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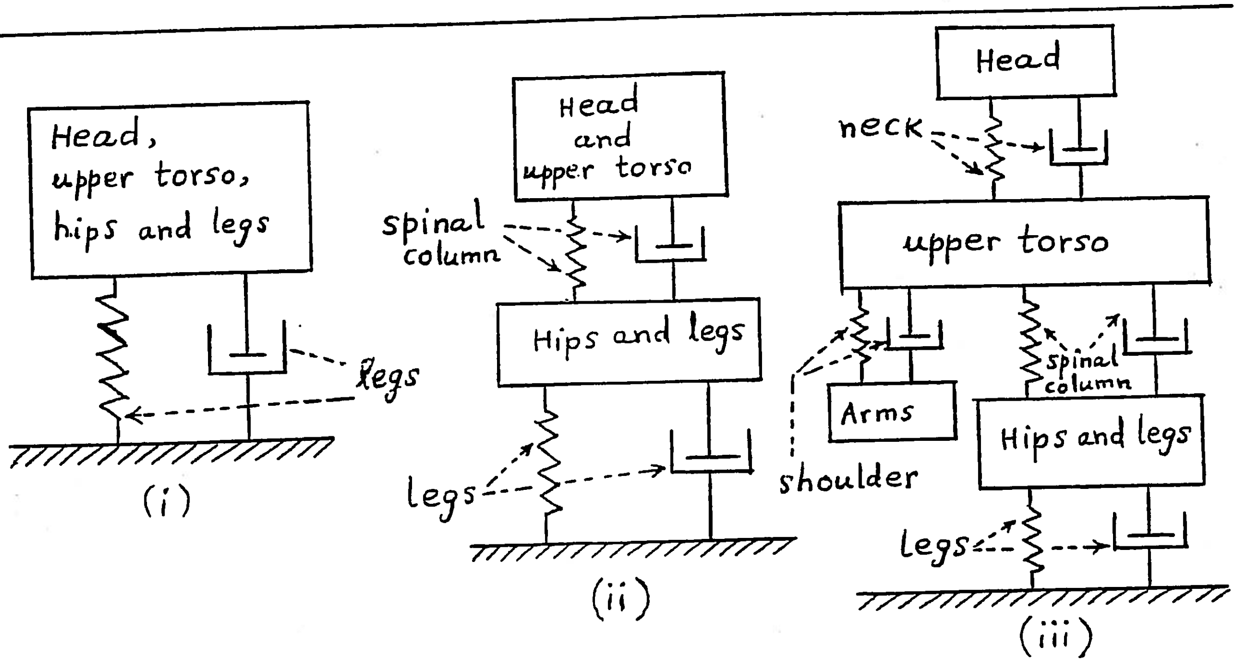
Chapter 1

Fundamentals of Vibration

1.1



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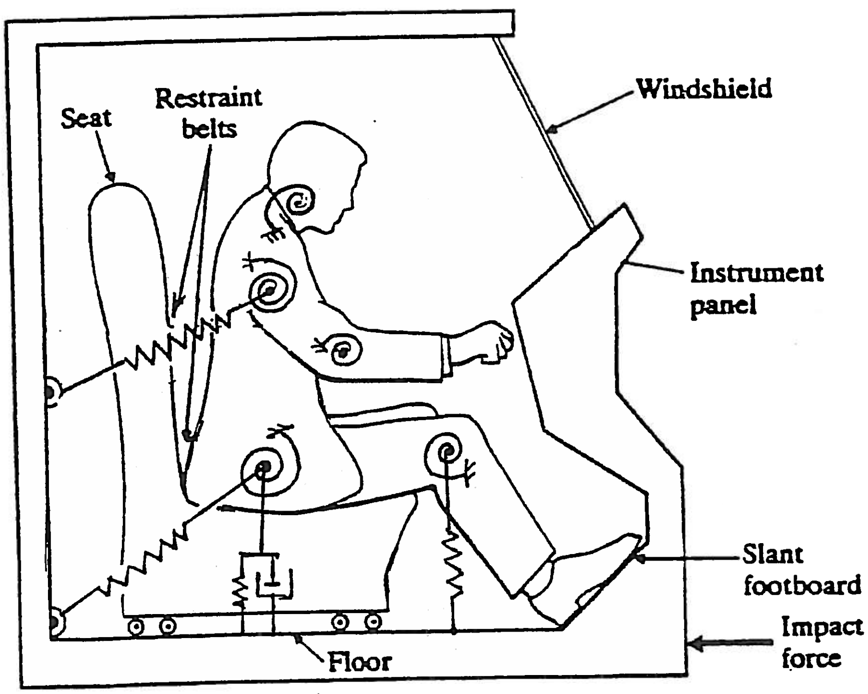

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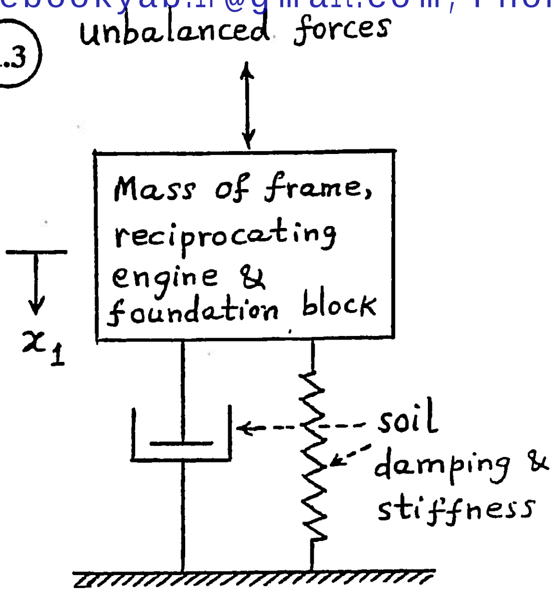
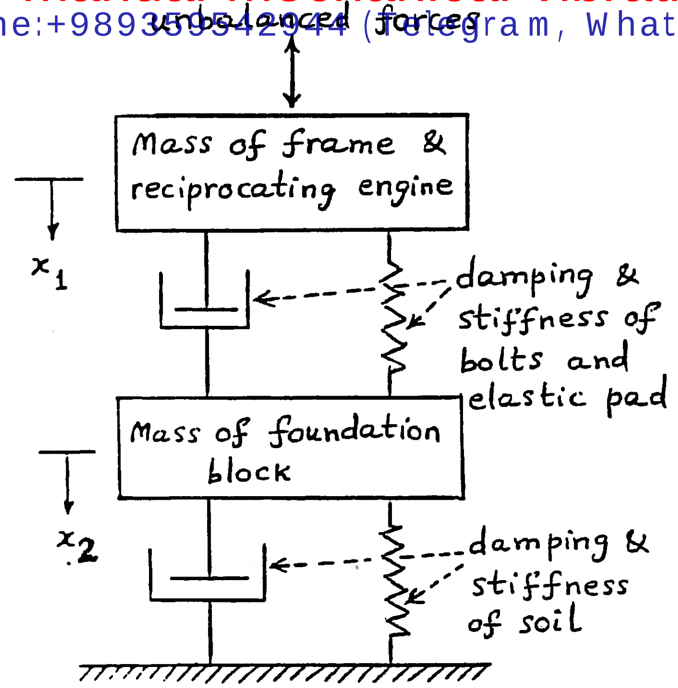
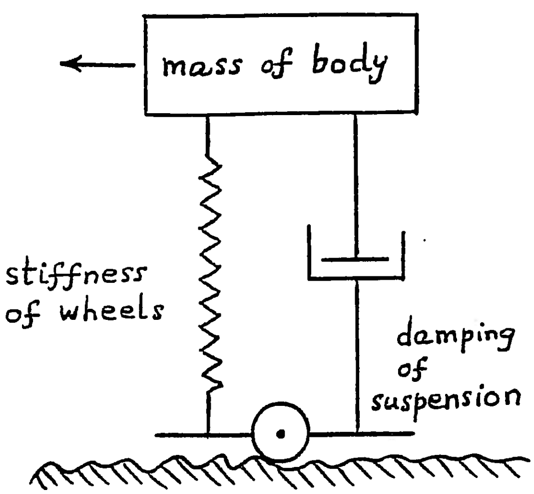


