\Delta X_{4-8} = 0 \text{ m}}$$

Displacement,  $t=0$  to  $8s$ :

$$\Delta X_{0-8} = \int_0^4 \frac{3}{2}t \, dt + \int_4^8 (-3t + 18) \, dt$$

$$\underline{\Delta X_{0-8} = 12 \text{ m}}$$

b) Distance traveled,  $t=0$  to  $4s$ :

$$\Delta S_{0-4} = |\Delta X_1| + |\Delta X_2| = 10 + 12 \quad \underline{\Delta S_{0-4} = 12 \text{ m}}$$

Distance traveled,  $t=4$  to  $8s$ :

$$\Delta S_{4-8} = |\Delta X_3| + |\Delta X_4| = |18-12| + |12-18| = 6 + 6$$

$$\underline{\Delta S_{4-8} = 12 \text{ m}}$$

Distance traveled,  $t=0$  to  $8s$ :

$$\Delta S_{0-8} = |\Delta X_1| + |\Delta X_2| + |\Delta X_3| = |12-0| + |18-12| + |12-18|$$

$$= 12 + 6 + 6 \quad \underline{\Delta S_{0-8} = 24 \text{ m}}$$

### ★ ALTERNATIVE SOLUTION

Displacement from  $t=0$  to  $t=4s$  is equal to the area of the  $v-t$  diagram. Therefore,

$$\Delta X_{0-4} = \frac{1}{2}(4)(6) = \underline{12 \text{ m}}$$

Displacement from  $t=4$  to  $t=8s$ :

$$\Delta X_{4-8} = \frac{1}{2}(6-4)(6) - \frac{1}{2}(8-6)(6) = \underline{0 \text{ m}}$$

$\therefore$  Displacement from  $t=0$  to  $t=8s$  is:

$$\Delta X_{0-8} = \Delta X_{0-4} + \Delta X_{4-8} = 12 + 0 = \underline{12 \text{ m}}$$

Distance traveled from  $t=0$  to  $t=4s$  is

$$\Delta S_{0-4} = \frac{1}{2}(4)(6) = \underline{12 \text{ m}}$$

Distance traveled from  $t=4$  to  $t=8s$  is

$$\Delta S_{4-8} = |\Delta X_{4-6}| + |\Delta X_{6-8}| = \frac{1}{2}(6-4)(6) + \frac{1}{2}(8-6)(6)$$

$$\therefore \Delta S_{4-8} = 6 + 6 = \underline{12 \text{ m}}$$

Distance traveled from  $t=0$  to  $t=8s$  is

$$\Delta S_{0-8} = \Delta S_{0-4} + \Delta S_{4-8}$$

$$= 12 \text{ m} + 12 \text{ m} \quad \therefore \underline{\Delta S_{0-8} = 24 \text{ m}}$$

13.37

GIVEN Figure from Problem 13.36

**FIND** a.) acceleration of part at  $t=2s$  +  $t=6s$   
b.) Discuss Significance of pts A and B

**SOLUTION**

a.) From  $v-t$  diagram:

$$v = \begin{cases} \frac{3}{2}t & 0 \leq t < 4 \\ -3t + 18 & 4 \leq t < 8 \\ -6 & t > 8 \end{cases}$$

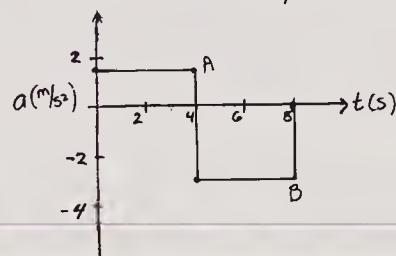
$$a = \frac{dv}{dt}$$

$$a = \begin{cases} \frac{3}{2} & 0 \leq t < 4 \\ -3 & 4 \leq t < 8 \\ 0 & t > 8 \end{cases}$$

$$\text{At } t=2s, \quad \underline{a = 1.5 \text{ m/s}^2}$$

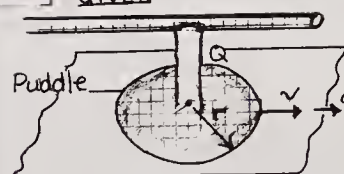
$$\text{At } t=6s, \quad \underline{a = -3 \text{ m/s}^2}$$

b.) By plotting an  $a-t$  diagram, it can be seen that points A and B are points at which the acceleration changes from  $1.5$  to  $-3 \text{ m/s}^2$  and from  $-3$  to  $0 \text{ m/s}^2$  instantaneously.



