2. Detailed Modeling Solutions: ALL Problems

[1] Step 1 Modeling
4 spring (truss) elements; 5 nodes; 1 DOF/node

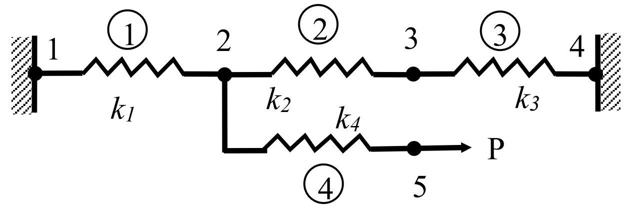

Figure P1

Step 2 Element equations
Element 1

$$[K]^{(1)} = \begin{bmatrix} k_1 & -k_1 \\ -k_1 & k_1 \end{bmatrix} \quad \{u\}^{(1)} = \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} \quad \{P\}^{(1)} = \begin{Bmatrix} F_1^{(1)} \\ F_2^{(1)} \end{Bmatrix}$$

we get $\begin{bmatrix} k_1 & -k_1 \\ -k_1 & k_1 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} = \begin{Bmatrix} F_1^{(1)} \\ F_2^{(1)} \end{Bmatrix}$

Similarly we have

For element 2 $\begin{bmatrix} k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix} \begin{Bmatrix} u_2 \\ u_3 \end{Bmatrix} = \begin{Bmatrix} F_2^{(2)} \\ F_3^{(2)} \end{Bmatrix}$ *For element 3* $\begin{bmatrix} k_3 & -k_3 \\ -k_3 & k_3 \end{bmatrix} \begin{Bmatrix} u_3 \\ u_4 \end{Bmatrix} = \begin{Bmatrix} F_3^{(3)} \\ F_4^{(3)} \end{Bmatrix}$

For element 4 $\begin{bmatrix} k_4 & -k_4 \\ -k_4 & k_4 \end{bmatrix} \begin{Bmatrix} u_2 \\ u_5 \end{Bmatrix} = \begin{Bmatrix} F_2^{(4)} \\ F_5^{(4)} \end{Bmatrix}$

Step 3 Assembly
Global stiffness matrix

$$[K] = \begin{bmatrix} k_1 & -k_1 & 0 & 0 & 0 \\ -k_1 & k_1+k_2+k_4 & -k_2 & 0 & -k_4 \\ 0 & -k_2 & k_2+k_3 & -k_3 & 0 \\ 0 & 0 & -k_3 & k_3 & 0 \\ 0 & -k_4 & 0 & 0 & k_4 \end{bmatrix}$$

Global unknowns vector and load vector

$$\{u\} = \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \end{Bmatrix} \qquad \{P\} = \begin{Bmatrix} F_1^{(1)} \\ F_2^{(1)} + F_2^{(2)} + F_2^{(4)} \\ F_3^{(2)} + F_3^{(3)} \\ F_4^{(3)} \\ F_5^{(4)} \end{Bmatrix} = \begin{Bmatrix} R_1 \\ 0 \\ 0 \\ R_4 \\ P \end{Bmatrix}$$

Global FEM equation

$$\begin{bmatrix} k_1 & -k_1 & 0 & 0 & 0 \\ -k_1 & k_1+k_2+k_4 & -k_2 & 0 & -k_4 \\ 0 & -k_2 & k_2+k_3 & -k_3 & 0 \\ 0 & 0 & -k_3 & k_3 & 0 \\ 0 & -k_4 & 0 & 0 & k_4 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \end{Bmatrix} = \begin{Bmatrix} R_1 \\ 0 \\ 0 \\ R_4 \\ P \end{Bmatrix}$$

Step 4 Boundary condition and solution
We have $u_1 = u_4 = 0$
Therefore, the **reduced** global FEM equations are:

$$\begin{bmatrix} k_1+k_2+k_4 & -k_2 & -k_4 \\ -k_2 & k_2+k_3 & 0 \\ -k_4 & 0 & k_4 \end{bmatrix} \begin{Bmatrix} u_2 \\ u_3 \\ u_5 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ P \end{Bmatrix}$$

The solution is

$$u_2 = \frac{k_2+k_3}{(k_1+k_2)(k_2+k_3)-k_2^2}P \;\; ; \;\; u_3 = \frac{k_2}{(k_1+k_2)(k_2+k_3)-k_2^2}P \;\; ; \;\; u_5 = [\frac{k_2+k_3}{(k_1+k_2)(k_2+k_3)-k_2^2} + \frac{1}{k_4}]P$$