(a) one degree of freedom model

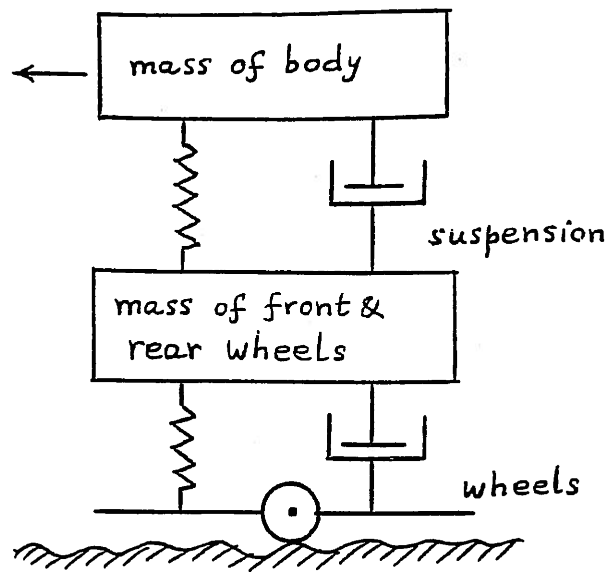


(b) Two degree of freedom model

1.4



(i)



(ii)

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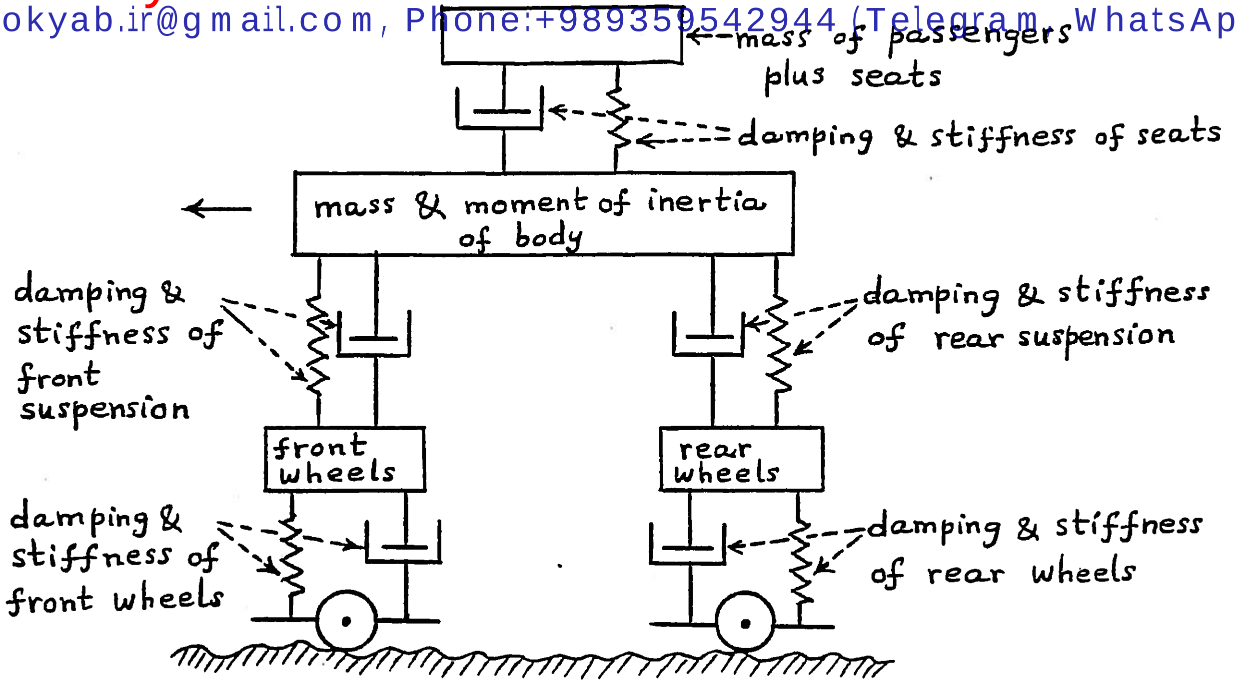

(iii)

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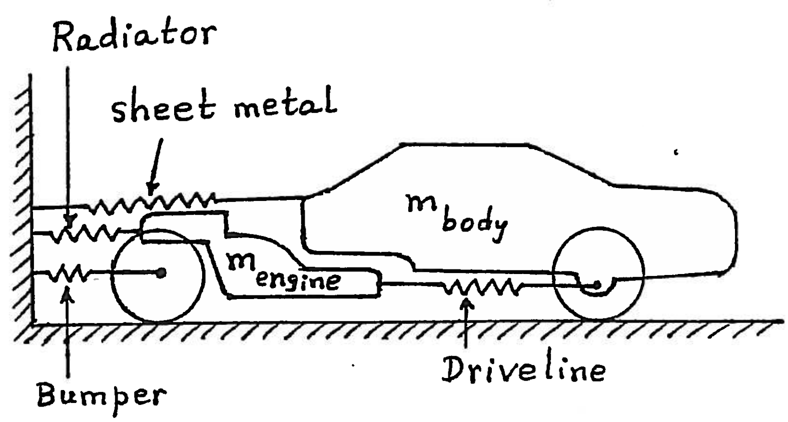

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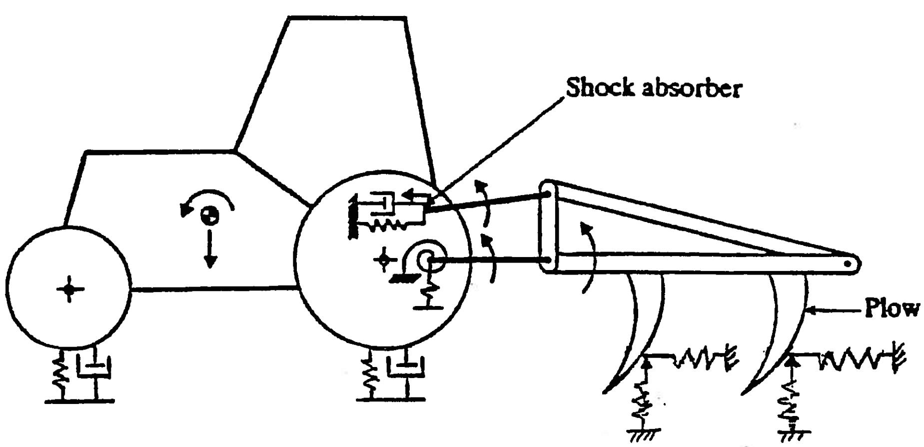

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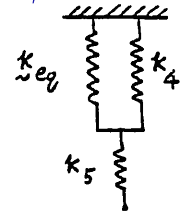
1.7

$$\frac{1}{\tilde{k}_{eq}} = \frac{1}{2k_1} + \frac{1}{k_2} + \frac{1}{2k_3} \quad ; \quad \tilde{k}_{eq} = \left(\frac{2k_1 k_2 k_3}{k_2 k_3 + 2k_1 k_3 + k_1 k_2} \right)$$



$$\frac{1}{k_{eq}} = \frac{1}{\tilde{k}_{eq} + k_4} + \frac{1}{k_5}$$

$$k_{eq} = \frac{k_5 (\tilde{k}_{eq} + k_4)}{k_5 + k_4 + \tilde{k}_{eq}} = \frac{k_2 k_3 k_4 k_5 + 2k_1 k_3 k_4 k_5 + k_1 k_2 k_4 k_5 + 2k_1 k_2 k_3 k_5}{k_2 k_3 k_4 + k_2 k_3 k_5 + 2k_1 k_3 k_4 + 2k_1 k_3 k_5 + k_1 k_2 k_4 + k_1 k_2 k_5 + 2k_1 k_2 k_3}$$

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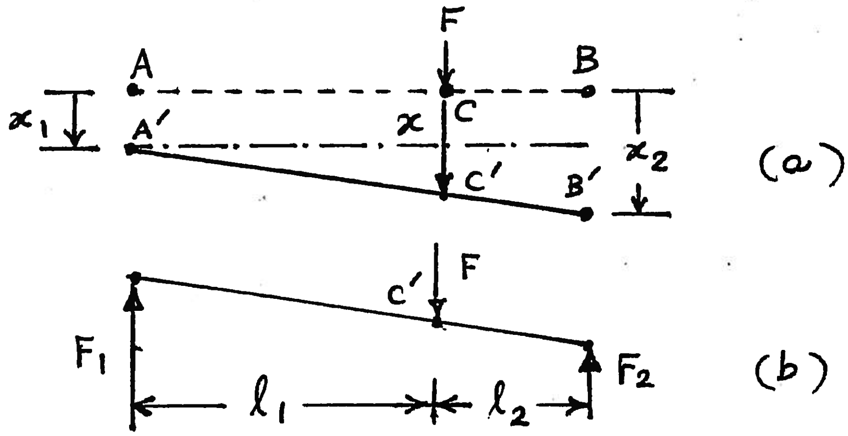
1.8



