Problem Solutions 4th edition

Chap 1

Determine the principal stresses for the stress state

$$\sigma_{ij} = \begin{vmatrix} 10 & -3 & 4 \\ -3 & 5 & 2 \\ 4 & 2 & 7 \end{vmatrix}.$$

Solution: $I_1 = 10+5+7=32$, $I_2 = -(50+35+70) +9 +4 +16 = -126$, $I_3 = 350 -48 -40 -80 -63 = 119$; $\sigma^3 - 22\sigma^2 -126\sigma -119 = 0$. A trial and error solution gives σ -= 13.04. Factoring out 13.04, $\sigma^2 -8.96\sigma + 9.16 = 0$. Solving; $\sigma_1 = 13.04$, $\sigma_2 = 7.785$, $\sigma_3 = 1.175$.

1-2 A 5-cm. diameter solid shaft is simultaneously subjected to an axial load of 80 kN and a torque of 400 Nm.

a. Determine the principal stresses at the surface assuming elastic behavior.

b. Find the largest shear stress.

Solution: a. The shear stress, τ, at a radius, r, is $\tau = \tau_s r/R$ where τ_s is the shear stress at the surface R is the radius of the rod. The torque, T, is given by $T = \int 2\pi\tau r^2 dr = (2\pi\tau_s /R)\int r^3 dr = \pi\tau_s R^3/2$. Solving for $= \tau_s$, $\tau_s = 2T/(\pi R^3) = 2(400N)/(\pi 0.025^3) = 16$ MPa.

The axial stress is $.08MN/(\pi 0.025^2) = 4.07$ MPa.

$\sigma_1,\sigma_2 = 4.07/2 \pm [(4.07/2)^2 + (16/2)^2)]^{1/2} = 1.029, -0.622$ MPa.

b. the largest shear stress is $(1.229 + 0.622)/2 = 0.925$ MPa.

A long thin-wall tube, capped on both ends is subjected to internal pressure. During elastic loading, does the tube length increase, decrease or remain constant?

Solution: Let y = hoop direction, x = axial direction, and z = radial direction. –

$e_x = e_2 = (1/E)[\sigma - \upsilon(\sigma_3 + \sigma_1)] = (1/E)[\sigma_2 - \upsilon(2\sigma_2)] = (\sigma_2/E)(1-2\upsilon)$

Since $\upsilon < 1/2$ for metals, $e_x = e_2$ is positive and the tube lengthens.

1-4 A solid 2-cm. diameter rod is subjected to a tensile force of 40 kN. An identical rod is subjected to a fluid pressure of 35 MPa and then to a tensile force of 40 kN. Which rod experiences the largest shear stress?

Solution: The shear stresses in both are identical because a hydrostatic pressure has no shear component.

1-5 Consider a long thin-wall, 5 cm in diameter tube, with a wall thickness of 0.25 mm that is capped on both ends. Find the three principal stresses when it is loaded under a tensile force of 400 kN and an internal pressure of 200 kPa.

Solution: $\sigma_x = PD/4t + F/(\pi Dt) = 12.2$ MPa.

$\sigma_y = PD/2t = 2.0$ MPa.

$\sigma_y = 0.$

1-6 Three strain gauges are mounted on the surface of a part. Gauge A is parallel to the x-axis and gauge C is parallel to the y-axis. The third gage, B, is at 30° to gauge A. When the part is loaded the gauges read

Gauge A	$3000\text{x}10^{-6}$
Gauge B	$3500\ \text{x}10^{-6}$
Gauge C	$1000\ \text{x}10^{-6}$

a. Find the value of γ_{xy}.
b. Find the principal strains in the plane of the surface.
c. Sketch the Mohr's circle diagram.