13.38

GIVEN



$h$  = thickness of puddle  
= constant

$Q$  = rate at which water leaks [ $\text{in}^3/\text{s}$ ]

**FIND** velocity,  $v$  and acceleration,  $a$  on edge of puddle as functions of  $r$ ,  $Q$  and  $h$ .

**SOLUTION**

$$\text{Volume of puddle} \Rightarrow V = \pi r^2 h$$

$$\text{By chain rule, } \frac{dV}{dt} = \frac{dV}{dr} \frac{dr}{dt} = 2\pi r h \frac{dr}{dt} = Q$$

$$\therefore v = \frac{dr}{dt} = \frac{Q}{2\pi r h}$$

Again, by chain rule:

$$a = \frac{dv}{dt} = \frac{dv}{dr} \frac{dr}{dt} = \left(-\frac{Q}{2\pi r^2 h}\right) v = \underline{\underline{-\frac{Q^2}{4\pi^2 h^2 r^3}}}$$

13.39

**GIVEN** Position of body given by:  
 $x = 3t^2 - 6t$   $x$  [m],  $t$  [s] (A)

- FIND**
- $v(t)$
  - Plot of time-velocity graph
  - Body's displacement for  $t=0$  to  $t=2s$  and  $t=0$  to  $t=4s$
  - Distance traveled for same intervals

**SOLUTION**

a)  $v = \frac{dx}{dt} = 6t - 6$  [m/s] (B)

- b) By Eq (B), the graph of  $v$  vs.  $t$  is shown in Figure a:

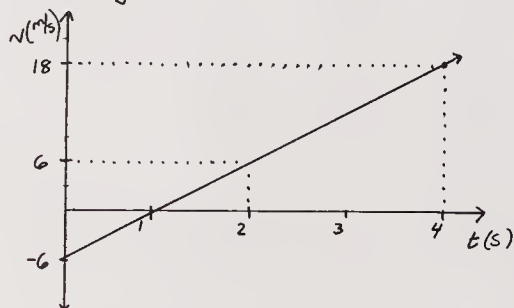


Figure (a)

- c) Displacement:  $\Delta x = x_{\text{final}} - x_{\text{initial}}$

For  $t=0$  to  $t=2s$ , the displacement is:

$$\Delta x_{0-2} = x(2) - x(0) \quad (C)$$

By Eq (A)

$$x(0) = 3(0)^2 - 6(0) = 0$$

$$x(2) = 3(2)^2 - 6(2) = 0$$

$$\therefore \Delta x_{0-2} = x(2) - x(0) = 0$$

For  $t=0$  to  $t=4s$ ,

$$\Delta x_{0-4} = x(4) - x(0)$$

$$x(4) = 3(4)^2 - 6(4) = 24 \text{ m}$$

$$\therefore \Delta x_{0-4} = 24 - 0 = 24 \text{ m}$$

- d) Distance traveled:  $\Delta s = |\Delta x_1| + |\Delta x_2|$

For  $t=0$  to  $t=2s$ , noting by Figure a that  $v=0$  at  $t=1s$ :

$$\Delta s_{0-2} = |\Delta x_{0-1}| + |\Delta x_{1-2}|$$

By Eq (A):

$$|\Delta x_{0-1}| = |x(1) - x(0)| = |-3 - 0| = 3 \text{ m}$$

$$|\Delta x_{1-2}| = |x(2) - x(1)| = |0 + 3| = 3 \text{ m}$$

$$\therefore \Delta s_{0-2} = 3 + 3 = 6 \text{ m}$$

For  $t=0$  to  $t=4s$ ;

$$\Delta s_{0-4} = |\Delta x_{0-1}| + |\Delta x_{1-4}|$$

By Eq (A), and since  $v$  is positive from  $t=1$  to  $t=4s$ ,

$$|\Delta x_{1-4}| = |x_4 - x_1| = |24 - (-3)| = 27 \text{ m}$$

$$\therefore \Delta s_{0-4} = 3 + 27 = 30 \text{ m}$$

**ALTERNATIVELY:**

By Figure(a.)