From Fig. (a), $x = x_1 + \frac{l_1}{l_1+l_2}(x_2 - x_1)$

$$= \frac{l_2}{l_1+l_2}\, x_1 + \frac{l_1}{l_1+l_2}\, x_2 \qquad (1)$$

Vertical force equilibrium from Fig. (b):

$$F = F_1 + F_2 \qquad (2)$$

Moment equilibrium about C' (Fig. (b)):

$$F_2\, l_2 = F_1\, l_1 \qquad (3)$$

Solution of Eqs. (2) and (3):

$$F_1 = \frac{F\, l_2}{l_1 + l_2}, \quad F_2 = \frac{F\, l_1}{l_1 + l_2} \qquad (4)$$

Displacements of springs k_1 and k_2 are given by

$$x_1 = \frac{F_1}{k_1} = \frac{F\, l_2}{k_1(l_1+l_2)}, \quad x_2 = \frac{F_2}{k_2} = \frac{F\, l_1}{k_2(l_1+l_2)} \qquad (5)$$

Displacement of force F can be found using Eqs. (5) in Eq. (1):

$$x = \frac{l_2}{l_1+l_2} \cdot \frac{F\, l_2}{k_1(l_1+l_2)} + \frac{l_1}{l_1+l_2} \cdot \frac{F\, l_1}{k_2(l_1+l_2)}$$

$$= \frac{F}{(l_1+l_2)^2} \left(\frac{l_1^2 k_1 + l_2^2 k_2}{k_1 k_2} \right) \qquad (6)$$

The equivalent spring constant of the system in the

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direction of x, k_e, is given by Eq. (6):

$$k_e = \frac{F}{x} = \frac{(l_1 + l_2)^2 \, k_1 k_2}{l_1^2 \, k_1 + l_2^2 \, k_2} \tag{7}$$

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1.9 Equivalence of potential energies gives

$$\frac{1}{2} k_{t1}\theta^2 + \frac{1}{2} k_{t2}\theta^2 + \frac{1}{2} k_1 (\theta \ell_1)^2 + \frac{1}{2} k_2 (\theta \ell_1)^2 + \frac{1}{2} k_3 (\theta \ell_2)^2 = \frac{1}{2} k_{eq}\theta^2$$

$$\therefore \; k_{eq} = k_{t1} + k_{t2} + k_1 \ell_1^2 + k_2 \ell_1^2 + k_3 \ell_2^2$$

1.10 k_{123} = for series springs k_1, k_2 and k_3 :

$$\frac{1}{k_{123}} = \frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_3} \; ; \qquad k_{123} = \frac{k_1 k_2 k_3}{k_1 k_2 + k_2 k_3 + k_3 k_1}$$

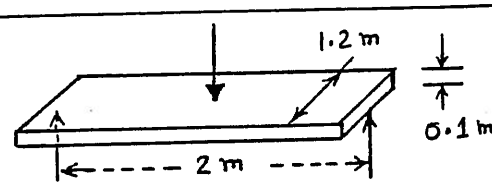
Using energy equivalence,

$$\frac{1}{2} k_{eq}\theta^2 = \frac{1}{2} k_4 \theta^2 + \frac{1}{2} k_{123}\theta^2 + \frac{1}{2} k_5 (\theta R)^2 + \frac{1}{2} k_6 (\theta R)^2$$

$$\therefore \; k_{eq} = k_4 + k_{123} + R^2 k_5 + R^2 k_6$$

$$= k_4 + \left(\frac{k_1 k_2 k_3}{k_1 k_2 + k_2 k_3 + k_3 k_1} \right) + R^2 (k_5 + k_6)$$

1.11 For simply supported beam, for load at middle,



$$k_1 = \frac{48 EI}{\ell^3} = \frac{48 (2.06 \times 10^{11})(10^{-4})}{8}$$

$$= 12.36 \times 10^7 \text{ N/m} \quad \text{where } I = \frac{1}{12}(1.2)(0.1)^3 = 10^{-4} \text{ m}^4 .$$

$$\delta_1 = \text{original deflection} = \frac{mg}{k_1} = \frac{500 \times 9.81}{12.36 \times 10^7} = 396.8447 \times 10^{-7} \text{ m}$$

When spring k is added, $k_{eq} = k + k_1$.

(a) New deflection $= \dfrac{mg}{k_{eq}} = \dfrac{\delta_1}{4}$; $k_{eq} = \dfrac{4\,mg}{\delta_1} = 4 k_1 = k + k_1$

$$\therefore \; k = 3 k_1 = 37.08 \times 10^7 \text{ N/m}$$

(b) New deflection $= \dfrac{mg}{k_{eq}} = \dfrac{\delta_1}{2}$; $k_{eq} = \dfrac{2\,mg}{\delta_1} = 2 k_1 = k + k_1$

$$\therefore \; k = k_1 = 12.36 \times 10^7 \text{ N/m}$$

(c) New deflection $= \dfrac{mg}{k_{eq}} = \dfrac{3}{4}\delta_1$; $k_{eq} = \dfrac{4\,mg}{3\,\delta_1} = \dfrac{4}{3} k_1 = k + k_1$

$$\therefore \; k = \frac{1}{3} k_1 = 4.12 \times 10^7 \text{ N/m}$$

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1.12 For a bar with length L, Young's modulus E and cross-section A, the axial stiffness (k) is given by

$$k = \frac{AE}{L} \qquad (1)$$

when cross-section is solid circular with diameter d,

$$\text{area} = A_1 = \pi d^2/4 \qquad (2)$$

when cross-section is square with side d,

$$\text{area} = A_2 = d^2 \qquad (3)$$

When cross-section is hollow circular with mean dia. d and wall thickness $t = 0.1d$,

$$\text{area} = \pi d t = \pi d (0.1 d) = 0.1 \pi d^2 \qquad (4)$$

For specified value of $k = \bar{k}$, cross-section area required is:

$$A = \frac{\bar{k} L}{E} = c \text{ (constant)} \qquad (5)$$

Weight of bar:

with solid circular section:

$$W_1 = \frac{\pi d^2}{4} L = c L \quad \text{with} \quad d^2 = \frac{4c}{\pi} \qquad (6)$$

with hollow circular section:

$$W_3 = 0.1 \pi d^2 L = 0.1 \pi \left(\frac{4c}{\pi}\right) L = 0.4 c L = 0.4 W_1 \qquad (7)$$

with square section:

$$W_2 = d^2 L = \frac{4c}{\pi} L = \frac{4}{\pi} W_1 = 1.2732 W_1 \qquad (8)$$

$\therefore$ The shaft with the hollow circular cross-section corresponds to minimum weight.

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1.13 Stiffness of a cantilever beam under a bending force at free end: $k = \frac{3EI}{l^3}$ (1)

For a specified value of $k = \bar{k}$,

$$I = \frac{\bar{k}\, l^3}{3E} = c = \text{constant} \tag{2}$$

For a solid circular section with diameter d,

$$I_1 = \frac{\pi d^4}{64} = c \Rightarrow d^4 = \frac{64\,c}{\pi} \quad \text{or} \quad d^2 = \sqrt{\frac{64\,c}{\pi}} \tag{3}$$

$$\text{weight of beam} = W_1 = \frac{\pi d^2 l}{4} = \frac{\pi l}{4}\sqrt{\frac{64\,c}{\pi}}$$

$$= 3.5449\, l \sqrt{c} \tag{4}$$

For a hollow circular section with mean diameter d and wall thickness $t = 0.1\, d$, weight of beam (W_2) is:

$$W_2 = \frac{\pi}{4}\left(d_o^4 - d_i^4\right) l = \frac{\pi l}{4}\left\{(d+t)^2 - (d-t)^2\right\}$$

$$= \frac{\pi l}{4}(4\,d\,t) = \pi\, d\, t\, l = \pi\, l\,(0.1\, d^2)$$

$$= 0.1\,\pi\, l \sqrt{\frac{64c}{\pi}} = 1.4180\, l\sqrt{c} \tag{5}$$

For a square section with side d, weight of the beam (W_3) is:

$$W_3 = d^2 l = l\sqrt{\frac{64\,c}{\pi}} = 4.5135\, l\sqrt{c} \tag{6}$$

By comparing Eqs. (4), (5) and (6), the minimum weight beam corresponds to the hollow circular cross-section.