Step 5 Post processing
Reaction force:

$$R_1 = -k_1u_2 = -\frac{k_1(k_2+k_3)}{(k_1+k_2)(k_2+k_3)-k_2^2}P \, ; \qquad R_4 = -k_3u_3 = -\frac{k_2k_3}{(k_1+k_2)(k_2+k_3)-k_2^2}P$$

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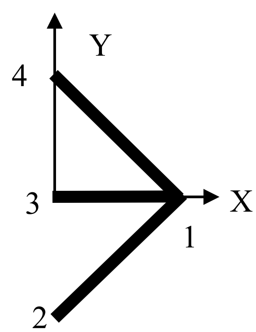
[2] Step 1 Modeling

3 truss elements; 2-nodes / element; 2 DOF/ node
For the shown coordinate system:



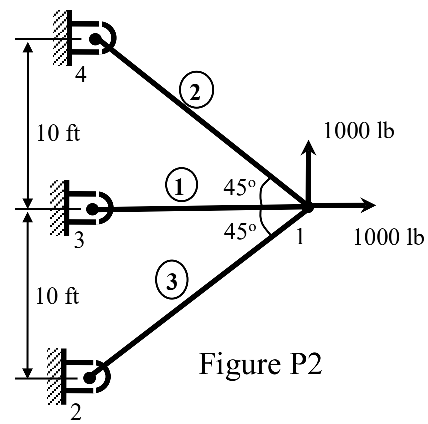


Figure P2

E	bar	α	cosα	sinα	$\cos^2\alpha$	$\sin^2\alpha$	cosαsinα
1	1-3	180°	-1	0	1	0	0
2	1-4	135°	$-1/\sqrt{2}$	$1/\sqrt{2}$	1/2	1/2	-1/2
3	1-2	225°	$-1/\sqrt{2}$	$-1/\sqrt{2}$	1/2	1/2	1/2

Step 2 Element equations
Here only stiffness matrices and unknowns vectors are provided.

$$[K]^{(1)} = \frac{30\times10^6\times2.5}{120}\begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \qquad \{u\}^{(1)} = \begin{Bmatrix} u_1 \\ v_1 \\ u_3 \\ v_3 \end{Bmatrix}$$

$$[K]^{(2)} = \frac{30\times10^6\times1.5}{120\sqrt{2}}\begin{bmatrix} 0.5 & -0.5 & -0.5 & 0.5 \\ -0.5 & 0.5 & 0.5 & -0.5 \\ -0.5 & 0.5 & 0.5 & -0.5 \\ 0.5 & -0.5 & -0.5 & 0.5 \end{bmatrix} \qquad \{u\}^{(2)} = \begin{Bmatrix} u_1 \\ v_1 \\ u_4 \\ v_4 \end{Bmatrix}$$

$$[K]^{(3)} = \frac{30\times10^6\times1.5}{120\sqrt{2}}\begin{bmatrix} 0.5 & 0.5 & -0.5 & -0.5 \\ 0.5 & 0.5 & -0.5 & -0.5 \\ -0.5 & -0.5 & 0.5 & 0.5 \\ -0.5 & -0.5 & 0.5 & 0.5 \end{bmatrix} \qquad \{u\}^{(3)} = \begin{Bmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \end{Bmatrix}$$

Step 3 Assembly
Here only global stiffness matrix and load vector are provided:

$$[K] = 2.5\times10^5\begin{bmatrix} 2.5+\frac{1.5}{2\sqrt{2}}+\frac{1.5}{2\sqrt{2}} & -\frac{1.5}{2\sqrt{2}}+\frac{1.5}{2\sqrt{2}} & -\frac{1.5}{2\sqrt{2}} & -\frac{1.5}{2\sqrt{2}} & -2.5 & 0 & -\frac{1.5}{2\sqrt{2}} & \frac{1.5}{2\sqrt{2}} \\ & \frac{1.5}{2\sqrt{2}}+\frac{1.5}{2\sqrt{2}} & -\frac{1.5}{2\sqrt{2}} & -\frac{1.5}{2\sqrt{2}} & 0 & 0 & \frac{1.5}{2\sqrt{2}} & -\frac{1.5}{2\sqrt{2}} \\ & & \frac{1.5}{2\sqrt{2}} & \frac{1.5}{2\sqrt{2}} & 0 & 0 & 0 & 0 \\ & & & \frac{1.5}{2\sqrt{2}} & 0 & 0 & 0 & 0 \\ & & & & 2.5 & 0 & 0 & 0 \\ & & & & & 0 & 0 & 0 \\ & & & & & & \frac{1.5}{2\sqrt{2}} & -\frac{1.5}{2\sqrt{2}} \\ & Sym & & & & & & \frac{1.5}{2\sqrt{2}} \end{bmatrix}$$