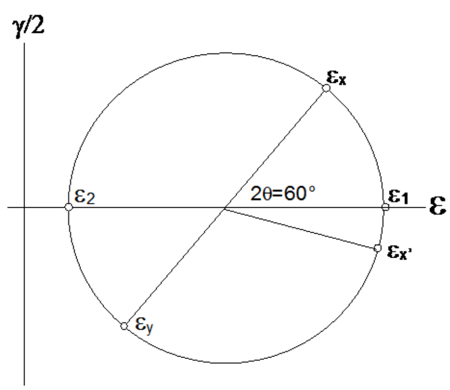
<u>Solution:</u> Let the B gauge be on the x' axis, the A gauge on the x-axis and the C gauge on the y-axis. $e_{x'x'} = e_{xx}\ell^2{}_{x'x'} + e_{yy}\ell^2{}_{x'y} + \gamma_{xy}\ell_{x'x}\ell_{x'y}$, where $\ell_{x'x}$ = cose_x = 30 = √3/2 and $\ell_{x'y}$ = cos 60 = ½. Substituting the measured strains,

$3500 = 3000(\sqrt{2/3})^2 - 1000(1/2)^2 + \gamma_{xy}(\sqrt{3}/2)(1/2)$

$\gamma_{xy} = (4/\sqrt{3}/2)\{3500-[3000-(1000(\sqrt{3}/2)^2+1000(1/2)^2]\} = 2{,}309\ (\text{x}10^{-6})$

b. $e_1,e_2 = (e_x + e_y)/2 \pm [(e_x-e_y)2 + \gamma_{xy}2]^{1/2}/2 = (3000+1000)/2 \pm [(3000-1000)^2 + 2309^2]^{1/2}/2$. $e_1 = 3530(\text{x}10^{-6})$, $e_2 = 470(\text{x}10^{-6})$, $e_3 = 0$.

c)



Find the principal stresses in the part of problem 1-6 if the elastic modulus of the part is 205 GPa and Poissons's ratio is 0.29.

Solution: $e_3 = 0 = (1/E)[0 - \nu(\sigma_1+\sigma_2)]$, $\sigma_1 = \sigma_2$

$e_1 = (1/E)(\sigma_1 - \nu\sigma_1)$; $\sigma_1 = Ee_1/(1-\nu) = 205\text{x}10^9(3530\text{x}10^{-6})/(1-.29^2) = 79$ MPa

Show that the true strain after elongation may be expressed as $\varepsilon = \ln(\frac{1}{1-r})$ where r is the reduction of area. $\varepsilon = \ln(\frac{1}{1-r})$.

<u>Solution</u>: $r = (A_o-A_1)/A_o = 1 - A_1/A_o = 1 - L_o/L_1$. $\varepsilon = \ln[1/(1-r)]$.

A thin sheet of steel, 1-mm thick, is bent as described in Example 1-11. assuming that E = is 205 GPa and $\nu = 0.29$, and that the neutral axis doesn't shift.

a. Find the state of stress on most of the outer surface.
b. Find the state of stress at the edge of the outer surface.

Solution: a. Substituting E = 205×10^9, t = 0.001, $\rho = 2.0$ and $\nu = 0.29$ into $\sigma_x = \dfrac{Et}{2\rho(1-\nu^2)}$ and $\sigma_y = \dfrac{\nu Et}{2\rho(1-\nu^2)}$, σ_x = 56 MPa, , σ_y = 16.2 MPa.

b. Now $\sigma_y = 0$, so $\sigma_y = \dfrac{\nu Et}{2\rho}$ = 51 MPa.

1-10 For an aluminum sheet, under plane stress loading $\varepsilon_x = 0.003$ and $\varepsilon_y = 0.001$. Assuming that E = is 68 GPa and $\nu = 0.30$, find ε_z.

Solution: $e_y = (1/E)(\sigma_y - \nu\sigma_y)$, $e_x = (1/E)(\sigma_x - \nu E e_y - \nu^2\sigma_x)$. Solving for σ_x,
$\sigma_x = [E/(1-\nu^2)]e_{y+}\ \nu e_y)$. Similarly, $\sigma_y = [E/(1-\nu^2)](e_{y+}\ \nu e_x)$. Substituting into
$e_z = (1/E)(-\nu\sigma_y - \nu\sigma_y) = (-\nu/E)(E/(1-\nu^2)[e_{y+}\ \nu e_y + e_{y+}\ \nu e_x) = [-\nu(1+\nu)//(1-\nu^2)](e_{y+}\ e_y) =$
0.29(-1.29/0.916)(0.004) = -0.00163.