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1.14 Spring force is given by $F = 800\,x + 40\,x^3$ (1)

static equilibrium of the rubber mounting (x^*) under the weight of the electronic instrument is given by

$$F = 200 = 800\,x^* + 40\,x^{*3}$$

or $$40\,x^{*3} + 800\,x^* - 200 = 0 \quad (2)$$

The roots of the cubic equation (2) can be found from MATLAB as

$$x^* = 0.2492,\quad -0.1246 \pm 4.4773\,i \quad (3)$$

Thus the static equilibrium position of the rubber mounting is given by the real root of Eq. (2):

$$x^* = 0.2492 \text{ in} \quad (4)$$

(a) Equivalent linear spring constant of rubber mounting at its static equilibrium position, using Eq. (1.7), is:

$$k_{eq} = \left.\frac{dF}{dx}\right|_{x^*} = 800 + 120\,x^{*2} = 800 + 1200\,(0.2492)^2$$

$$= 807.4521 \text{ lb/in} \quad (5)$$

(b) Deflection of rubber mounting corresponding to the equivalent linear spring constant is:

$$k = \frac{F}{k_{eq}} = \frac{200}{807.4521} = 0.2477 \text{ in} \quad (6)$$

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1.15

$$F(x) = 200\,x + 50\,x^2 + 10\,x^3 \qquad (1)$$

When the spring undergoes a steady deflection of $x^* = 0.5$ in during the operation of the engine, the force exerted on the spring can be found as

$$F = 200(0.5) + 50\,(0.5)^2 + 10\,(0.5)^3 = 113.75 \text{ lb} \qquad (2)$$

Equivalent linear spring constant at its steady deflection is given by Eq. (1.7):

$$k_{eq} = \left.\frac{dF}{dx}\right|_{x=x^*} = 200 + 100\,x^* + 30\,x^{*2}$$

$$= 200 + 100\,(0.5) + 30\,(0.5)^2$$

$$= 253.75 \text{ lb/in}$$

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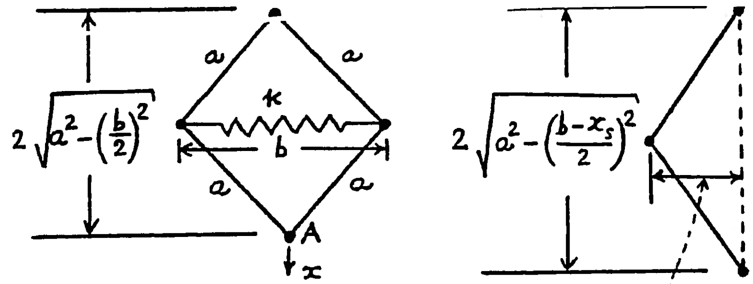
1.16 (a) x = downward deflection of point A,
x_s = resulting deformation of spring



Potential energy equivalence gives $\frac{1}{2} k_{eq}\, x^2 = \frac{1}{2} k\, x_s^2$

$$k_{eq} = k \left(\frac{x_s}{x}\right)^2$$

But
$$x = 2\left[\sqrt{a^2-\left(\frac{b-x_s}{2}\right)^2} - \sqrt{a^2-\left(\frac{b}{2}\right)^2}\right]$$

$$= 2\sqrt{a^2-\left(\frac{b}{2}\right)^2}\left[\left\{\frac{a^2-\left\{\frac{b}{2}\left(1-\frac{x_s}{b}\right)\right\}^2}{a^2-\left(\frac{b}{2}\right)^2}\right\}^{1/2} - 1\right]$$

$$= 2\sqrt{a^2-\frac{b^2}{4}}\left[\left\{\frac{\left(a^2-\frac{b^2}{4}-\frac{x_s^2}{4}+\frac{b\,x_s}{2}\right)}{\left(a^2-\frac{b^2}{4}\right)}\right\}^{1/2} - 1\right]$$

$$= 2\sqrt{a^2-\frac{b^2}{4}}\left[\left\{1 - \frac{\overbrace{\cdots x_s^2}^{\approx 0}}{\left(a^2-\frac{b^2}{4}\right)} + \frac{b\,x_s}{2\left(a^2-\frac{b^2}{4}\right)}\right\}^{1/2} - 1\right]$$

Using the relation $(1+\theta)^{1/2} \simeq 1+\frac{\theta}{2}$, we obtain

$$x = 2\left(a^2-\frac{b^2}{4}\right)^{1/2}\left[1+\frac{b\,x_s}{4\left(a^2-\frac{b^2}{4}\right)} - 1\right] = \frac{b\,x_s}{2\left(a^2-\frac{b^2}{4}\right)^{1/2}}$$

$$\therefore\ k_{eq} = k\left(\frac{x_s}{x}\right)^2 = 4k\left(\frac{a^2-\frac{b^2}{4}}{b^2}\right) = k\left(\frac{4a^2-b^2}{b^2}\right)$$

(b) Here $x = x_s$ (spring deflection)

$$\therefore\ k_{eq} = k$$

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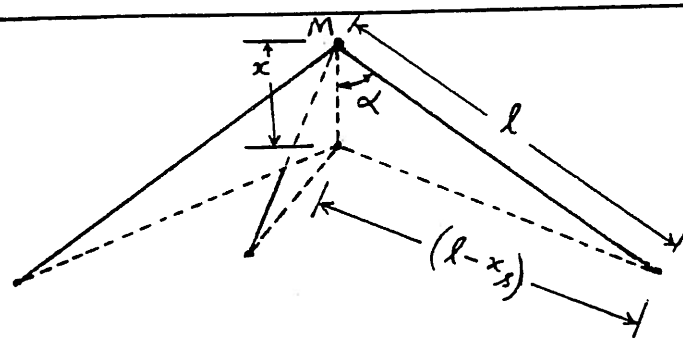
1.17 Let x = vertical displacement of mass M,
x_s = resulting deformation of each inclined spring.



From equivalence of potential energy,

$$\frac{1}{2} k_{eq} x^2 = 3 \left(\frac{1}{2} k x_s^2\right) \;; \qquad k_{eq} = 3 k \left(\frac{x_s}{x}\right)^2$$

From geometry,

$$(\ell - x_s)^2 = \ell^2 + x^2 - 2 \ell x \cos\alpha$$
$$x^2 - 2x \ell \cos\alpha + 2\ell x_s - x_s^2 = 0 \qquad (E_1)$$

Solving (E_1),

$$x = \ell \cos\alpha \left[1 \pm \left\{1 - \frac{(2\ell x_s - x_s^2)}{\ell^2 \cos^2\alpha}\right\}^{1/2}\right] \qquad (E_2)$$

Using the relation $\sqrt{1-\theta} \approx 1 - \frac{\theta}{2}$, (E_2) can be rewritten as

$$x = \ell \cos\alpha \left[1 \pm \left\{1 - \left(\frac{2\ell x_s - x_s^2}{2\ell^2 \cos^2\alpha}\right)\right\}\right] \qquad (E_3)$$

Assuming x to be small, we use minus sign and neglect x_s^2 compared to $2\ell x_s$ in (E_3). This gives

$$x = \frac{x_s}{\cos\alpha}$$

$$\therefore \quad k_{eq} = 3 k \cos^2\alpha$$

In a similar manner, $c_{eq} = 3 c \cos^2\alpha$

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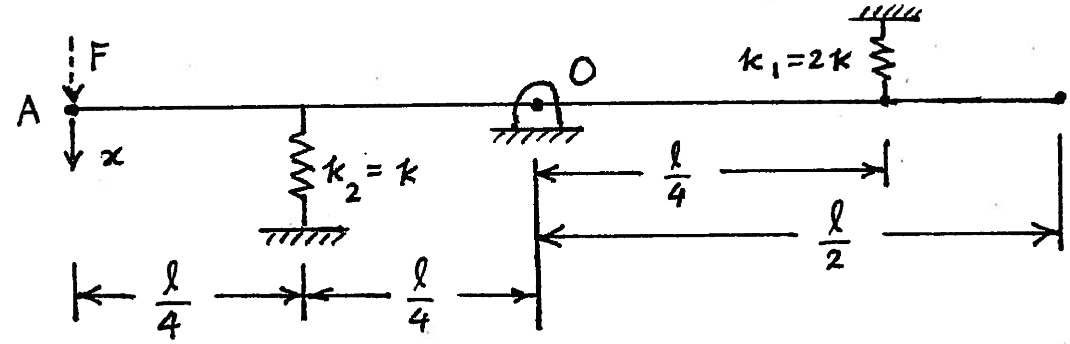
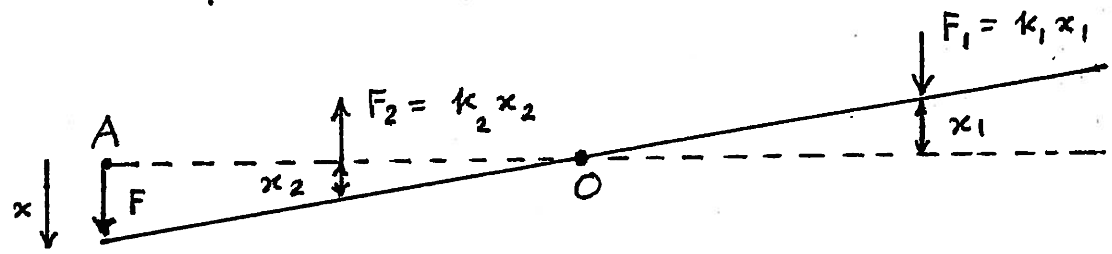
1.18





$$x_2 = \frac{x}{2}, \quad x_1 = \frac{x}{2}$$

$$F_2 = k_2 x_2 = \frac{k x}{2}, \quad F_1 = k_1 x_1 = 2k\left(\frac{x}{2}\right) = k x$$