$$\Delta x_{0-2} = \frac{1}{2}(-6)(1) + \frac{1}{2}(6)(2-1) = 0$$

$$\Delta x_{0-4} = \frac{1}{2}(-6)(1) + \frac{1}{2}(18)(4-1) = 24 \text{ m}$$

$$\Delta s_{0-2} = |\frac{1}{2}(-6)(1)| + |\frac{1}{2}(6)(2-1)| = 6 \text{ m}$$

$$\Delta s_{0-4} = |\frac{1}{2}(-6)(1)| + |\frac{1}{2}(18)(4-1)| = 30 \text{ m}$$

13.40

**GIVEN** First ball thrown upward with velocity  $v_0$ . Second ball thrown  $T$ -seconds later with same velocity.

**FIND**

Derive a formula for time  $t$  after the second ball is thrown at which the balls pass each other.

**SOLUTION**

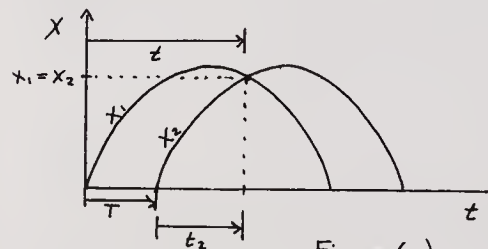


Figure (a.)

**Ball one**

$$a = -g \quad v_1 = \int a dt = -gt + v_0$$

$$x_1 = \int v_1 dt = -\frac{1}{2}gt^2 + v_0t + x_0$$

$$x_1 = -\frac{1}{2}gt^2 + v_0t \quad (A)$$

**Ball two** - thrown  $T$  seconds later,  $\therefore t_2 = (t - T)$   
 (See Fig. (a.))

$$a = -g \quad v_2 = \int a dt = -gt_2 + v_0$$

$$x_2 = \int v_2 dt = -\frac{1}{2}gt_2^2 + v_0t_2 + x_0$$

$$x_2 = -\frac{1}{2}gt_2^2 + v_0t_2 = -\frac{1}{2}g(t-T)^2 + v_0(t-T) \quad (B)$$

Balls pass each other when  $x_2 = x_1$  (See Fig (a.))

Equating (A) + (B) gives:

$$-\frac{1}{2}gt^2 + v_0t = -\frac{1}{2}g(t-T)^2 + v_0(t-T)$$

Simplifying gives:

$$0 = gTt - \frac{1}{2}gT^2 - v_0T \quad \text{or} \quad t = \frac{1}{2}T + \frac{v_0}{g}$$

$$\text{and } t_2 = t - T$$

$$\therefore t_2 = \frac{1}{2}T + \frac{v_0}{g} - T$$

$$t_2 = \frac{v_0}{g} - \frac{1}{2}T$$

13.41 **GIVEN** Time-velocity graph of racing car as shown (Fig. (a))

Note  $1 \text{ in} = 10 \text{ s}$  on time scale,  
 $1 \text{ in} = 5 \text{ ft/s}$  on velocity scale

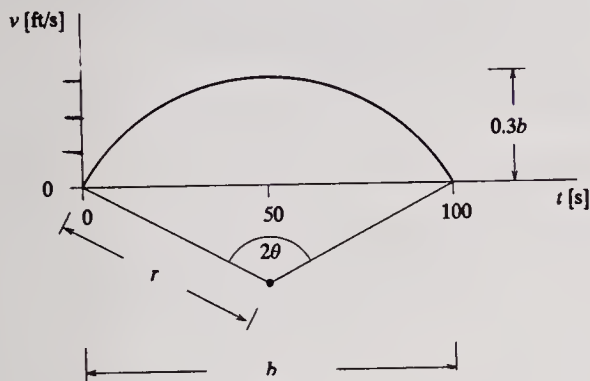


Figure (a)

**FIND** Distance traveled by car from  $t=0$  to  $t=100 \text{ s}$ .

**SOLUTION**

$$b = 100 \text{ s} \left( \frac{1 \text{ in}}{10 \text{ s}} \right) = 10 \text{ in}$$

$$0.3b = 0.3(10 \text{ in}) = 3 \text{ in}$$

$$v_{\text{max}} = 3 \text{ in} \left( \frac{5 \text{ ft/s}}{1 \text{ in}} \right) = 15 \text{ ft/s}$$