1-11 A piece of steel is elastically loaded under principal stresses, σ_1 = 300 MPa, σ_2 = 250 MPa and σ_3 = -200 MPa. Assuming that E = is 205 GPa and $\nu = 0.29$ find the stored elastic energy per volume.

Solution: $w = (1/2)(\sigma_1 e_1 + \sigma_2 e_2 + \sigma_3 e_3)$. Substituting $e_1 = (1/E)[\sigma_1 - \nu(\sigma_2 + \sigma_3)]$,
$e_2 = (1/E)[\sigma_2 - \nu(\sigma_3 + \sigma_1)]$ and $e_3 = (1/E)[\sigma_3 - \nu(\sigma_1 + \sigma_2)]$,
$w = 1/(2E)[\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - 2\nu(\sigma_2\sigma_3 + \sigma_3\sigma_1 + \sigma_1\sigma_2)] =$
$(1/(2 \times 205 \times 10^9)[300^2 + 250^2 + 200^2 - (2 \times 0.29)(-200 \times 250 - 300 \times 250 + 250 + 300)] \times 10^{12} =$
$400 J/m^3$.

1-12 A slab of metal is subjected to plane-strain deformation (e_2=0) such that σ_1 = 40 ksi and $\sigma_3 = 0$. Assume that the loading is elastic and that E = is 205 GPa and $\nu = 0.29$ (Note the mixed units.) Find
a. the three normal strains.
b. the strain energy per volume.

Solution: $w = (1/2)(\sigma_1 e_1 + \sigma_2 e_2 + \sigma_3 e_3) = (1/2)(\sigma_1 e_1 + 0 + 0) = \sigma_1 e_1/2$,
σ_1 = 40ksi(6.89MPa/ksi) = 276 MPa.
$0 = e_2 = (1/E)[\sigma_2 - \nu\sigma_1]$, $\sigma_2 = \nu\sigma_1 = 0.29 \times 276 = 80$ MPa.
$e_1 = (1/E)(\sigma_1 - \nu\sigma_2) = (1/205 \times 10^3)[276 - .29(80)] = 0.00121$.
$w = (276 \times 10^6)(0.00121)/2 = 167\ kJ/m^3$.

Chapter 2

a) If the principal stresses on a material with a yield stress in shear are σ_1 = 175 MPa and σ_2 = 350 MPa, what tensile stress σ_3 must be applied to cause yielding according to the Tresca criterion?

b) If the stresses in (a) were compressive, what tensile stress σ_3 must be applied to cause yielding according to the Tresca criterion?

Solution: a) $\sigma_1 - \sigma_3 = 2k$, $\sigma_3 = 2k - \sigma_1 = 400 - 350 = 50$ MPa.

b) $\sigma_3 = 2k - \sigma_1 = 400 - (350) = 50$ MPa.

Consider a 6-cm diameter tube with 1-mm thick wall with closed ends made from a metal with a tensile yield strength of 25 MPa. After applying a compressive load of 2000 N to the ends. What internal pressure is required to cause yielding according to a) the Tresca criterion. B) the von Mises criterion?

Solution: a) The ratio of the tube diameter to wall thickness is very large, so it can be treated as a thin wall tube. The stress caused by the pressure can be found by x- and y- direction force balances.