Equivalent spring constant of the system (k_{eq}) at point A can be determined by considering the moment equilibrium of forces about the pivot point O:

$$F\left(\frac{l}{2}\right) - F_2\left(\frac{l}{4}\right) - F_1\left(\frac{l}{4}\right) = 0$$

$$F = \frac{F_2}{2} + \frac{F_1}{2} = \frac{k x}{4} + \frac{k x}{2} = \frac{3}{4} k x$$

$$= k_{eq}\, x$$

$$\therefore \quad k_{eq} = \frac{3}{4} k$$

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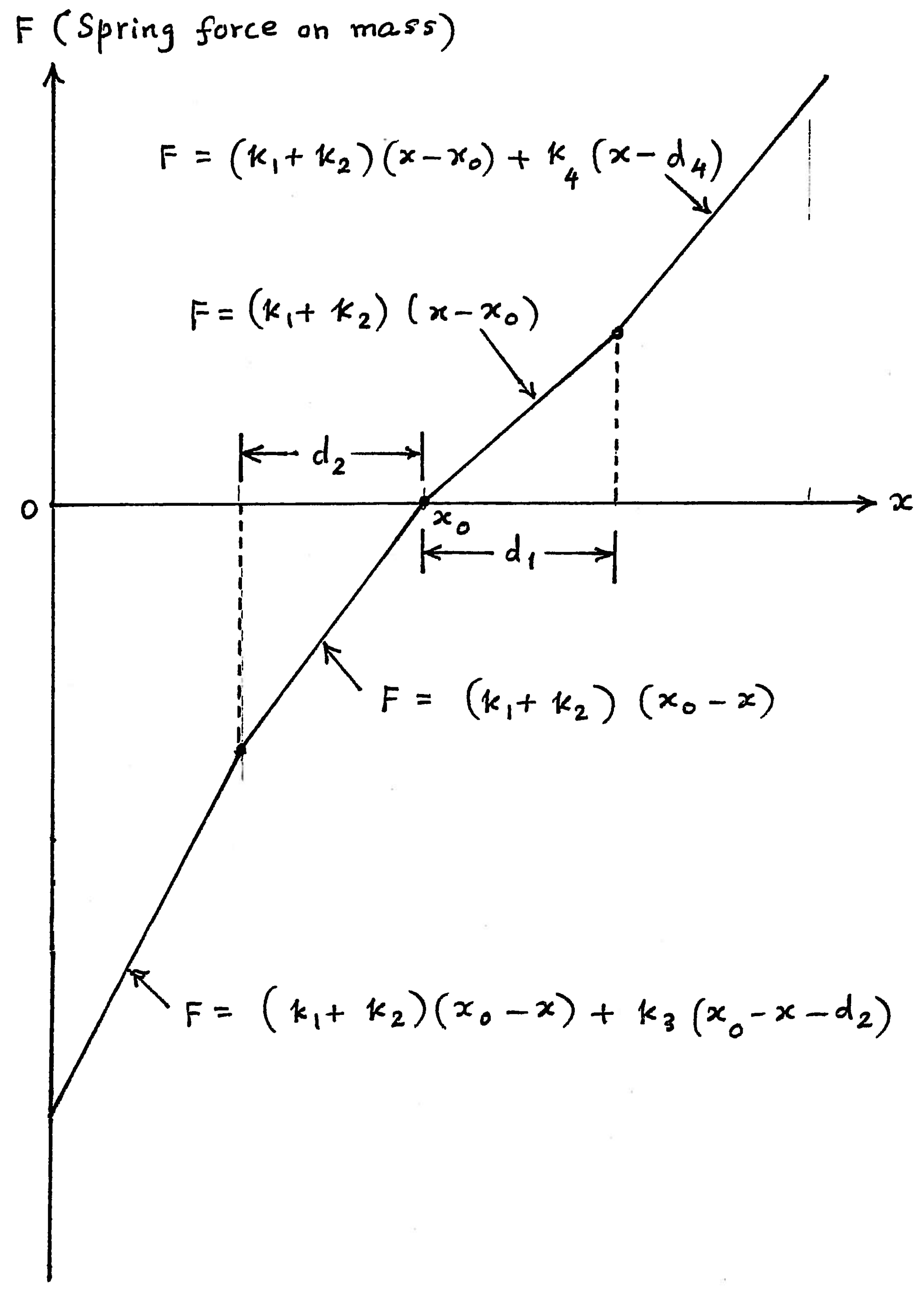
1.19



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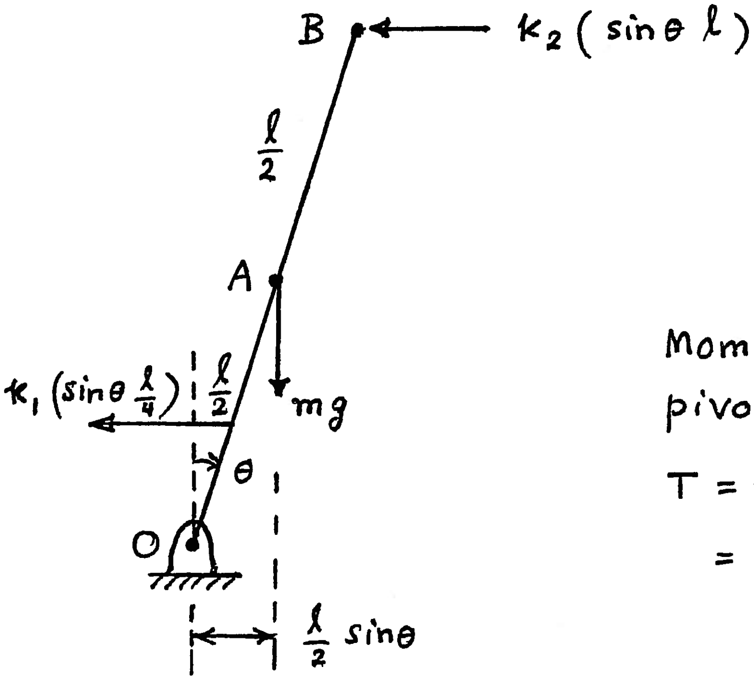
1.20



Moment about the pivot point O:

$$T = \text{moment}$$

$$= mg \frac{l}{2} \sin\theta - \left(k_1 \frac{l}{4} \sin\theta\right) \frac{l}{4} - \left(k_2 \, l \sin\theta\right) l$$

$$\simeq \left(\frac{mgl}{2} - k_1 \frac{l^2}{16} - k_2 l^2\right) \theta \qquad (1)$$