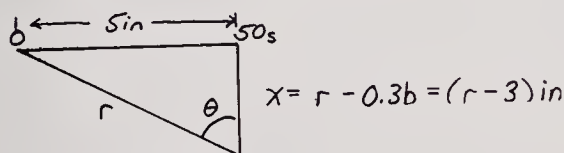


Figure (b)

By Fig. (b):

$$r^2 = 5^2 + x^2 = 25 + (r-3)^2$$

$$r = 17/3 \text{ in}$$

Also by Fig. (b):

$$5 = r \sin \theta \rightarrow \theta = \sin^{-1} \left( \frac{5}{17/3} \right)$$

$$\theta = 61.93^\circ = 1.0808 \text{ rad}$$

By Fig. (a.), the area of the circular section is:  $A_{cs} = \theta r^2$

$$A_{cs} = (1.0808) \left( \frac{17}{3} \right)^2 = 34.706 \text{ in}^2$$

The area of the triangle is:

$$A_{tri} = 2 \left[ \frac{1}{2} (5 \text{ in}) \left( \frac{17}{3} - 3 \right) \right] = 13.333 \text{ in}^2$$

The net area between the  $t$ -axis and arc is:

$$A_{\text{net}} = A_{cs} - A_{tri} = 34.706 - 13.333 = 21.373 \text{ in}^2$$

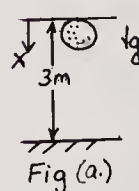
Therefore, the distance traveled by the car is:

$$S = (21.373 \text{ in}^2) \left( \frac{50 \text{ ft}}{\text{in}^2} \right) = \underline{\underline{1069 \text{ ft}}}$$

**GIVEN** From the instant that the ball is dropped from rest 3m above the floor to the instant that it rebounds, 2m, the time elapsed is 1.48 s. Also,  $g = 9.81 \text{ m/s}^2$ .

**FIND** Time [s] that ball remains in contact with the floor.

**SOLUTION**

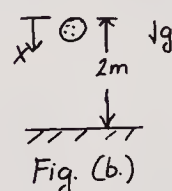


Ball is released at time  $t=0$  and drops  $x=3 \text{ m}$  (Fig. (a)).

With the relationship  $x = \frac{1}{2} a t^2 + v_0 t + x_0$  (A)

and  $t_1 =$  time the instant before the ball touches the ground,

$$3 = \frac{9.81}{2} t_1^2 + 0(t_1) + 0 \quad t_1 = 0.782 \text{ s}$$



The ball rebounds to a height 2m above the floor (Fig. (b)).

Therefore, at  $x=2 \text{ m}$  above floor,  $v=0$ .

The time  $t_2$  that it takes the ball to rise 2m is the same time it takes to fall 2m, starting from rest, ( $x_0 = v_0 = 0$ ). Therefore, by Eq. (A) and Fig. (b.):

$$2 \text{ m} = \frac{1}{2} g t_2^2 = \frac{1}{2} (9.81) t_2^2$$

$$\text{OR } t_2 = 0.6385 \text{ s}$$

Hence, the time that the ball is in contact with the floor is:

$$t = 1.48 \text{ s} - t_1 - t_2 = 1.48 \text{ s} - 0.7821 \text{ s} - 0.6385 \text{ s}$$

$$\therefore \underline{\underline{t = 0.059 \text{ s}}}$$

13.43 **GIVEN**  $a = -3t \text{ [m/s}^2]$

at  $t=0$ ,  $x=6 \text{ m} (=x_0)$   $v=4 \text{ m/s} (=v_0)$

**FIND** a)  $x$ ,  $v$ , &  $a$  at  $t=2 \text{ s}$

b) Displacement and distance traveled in intervals  $t=0-2 \text{ s}$  and  $t=0-4 \text{ s}$ .