From pressure, $\sigma_x = Pd/(2t) = 60P$ and $\sigma_y = Pd/(4t) = 30P$. The stress caused by the axial load is $\sigma_y = F/(dt) = -2000N/[\pi(0.060)(0.001)] = -10.6$ MPa, so the total stress, $\sigma_y = 30P - 10.6$ MPa

a) $\sigma_x = 60P = \sigma_{max}$ is the largest stress, $\sigma_y = 30P - 10.6$ MPa and $\sigma_z = 0$. There are two possibilities which must be checked.

i. If $\sigma_z < \sigma_y$, $\sigma_z = \sigma_{min}$, yielding will occur when 60P-0 = Y, or P=Y/60 =25/60 = 0.416 MPa

ii. If $\sigma_y < \sigma_z$, $\sigma_y = \sigma_{min}$, and yielding will occur when

60P-(30P-10.6) = Y, or 30P = Y + 10.6, P = (Y+10.6)/30 = 35.6/30 = 1.1187 MPa

Yielding will occur when the smaller of the two values is reached, and therefore the smaller one is appropriate. P = 0.415 MPa.

b) Substituting into eq. 2-7 (in MPa),

$2(25)^2 = [60P-(30P-10.6)]^2 + [(30P-10.6)-0]^2 + [0-60P]^2$,

$1250 = 5400P^2 + 224$, p = 0.436 MPa.

2-3 Consider a 0.5 m-diameter cylindrical pressure vessel with hemispherical ends made from a metal for which *k* = 500 MPa. If no section of the pressure vessel is to yield under an internal pressure of 35 MPa, what is the minimum wall thickness according to a) the Tresca criterion? b) the von Mises criterion?

Solution: A force balance in the hemispherical ends gives σ_x ($=\sigma_y$) = PD/(4t).

A force balance in the cylindrical section gives σ_x = PD/(2t). σ_y = PD/(4t) so this section has the greatest stress.

a. $\sigma_{max} - \sigma_{min} = 2k$, PD/2t – 0 = 2k, t = PD/(4k) = 35(0.5)/(4x500) = 8.75 mm

b. $(\sigma_x/2 - 0)^2 + (0 - \sigma_x)^2 + (\sigma_x - \sigma_x/2)^2 = 6k^2$, $(3/2)\sigma_x^2 = 6k^2$, $\sigma_x = 2k = PD/(2t)$, t = PD/(4k) which is identical to part a. t = 8.75 mm

$$\bar{\varepsilon} = \sqrt{2(\varepsilon_x^{\ 2} + \varepsilon_y^{\ 2})}/3$$

2-4 A thin-wall tube is subjected to combined tensile and torsional loading. Find the relationship between the axial stress, σ, the shear stress, τ, and the tensile yield strength, *Y*, to cause yielding according to a) the Tresca criterion, b) the von Mises criterion.

Solution: a) $\sigma_1, \sigma_2 = \sigma/2 \pm \sqrt{(\sigma/2)^2 + \tau^2}$ If $\sigma/2 - \sqrt{(\sigma/2)^2 + \tau^2} > 0$, $\sigma_{min} = 0$, so the Tresca criterion predicts yielding when $\sigma/2 \pm \sqrt{(\sigma/2)^2 + \tau^2} = Y$. If $\sigma/2 - \sqrt{(\sigma/2)^2 + \tau^2} <$ 0, $\sigma_{min} = -\sqrt{(\sigma/2)^2 + \tau^2}$, so the Tresca criterion predicts yielding when $2\sqrt{(\sigma/2)^2 + \tau^2}$

b) $\{2[\sigma/2 - \sqrt{(\sigma/2)^2 + \tau^2}\,]^2 + [2\sqrt{(\sigma/2)^2 + \tau^2}\,]^2\}^{1/2} = \sqrt{2}Y$

Consider a plane-strain compression test with a compressive load, F_y, a strip width, w, an indenter width, b, and a strip thickness, t. Using the von Mises criterion, find:

a) $\bar{\varepsilon}$ as a function of ε_y.

b) $\bar{\sigma}$ as a function of σ_y.

c) an expression for the work per volume in terms of ε_y and σ_y.

d) an expression in the form of $\sigma_y = f(K, \varepsilon_y, n)$ assuming $\bar{\sigma} = K\bar{\varepsilon}^n$.