Denoting the equivalent torsional spring constant of the system as k_t, the moment T can be expressed as

$$T = k_t \, \theta \qquad (2)$$

By equating Eqs. (1) and (2), we obtain

$$k_t = \frac{mgl}{2} - \frac{k_1 l^2}{16} - k_2 l^2 \qquad (3)$$

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1.21 When mercury is displaced by an amount x in one leg of the manometer (Fig. 1.77), the mercury column will undergo a total displacement of $2x$. The magnitude of the force, due to the weight of the displaced mercury, acts on the rest of the fluid. The restoring force is given by

$$F = 2\gamma A x \quad (1)$$

where γ is the specific weight of mercury and A is the cross-sectional area of the manometer tube. If k_{eq} denotes the spring constant associated with the restoring force, the restoring force can be expressed as

$$F = k_{eq}\, x \quad (2)$$

Equations (1) and (2) yield the equivalent spring constant as

$$k_{eq} = 2\gamma A \quad (3)$$

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1.22 When the drum is displaced by an amount x from its static equilibrium position, the weight of the fluid (sea water) displaced is given by

$$W = \rho_w g \left(\frac{\pi d^2}{4}\right) x \qquad (1)$$

where ρ_w is the density of sea water and g is the acceleration due to gravity. The weight, W, given by Eq. (1) also denotes the restoring force F. By expressing the restoring force as

$$F = k_{eq}\, x \qquad (2)$$

where k_{eq} denotes the equivalent spring constant associated with the restoring force. Equating (1) and (2), we obtain

$$k_{eq} = \rho_w g \frac{\pi d^2}{4} \qquad (3)$$

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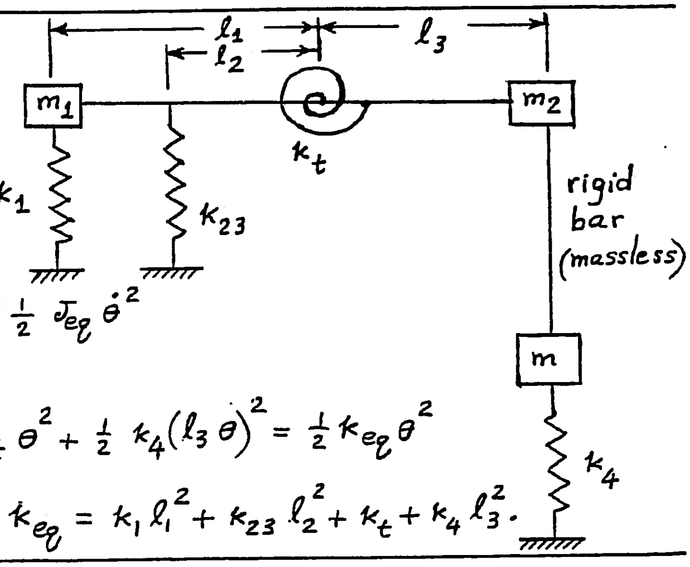
1.23 $k_{23} = \dfrac{k_2 k_3}{k_2 + k_3}$



$k_4 = A \rho g = \dfrac{\pi d^2}{4} \rho g$

From kinetic energy,

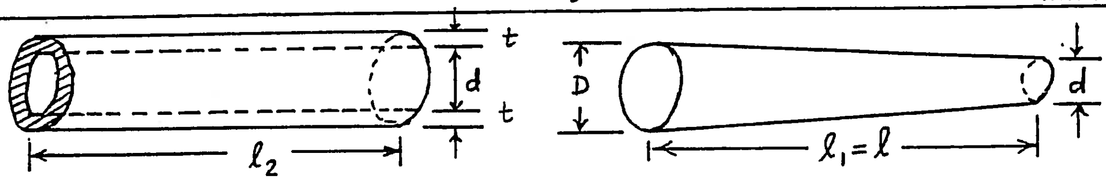
$\frac{1}{2} m_1 (l_1 \dot{\theta})^2 + \frac{1}{2}(m_2 + m)(l_3 \dot{\theta})^2 = \frac{1}{2} J_{eq} \dot{\theta}^2$

From potential energy,

$\frac{1}{2} k_1 (l_1 \theta)^2 + \frac{1}{2} k_{23} (l_2 \theta)^2 + \frac{1}{2} k_t \theta^2 + \frac{1}{2} k_4 (l_3 \theta)^2 = \frac{1}{2} k_{eq} \theta^2$

$\therefore \; J_{eq} = m_1 l_1^2 + (m_2 + m) l_3^2 \; ; \quad k_{eq} = k_1 l_1^2 + k_{23} l_2^2 + k_t + k_4 l_3^2 .$

1.24



$k_2 = \dfrac{E A}{l_2} = \dfrac{\pi E t (d + t)}{l_2}$ $\qquad k_1 = \dfrac{\pi E D d}{4 l}$

$k_2 = k_1$ gives $\quad l_2 = \dfrac{4 t (d + t)}{D d}$

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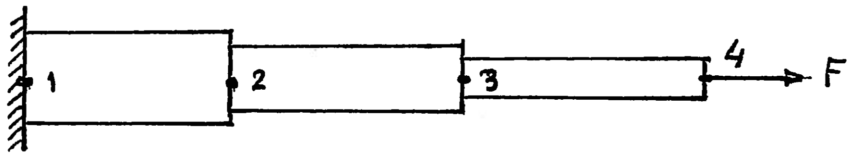
1.25 (a) Spring constant (stiffness) of step i in the axial direction:

$$k_i = \frac{A_i E_i}{l_i} = \frac{A_i E}{l_i}, \quad i = 1,2,3 \qquad (1)$$

(b)



The reaction at any point along the stepped shaft due to an axial force (F) applied at point 4 will be same as F. Hence the springs (stiffnesses) corresponding to the three steps 12, 23 and 34 are to be considered as series springs. In view of Eq.(1), the equivalent spring constant given by Eq. (1.17) becomes

$$\frac{1}{k_{eq}} = \frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_3} = \frac{1}{E}\left(\frac{l_1}{A_1} + \frac{l_2}{A_2} + \frac{l_3}{A_3}\right)$$

$$= \frac{1}{E}\,\frac{(l_1 A_2 A_3 + l_2 A_1 A_3 + l_3 A_1 A_2)}{A_1 A_2 A_3}$$

or

$$k_{eq} = \frac{E A_1 A_2 A_3}{l_1 A_2 A_3 + l_2 A_1 A_3 + l_3 A_1 A_2} \qquad (2)$$

(c) steps behave as series springs.

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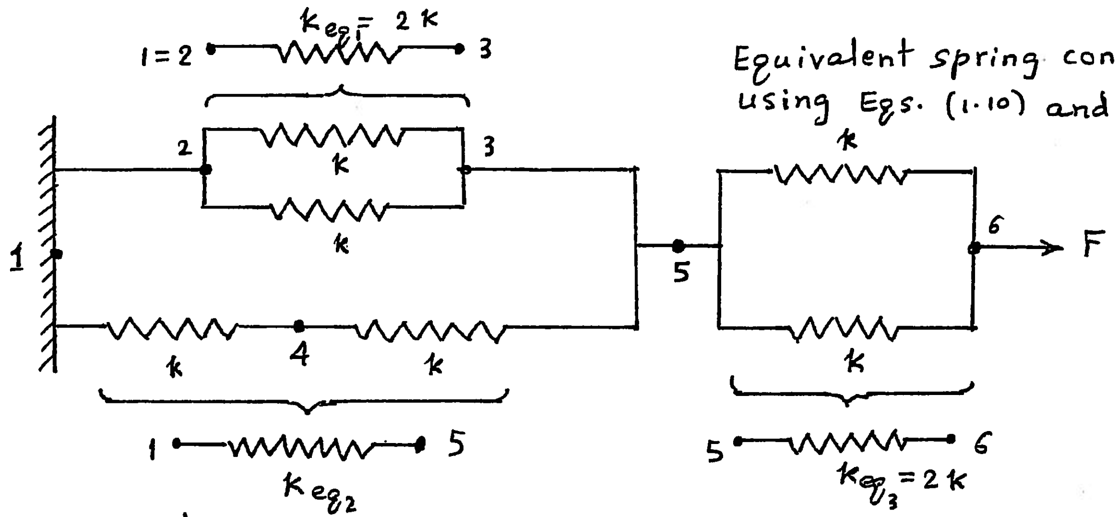
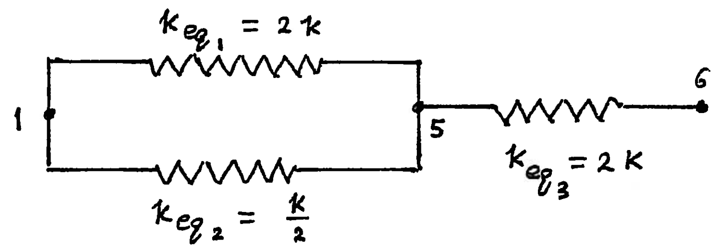
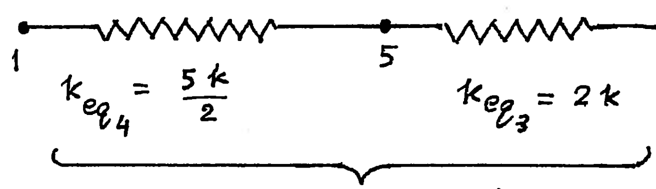
1.26

Equivalent spring constants using Eqs. (1.10) and (1.16).