**SOLUTION**  $v(t) = \int a dt = \int -3t dt = -\frac{3}{2} t^2 + v_0$

Since  $v_0 = 4$ ,  $v(t) = -\frac{3}{2} t^2 + 4$  (A)

$$x(t) = \int v dt = \int \left( -\frac{3}{2} t^2 + 4 \right) dt = -\frac{1}{2} t^3 + 4t + x_0$$

Since  $x_0 = 6 \text{ m}$ ,  $x(t) = -\frac{1}{2} t^3 + 4t + 6$  (B)

a.) By Eqs. (B) and (A),

$$x(t=2) = \underline{\underline{10 \text{ m}}}$$

$$v(t=2) = \underline{\underline{-2 \text{ m/s}}}$$

$$a(t=2) = \underline{\underline{-6 \text{ m/s}^2}}$$

(Continued)

b.) Displacement

For  $t=0$  to  $t=2s$ ,

$$\Delta X_{0-2} = X(2s) - X(0), \text{ where } X(0) = X_0 = 6m$$

$$\text{By Eq. (B): } X(2s) = -\frac{1}{2}(2)^3 + 4(2) + 6 = 10m$$

$$\therefore \Delta X_{0-2} = 10 - 6 = 4m$$

For  $t=0$  to  $t=4s$ ,

$$\Delta X_{0-4} = X(4s) - X_0$$

$$\text{By Eq. (B): } X(4s) = -\frac{1}{2}(4)^3 + 4(4) + 6 = -10m$$

$$\therefore \Delta X_{0-4} = -10 - 6 = -16m$$

Distances

Note that, by Eq (A),  $v$  is positive from  $t=0$  to  $t=\sqrt{8/3} s = 1.630s$ . From

$t=1.630s$  to  $t=2s$ ,  $v$  is negative. Thus,

$$\Delta S_{0-2} = |\Delta X_{0-1.630}| + |\Delta X_{1.630-2}|$$

where, By Eq (B),

$$\Delta X_{0-1.630} = X(1.630s) - X(0) = 10.343 - 6 = 4.343$$

$$\Delta X_{1.630-2} = X(2s) - X(1.630) = 10 - 10.343 = -0.343$$

$$\therefore \Delta S_{0-2} = |4.343| + |-0.343| = 4.69m$$

From  $t=0$  to  $t=4s$ ,  $v$  is negative from  $t=1.630s$  to  $t=4s$ .

$$\therefore \Delta S_{0-4} = |\Delta X_{0-1.630}| + |\Delta X_{1.630-4}|$$

where  $\Delta X_{0-1.630} = 4.343m$  as before and by Eq (B):

$$\Delta X_{1.630-4} = X(4s) - X(1.630) = -10 - 10.343 = -20.343$$

$$\therefore \Delta S_{0-4} = |4.343| + |-20.343| = 24.69m$$

Therefore:  $2r\dot{r} = 2X\dot{X}$ , or  $r v_r = X v$ , ( $v_r = 1 \text{ ft/s}$ )

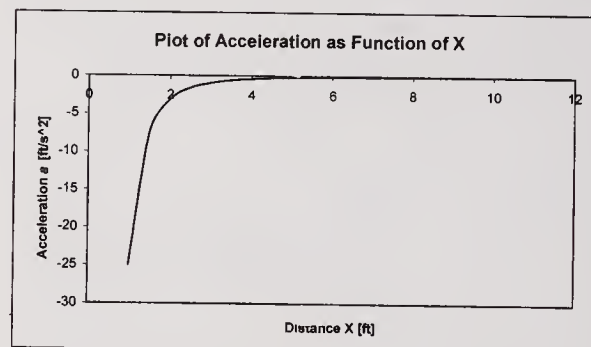
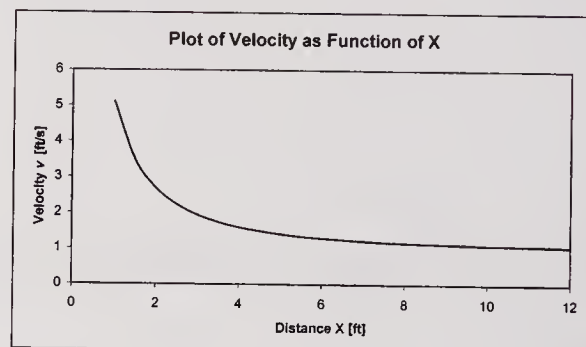
$$\text{So, } v = \frac{r}{X} = \sqrt{1 + \frac{25}{X^2}} \text{ [ft/s]} \quad (A)$$

Note that as  $X \rightarrow 0$ ,  $v \rightarrow \infty$  (see plot).