Solution: a. If $\varepsilon_z = 0$, $\varepsilon_y = -e_x$ $\bar{\varepsilon} = \sqrt{2(\varepsilon_x{}^2 + \varepsilon_y{}^2)/3} = = 1.154\varepsilon_y$

b. $\sigma_x = 0$, $\sigma_z = -(1/2)\sigma_y$; $\bar{\sigma} = \sqrt{(1/2)[(\sigma_y - \sigma_y/2)^2 + (\sigma_y/2 - 0)^2 + (0 - \sigma_y)^2]} = \sigma_y/1.154$

c. $w = \int \sigma_y d\varepsilon_y$

d. $\sigma_y = \sqrt{4/3}\bar{\sigma} = \sqrt{4/3}K\bar{\varepsilon}^n = \sqrt{4/3}K(\sqrt{4/3})\varepsilon_y)^n = (4/3)^{n+1/2}\,\varepsilon_y$

The following yield criterion has been proposed: "Yielding will occur when the sum of the two largest shear stresses reaches a critical value. " Stated mathematically $(\sigma_1 - \sigma_3) + (\sigma_1 - \sigma_2) = C$ if $(\sigma_1 - \sigma_2) > (\sigma_2 - \sigma_3)$ or $(\sigma_2 - \sigma_3) + (\sigma_1 - \sigma_2) = C$ if $(\sigma_1 - \sigma_2) \leq (\sigma_2 - \sigma_3)$ where $\sigma_1 > \sigma_2 > \sigma_3$, $C = 2Y$ and Y = tensile yield strength.

Solution: a) Yes. The value of the left hand sides are not affected if each principal stress is increased the same amount.

b) First find the constant C. Consider an x-direction tension test. At yielding, $\sigma_x = \sigma_1 = Y$, $\sigma_y = \sigma_z = \sigma_2 = \sigma_3 = 0$. Therefore $(\sigma_1 - \sigma_2) > (\sigma_2 - \sigma_3)$ so criterion I applies, and $C = (\sigma_1 - \sigma_3) + (\sigma_1 - \sigma_2) = 2Y$. Therefore $C = 2Y$.

We can also think about an x-direction compression test. At yielding, $\sigma_x = \sigma_3 = -Y$, $\sigma_y = \sigma_z = \sigma_2 = \sigma_3 = 0$. Therefore $(\sigma_2 - \sigma_3) > (\sigma_1 - \sigma_2)>$ so criterion II applies, and $C = (\sigma_1 - \sigma_3) + (\sigma_2 - \sigma_3) = -(-2Y)$ or again $C = 2Y$.

Now consider several loading paths:

In region A, $\sigma_x = \sigma_1$, $\sigma_y = \sigma_2$, $\sigma_z = \sigma_3 = 0$ and $\sigma_x > 2\sigma_y$ so $(\sigma_1 - \sigma_3) > (\sigma_1 - \sigma_2)$

Therefore criterion I, $(\sigma_x - 0) + (\sigma_x - \sigma_y) = 2Y$, or $\sigma_x = Y + \sigma_y/2$

In region B, $\sigma_x = \sigma_1$, $\sigma_y = \sigma_2$, $\sigma_z = \sigma_3 = 0$ but $\sigma_x < 2\sigma_y$ so $(\sigma_1 - \sigma_3) < (\sigma_1 - \sigma_2)$

Therefore criterion II, $(\sigma_x - 0) + (\sigma_y - 0) = 2Y$, or $\sigma_x = 2Y - \sigma_y$

In region C, $\sigma_y = \sigma_1$, $\sigma_x = \sigma_2$, $\sigma_z = \sigma_3 = 0$ but $\sigma_y < 2\sigma_x$ so $(\sigma_1 - \sigma_3) < (\sigma_1 - \sigma_2)$

Therefore criterion II, $(\sigma_y - 0) + (\sigma_x - 0) = 2Y$, or $\sigma_y = 2Y - \sigma_x$

In region D, $\sigma_y = \sigma_1$, $\sigma_x = \sigma_2$, $\sigma_z = \sigma_3 = 0$ and $\sigma_y > 2\sigma_x$ so $(\sigma_1 - \sigma_2) > (\sigma_2 - \sigma_3)$

Therefore criterion I, $(\sigma_y - 0) + (\sigma_y - \sigma_x) = 2Y$, or $\sigma_y = Y + \sigma_x/2$