$$\frac{1}{k_{eq_2}} = \frac{1}{k} + \frac{1}{k} \Rightarrow k_{eq_2} = \frac{k}{2}$$





$$k_{eq\,5} \Rightarrow \frac{1}{k_{eq_5}} = \frac{1}{k_{eq\,4}} + \frac{1}{k_{eq_3}} = \frac{2}{5k} + \frac{1}{2k}$$

$$k_{eq_5} = \frac{10}{9}\,k$$

$$k_{eq} = k_{eq_5} = \frac{10}{9}\,k$$

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1.27 (a) Torsional spring constant or stiffness of step i is

$$k_{t_i} = \frac{G_i J_i}{l_i} = \frac{G_i \pi D_i^4}{32 \, l_i}, \quad i = 1, 2, 3 \qquad (1)$$

(b) The reactive torque at any point along the stepped shaft due to an applied torque T at the free end will be T. Hence the torsional stiffnesses (springs) corresponding to the three steps 12, 23 and 34 are to be considered as series springs. In view of Eq.(1), the equivalent torsional spring constant given by Eq. (1.17) becomes (Eq. (1.17) is to be interpreted for torsional springs):

$$\frac{1}{k_{eq}} = \frac{1}{k_{t1}} + \frac{1}{k_{t2}} + \frac{1}{k_{t3}} = \frac{32}{\pi G}\left(\frac{l_1}{D_1^4} + \frac{l_2}{D_2^4} + \frac{l_3}{D_3^4}\right)$$

$$= \frac{32}{\pi G}\left(\frac{l_1 D_2^4 D_3^4 + l_2 D_1^4 D_3^4 + l_3 D_1^4 D_2^4}{D_1^4 D_2^4 D_3^4}\right)$$

or

$$k_{eq} = \frac{\pi G D_1^4 D_2^4 D_3^4}{32\left(l_1 D_2^4 D_3^4 + l_2 D_1^4 D_3^4 + l_3 D_1^4 D_2^4\right)} \qquad (2)$$

(c) steps behave as series springs.

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1.28 (a) $F \simeq F\big|_{x_0} + \frac{dF}{dx}\big|_{x_0} \cdot (x - x_0) = \left(500\,x + 2x^3\right)_{x=10} + \left(500 + 6x^2\right)_{x=10} \cdot (x-10)$

$\simeq 1100\,x - 4000$

(b) at $x = 9$ mm :

Exact $F_9 = 500 \times 9 + 2\,(9)^3 = 5958$ N

Approximate $F_9 = 1100 \times 9 - 4000 = 5900$ N

Error $= -0.9735\,\%$

(c) at $x = 11$ mm :

Exact $F_{11} = 500 \times 11 + 2\,(11)^3 = 8162$ N

Approximate $F_{11} = 1100 \times 11 - 4000 = 8100$ N

Error $= +0.7596\,\%$

1.29 $p v^\gamma = \text{constant}$ --- (E_1) ; Differentiation of (E_1) gives

$$dp\; v^\gamma + p\,\gamma\, v^{\gamma-1}\; dv = 0$$

$$dp = -\frac{p\gamma}{v}\; dv \quad \text{---}(E_2)$$

change in volume when mass moves by dx, $dv = -A \cdot dx$ --- (E_3)

Eqs. (E_2) and (E_3) give $dp = \frac{p\gamma A}{v}\; dx$

Force due to pressure change $= dF = dp \cdot A = \frac{p\gamma A^2}{v} \cdot dx$

spring constant of air spring $= k = \frac{dF}{dx} = \left(\frac{p\gamma A^2}{v}\right).$

1.30 Equivalent spring constants in diffent directions are

$$k_{e1} = \left(\frac{k_5\,k_6\,k_7}{k_5 k_6 + k_5 k_7 + k_6 k_7}\right), \qquad k_{e2} = \left(\frac{k_8\,k_9}{k_8 + k_9}\right),$$

$$k_{e3} = \left(\frac{k_1\,k_2}{k_1 + k_2}\right), \qquad k_{e4} = \left(\frac{k_3\,k_4}{k_3 + k_4}\right)$$

If the force P moves by x, spring located at θ_i undergoes a displacement of $x_i = x \cos\theta_i$ (derivation as in problem 1.17).

Equivalence of potential energy gives $\frac{1}{2}\, k_{eq}\, x^2 = \frac{1}{2} \sum_{i=1}^{4} k_{ei}\, x_i^2$

$$k_{eq} = \sum_{i=1}^{4} \left(k_{ei}\, \cos^2\theta_i\right)$$

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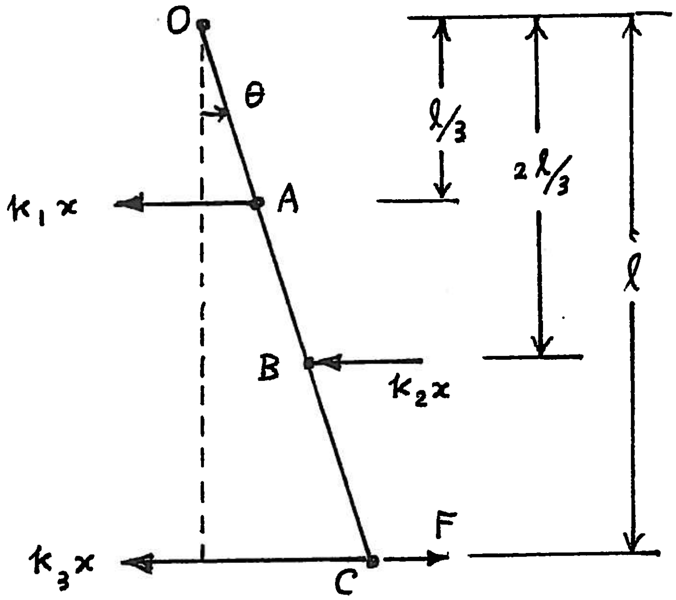
1.31



Let the link OABC undergo a small angular displacement θ as shown in above figure. The spring reaction forces are also indicated in the figure. Equilibrium of moments about the pivot point O gives:

$$-k_1 x \left(\frac{l}{3}\right) - k_3 x (l) - k_2 x \left(\frac{2l}{3}\right) + F(l) = 0$$

or $$F = \left(\frac{k_1}{3} + \frac{2}{3} k_2 + k_3\right) x \qquad (1)$$

If k_{eq} denotes the equivalent spring constant of the link along the direction of F at point C, we have

$$F = k_{eq}\, x \qquad (2)$$

Equations (1) and (2) give

$$k_{eq} = \frac{k_1}{3} + \frac{2 k_2}{3} + k_3 = \frac{k}{3} + \frac{2}{3}(2k) + (3k)$$

$$\therefore\ k_{eq} = \frac{14}{3} k \qquad (3)$$

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1.32 Spring constant of a helical spring is

$$k = \frac{G d^4}{8 N D^3} \qquad (1)$$

Assuming the shear modulus of steel as $G = 79.3$ GPa, Eq. (1) gives, for $D = 0.2$ m, $d = 0.005$ m and $N = 10$,

$$k = \frac{(79.3 \times 10^9)(0.005)^4}{8\,(10)\,(0.2)^3} = 77.4414 \text{ N/m}$$

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1.33 (a) D and d: same for both helical springs

Weight of a helical spring is:

$$W = \pi D \left(\frac{\pi d^2}{4}\right) N \gamma \qquad (1)$$

where γ = specific weight of material of spring.

For a steel spring with $\gamma_s = 76.5$ kN/m^3, the weight is (for $N_s = 10$):

$$W_s = \pi D \left(\frac{\pi d^2}{4}\right) N_s \gamma_s = \frac{\pi^2 D d^2}{4} (10)(76.5 \times 10^3)$$

$$= 19.125 \times 10^4 \pi^2 D d^2 \qquad (2)$$

For an aluminum spring with $\gamma_a = 26.6$ kN/m^3, the weight is (for number of turns N_a),

$$W_a = \pi D \left(\frac{\pi d^2}{4}\right) N_a \gamma_a = \frac{\pi^2 D d^2 N_a}{4} (26.6 \times 10^3)$$

$$= 6.65 \times 10^3 \pi^2 D d^2 N_a \qquad (3)$$

Equating (2) and (3),

$$19.125 \times 10^4 \pi^2 D d^2 = 6.65 \times 10^3 \pi^2 D d^2 N_a$$

or $$N_a = \frac{19.125 \times 10^4}{6.65 \times 10^3} = 28.7594 \qquad (4)$$

(b) Spring constant of a helical spring is:

$$k = G d^4 / (8 N D^3)$$

For a steel spring with $G = 79.3$ GPa,

$$k_s = (79.3 \times 10^9) d^4 / \{8 (10) D^3\}$$

$$= 0.99125 \times 10^9 d^4 / D^3 \qquad (5)$$

For an aluminum spring with $G = 26.2$ GPa,

$$k_a = (26.2 \times 10^9) d^4 / \{8 (28.7594) D^3\}$$

$$= 0.1139 \times 10^9 d^4 / D^3 \qquad (6)$$

Eqs. (5) and (6) indicate that the spring constant of steel spring is $0.99125/0.1139 = 8.7046$ times larger than that of aluminum spring.