Differentiation of Eq (A) yields the acceleration. By chain rule:

$$a = \frac{dv}{dx} \frac{dx}{dt} = v \frac{dv}{dX} = -\frac{25}{X^3} \text{ [ft/s}^2\text{]} \quad (B)$$

Again, note that as  $X \rightarrow 0$ ,  $a \rightarrow \infty$  (see plot).



b.) This method is not a good one, since velocity and acceleration  $\rightarrow \infty$  as  $X \rightarrow 0$ .  $\therefore$  The car runs the risk of running into the pulley pole at such a high speed.

13.44 Computer Problem

Given

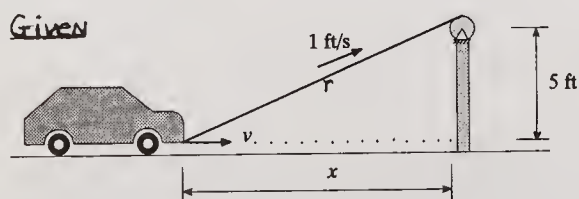


Figure (a)

Find a.) Plot speed,  $v$  [ft/s] and acceleration  $a$  [ft/s<sup>2</sup>] of car as function of  $x$  for  $1 \leq x \leq 12$  ft.

b.) Is this method of pulling the car a good one? Explain.

Solution

$$\text{By Fig (a), } r^2 = x^2 + 5^2$$

13.45 Computer Problem

GIVEN Jet propelled boat moves in straight line such that position is  $X = t^3 + 6t^2 + 5$  [ft] where  $t$  denotes time in seconds.

Find a.) Plot curves for position, velocity, and acceleration as functions of  $t$  for  $0 \leq t \leq 4s$ .

b.)  $v_{av}$  &  $a_{av}$  for  $0 \leq t \leq 4s$  and plot on graphs of part a.

c.) Est. time for which  $v = v_{av}$ ;  $a = a_{av}$ .

d.) Verify by calculation.

(Continued)

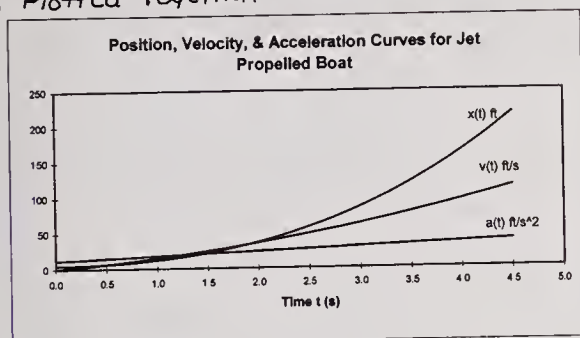
13.45 Cont.

**SOLUTION**  $x = t^3 + 6t^2 + 5$  [ft]

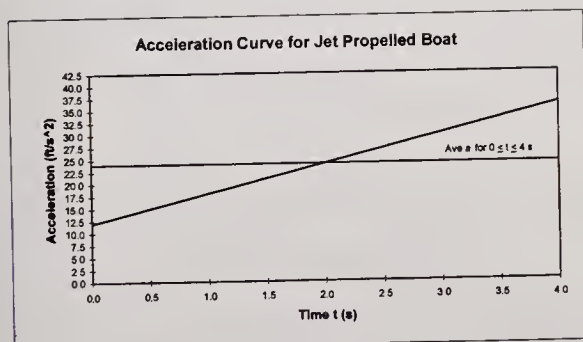
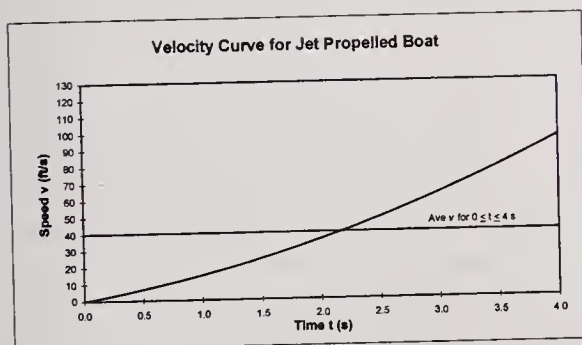
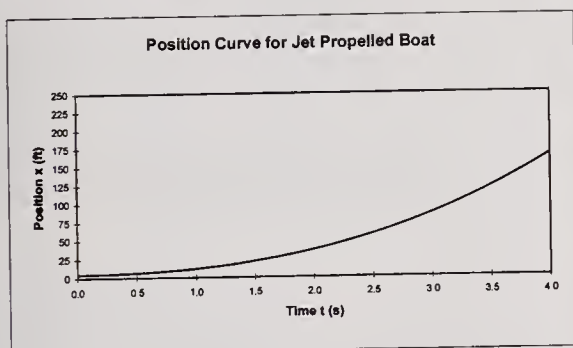
$\therefore v(t) = 3t^2 + 12t$  [ft/s]