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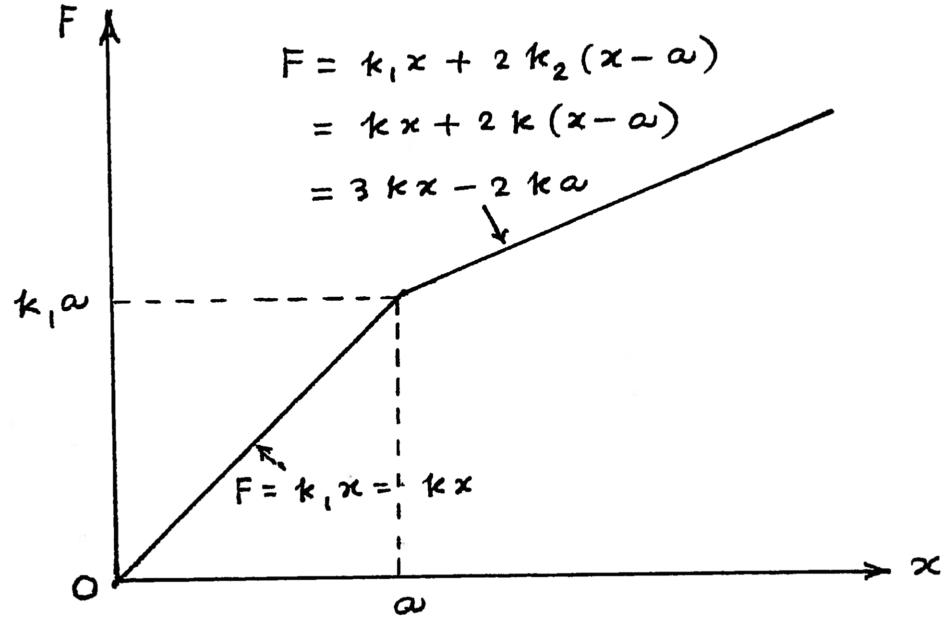
1.34



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1.35 From Problem 1.29, $k = \frac{p \gamma A^2}{v}$ with $\gamma = 1.4$ for air

Let $p = 200$ psi

$k = 75$ lb/in $= \frac{(200)(1.4) A^2}{v} \Rightarrow \frac{A^2}{v} = 0.2679$

Let diameter of piston $= d = 2$ inch ; $A = \frac{\pi}{4} (2)^2 = 3.1416$ in^2

$v = A^2/0.2679 = 36.8408$ in^3

Let $h = 2$ inch ; $\frac{\pi}{4} D^2 (2) = v \Rightarrow D = 4.8429$ inch

1.36 $F = a\,x + b\,x^3 = 2\,(10^4)\,x + 4\,(10^7)\,x^3$

Around x^*: $F(x) \approx F(x^*) + \frac{dF}{dx}\big|_{x^*} (x - x^*)$

When $x^* = 10^{-2}$ m, $F(x^*) = 2\,(10^4)\,(10^{-2}) + 4\,(10^7)\,(10^{-6}) = 240$ N

$\frac{dF}{dx}\big|_{x^*} = a + 3\,b\,x^2 = 2\,(10^4) + 3\,(4)\,(10^7)\,(10^{-4}) = 32000$

Hence $F(x) = 240 + 32000\,(x - 0.01) = (32000\,x - 80)$ N

Since the linearized spring constant is given by $F(x) = k_{eq}\,x$, we have $k_{eq} = 32{,}000$ N/m.

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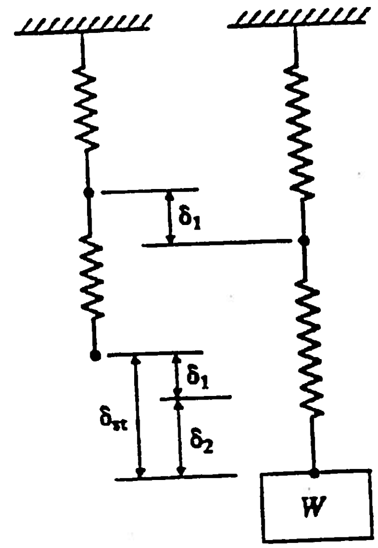
1.37 $F_i = a_i\, x_i + b_i\, x_i^3 \quad ; \; i = 1, 2$

Springs in series:

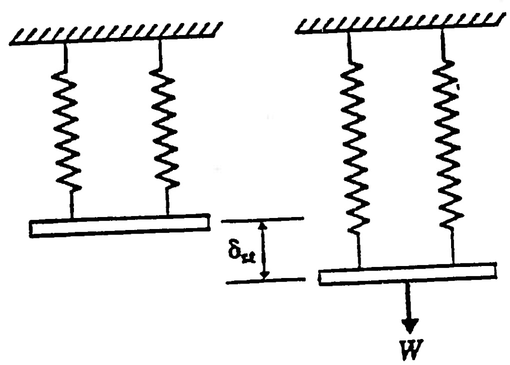
$$W = a_1\, \delta_1 + b_1\, \delta_1^3 \qquad (1)$$
$$W = a_2\, \delta_2 + b_2\, \delta_2^3 \qquad (2)$$
$$W = k_{eq}\, \delta_{st} \qquad (3)$$
$$\delta_{st} = \delta_1 + \delta_2 \qquad (4)$$



solve Eqs. (1) and (2) for δ_1 and δ_2, respectively. Substitute the result in Eq. (4) and then in Eq. (3) to find k_{eq}.

Springs in parallel:

$$W = F_1 + F_2$$
$$= a_1\, \delta_{st} + b_1\, \delta_{st}^3 + a_2\, \delta_{st} + b_2\, \delta_{st}^3$$
$$= k_{eq}\, \delta_{st}$$



$$k_{eq} = a_1 + b_1\, \delta_{st}^2 + a_2 + b_2\, \delta_{st}^2$$

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1.38 $k = \dfrac{G d^4}{8 D^3 N} \geq 8 \times 10^6$ N/m ; $\dfrac{D}{d} \geq 6$; $N \geq 10$

$W = \pi D N \rho \left(\dfrac{\pi d^2}{4}\right)$ where ρ = weight per unit volume

$$f_1 = \frac{1}{2}\sqrt{\frac{kg}{W}} = \frac{1}{2}\sqrt{\frac{G d^2 g}{2 \pi^2 D^4 N^2 \rho}} \geq 0.4 \text{ Hz}$$

Using $G = 73.1 \times 10^9$ N/m^2, $\rho = 76000$ N/m^3, $g = 9.81$ m/sec^2, $\dfrac{D}{d} = 6, 8, 10$; $N = 10, 15, 20$; $d = 0.4, 0.6, \ldots,$ values of k and f_1 are computed.

Combination of $\dfrac{D}{d} = 6$, $N = 10$ and $d = 2.0$ m, corresponding to $k = 8.4606 \times 10^6$ N/m and $f_1 = 0.4801$ Hz, can be taken as an acceptable design.

1.39 Total elongation (strain) is same in each material:

$$\epsilon_s = \epsilon_a = \frac{x}{\ell} \tag{1}$$

where x is the total elongation. Equation (1) can be expressed as

$$\frac{\sigma_s}{E_s} = \frac{\sigma_a}{E_a} = \frac{x}{\ell} \tag{2}$$

$$\text{or} \quad \sigma_s = \frac{E_s\, x}{\ell} \tag{3}$$

$$\sigma_a = \frac{E_a\, x}{\ell} \tag{4}$$

Total axial force is:

$$F = F_s + F_a = \sigma_s A_s + \sigma_a A_a \tag{5}$$

where F_s and F_a denote the axial forces acting on steel and aluminum, respectively, and A_s and A_a represent the cross-sectional areas of the two materials. Equating F to k_{eq} x where k_{eq} denotes the equivalent spring constant of the bimetallic bar, we obtain from Eqs. (3) to (5):

$$F = k_{eq}\, x = \left(\frac{E_s\, x}{\ell}\right) A_s + \left(\frac{E_a\, x}{\ell}\right) A_a$$

$$\text{or} \quad k_{eq} = \frac{E_s A_s}{\ell} + \frac{E_a A_a}{\ell} \tag{6}$$