$a(t) = 6t + 12$  [ft/s<sup>2</sup>]

a.) Plotted Together:



Plotted Separately:



b.) Average speed and acceleration:

$v_{ave} = \frac{1}{4} \int_0^4 [3t^2 + 12t] dt = 40 \text{ ft/s}$

→ plotted on v-t graph above

$a_{ave} = \frac{1}{4} \int_0^4 (6t + 12) dt = 24 \text{ ft/s}^2$

→ plotted on a-t graph above

c.) From plots obtained in part a.) and averages obtained in part b.) :

$t$  for instantaneous velocity = ave. velocity  
 $\approx 2.15 \text{ s}$

$t$  for instantaneous acceleration = ave acceleration  
 $\approx 2 \text{ s}$

d.) These results can be verified by calculation using the average values found in part b.) and solving for  $t$ :

$v_{ave} = 40 \text{ ft/s} = 3t^2 + 12t$   $t = 2.16 \text{ s}$

$a_{ave} = 24 \text{ ft/s}^2 = 6t + 12$   $t = 2 \text{ s}$

13.46

**GIVEN** Particle moves on straight line with velocity shown on time-velocity graph shown in Figure (a):

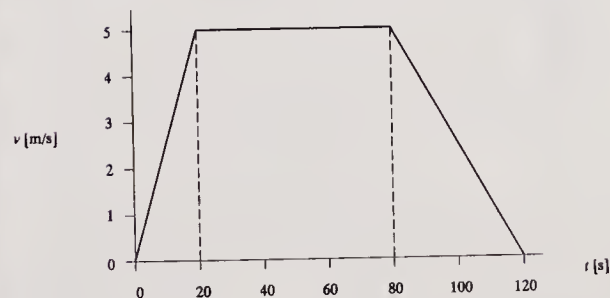


Figure (a)

**FIND** Plot time-acceleration and time-position graphs and from these plots, determine the acceleration and position for  $t = 20 \text{ s}$ ,  $60 \text{ s}$ ,  $80 \text{ s}$ , and  $120 \text{ s}$ .

**SOLUTION** By Figure (a.)

$$v(\text{m/s}) = \begin{cases} \frac{1}{4}t, & 0 \leq t \leq 20 \\ 5, & 20 \leq t \leq 80 \\ -\frac{1}{8}t + 15, & 80 \leq t \leq 120 \end{cases}$$

Also by Figure (a.), the acceleration is the slope of the velocity curve (i.e.  $a = dv/dt$ )  
Therefore,

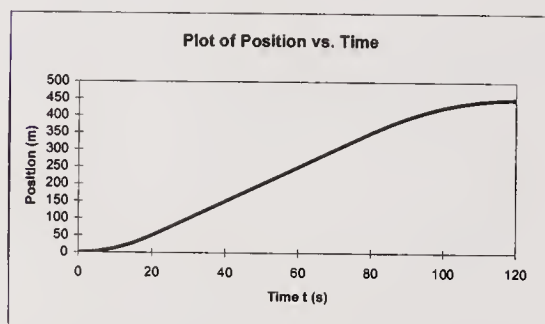
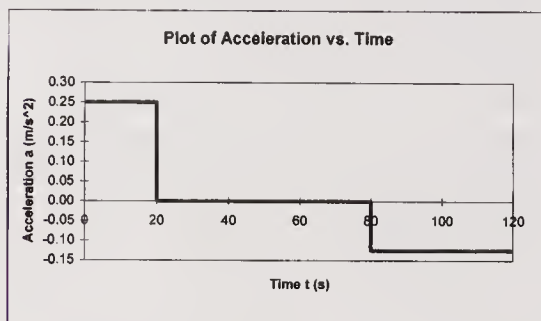
$$a(\text{m/s}^2) = \begin{cases} \frac{1}{4}, & 0 \leq t \leq 20 \\ 0, & 20 \leq t \leq 80 \\ -\frac{1}{8}, & 80 \leq t \leq 120 \end{cases}$$

(Continued)

The displacement,  $x$ , is obtained by integration:  
i.e.  $x = \int v dt$ . Therefore, with Eq (A),

$$x(m) = \begin{cases} \frac{1}{8} t^2 & 0 \leq t \leq 20 \\ 5t - 50 & 20 \leq t \leq 80 \\ -\frac{1}{16} t^2 + 15t - 450 & 80 \leq t \leq 120 \end{cases} \quad (C)$$

With Eqs (B) & (C), the time-acceleration and time-position plots are as follows:



From these plots, it can be determined that:

@  $t = 20s$ :  $a = \frac{1}{4} m/s^2$ ,  $x = 50 m$

@  $t = 60s$ :  $a = 0 m/s^2$ ,  $x = 250 m$

@  $t = 80s$ :  $a = -\frac{1}{8} m/s^2$ ,  $x = 350 m$

@  $t = 120s$ :  $a = -\frac{1}{8} m/s^2$ ,  $x = 450 m$

b.) Geometry of Path in  $(x, v)$  plane

By Eq (A):  $v^2 + 16x^2 = 400 \quad (c)$

or  $\frac{v^2}{20^2} + \frac{x^2}{5^2} = 1 \quad (D)$

Equation (D) is the equation of an ellipse (see Fig. (a.))

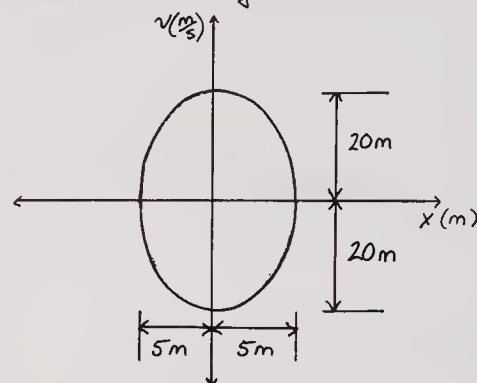


Figure (a.)

c.) The distance traveled by the particle from  $x=0$  to  $x$  at  $v = -10 m/s$  for first time:

when  $x=0$ ,  $v=20 m/s$  (Given) and,

when  $v=0$ ,  $x=5$  (by Eq. (D))

Hence, velocity changes signs (direction) at  $x=5 m$ . By Eq. (c):

$$v = \pm \sqrt{400 - 16x^2} \quad (E)$$

Thus, for  $v = -10 m$ , Eq. (E) yields:

$$-10 = -\sqrt{400 - 16x^2} \quad x = 4.33 m$$

So, the distance traveled from  $x=0$  to position when  $v=-10 m/s$  is

$$\Delta s = |x_1| + |x_2| = 5 - 0 + 4.33 = 9.33 m$$

13.47 GIVEN  $a = -16x (m/s^2)$  when  $x=0$ ,  $v=20 m/s$

FIND a.)  $x$  when  $v=0$

b.) Plot the geometric path in  $x, v$  plane

c.) Determine distance traveled from  $x=0$  to position when  $v=-10 m/s$  for first time.

SOLUTION

By the chain rule,  $a = \frac{dv}{dt} = \frac{dv}{dx} \frac{dx}{dt} = v \frac{dv}{dx}$

$$\text{or } \int_0^x a dx = \int_0^x (-16x) dx = -8x^2 = \int_{20}^v v dv$$

Integration yields:

$$-8x^2 = \frac{1}{2} (v^2 - 400) \quad (A)$$

For  $v=0$ , Eq (A) yields:  $x = 5 m \quad (B)$

13.48 GIVEN  $a = \frac{x^3}{2}$ ; when  $t=1$ ,  $x=1$ ,  $v=0$  and  $v \geq 0$ .

FIND  $v(x)$  and  $t(x)$  and  $v(t)$ ,  $x(t)$ ,  $a(t)$

Sketch graphs of all 5.

SOLUTION

By the chain rule,  $a = \frac{dv}{dt} = \frac{dv}{dx} \frac{dx}{dt} = v \frac{dv}{dx}$

$$\therefore \int_0^v v dv = \frac{1}{2} \int_0^x x^3 dx$$

$$\text{or } \frac{v^2}{2} = \frac{x^4}{8} \Rightarrow v = \pm \frac{x^2}{2}$$

$$\text{Since } v \geq 0, \quad v = \frac{x^2}{2} \quad (A)$$

(Continued)