$$= 2.5\times10^5\begin{bmatrix} 3.56 & 0 & -0.53 & -0.53 & -2.5 & 0 & -0.53 & 0.53 \\ & 1.06 & -0.53 & -0.53 & 0 & 0 & 0.53 & -0.53 \\ & & 0.53 & 0.53 & 0 & 0 & 0 & 0 \\ & & & 0.53 & 0 & 0 & 0 & 0 \\ & & & & 2.5 & 0 & 0 & 0 \\ & & & & & 0 & 0 & 0 \\ & & & & & & 0.53 & -0.53 \\ Sym & & & & & & & 0.53 \end{bmatrix} \qquad \{P\} = \begin{Bmatrix} 1000 \\ 1000 \\ R_{2x} \\ R_{2y} \\ R_{3x} \\ R_{3y} \\ R_{4x} \\ R_{4y} \end{Bmatrix}$$

Step 4 Boundary condition and solution
We have $u_2 = u_3 = u_4 = v_2 = v_3 = v_4 = 0$, therefore the reduced global FEM equation is

$$2.5\times10^5\times\begin{bmatrix} 3.56 & 0 \\ 0 & 1.06 \end{bmatrix}\begin{Bmatrix} u_1 \\ v_1 \end{Bmatrix} = \begin{Bmatrix} 1000 \\ 1000 \end{Bmatrix}$$

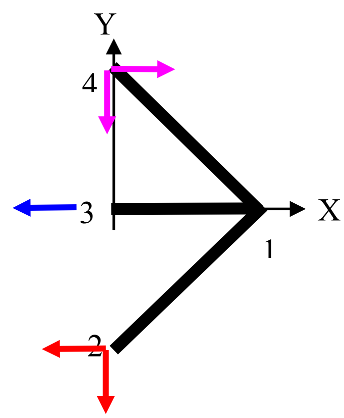
The solution is $u_1 = 1.124\times10^{-3}$ inch $\quad v_1 = 3.774\times10^{-3}$ inch

Step 5 Postprocessing

$$\{P\}=\begin{Bmatrix}1000\\1000\\R_{2x}\\R_{2y}\\R_{3x}\\R_{3y}\\R_{4x}\\R_{4y}\end{Bmatrix}=2.5\times10^{5}\begin{bmatrix}3.56&0&-0.53&-0.53&-2.5&0&-0.53&0.53\\0&1.06&-0.53&-0.53&0&0&0.53&-0.53\\-0.53&-0.53&0.53&0.53&0&0&0&0\\-0.53&-0.53&0.53&0.53&0&0&0&0\\-2.5&0&0&0&2.5&0&0&0\\0&0&0&0&0&0&0&0\\-0.53&0.53&0&0&0&0&0.53&-0.53\\0.53&-0.53&0&0&0&0&-0.53&0.53\end{bmatrix}\begin{Bmatrix}1.124\times10^{-3}\\3.774\times10^{-3}\\0\\0\\0\\0\\0\\0\end{Bmatrix}$$

$$=\{1000\quad 1000\quad -649\quad -649\quad -702\quad 0\quad 351\quad -351\}^{T}$$

The forces are shown in the following figure.



Stress in bar 1-3: $\sigma_{1-3}=\dfrac{702}{2.5}=280.8\,psi$

Stress in bar 1-2: $\sigma_{1-2}=\dfrac{\sqrt{649^2+649^2}}{1.5}=611.9\,psi$

Stress in bar 1-4: $\sigma_{1-4}=-\dfrac{\sqrt{351^2+(-351)^2}}{1.5}=-331.1\,psi$

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[3] Step 1 Modeling

3 truss elements; 2-nodes / element; 2 DOF/ node

$L_1 = L_3 = 10$ ft = 120 in, $L_2 = 10$ ft $=120\sqrt{2}$ in

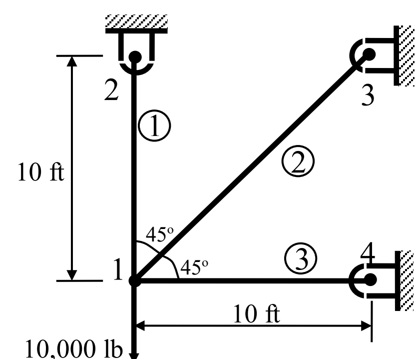

Figure P3

Step 2 Element equations

Element 1:(nodes 1-2)

The angle between the element's local coordinates and the global coordinates is θ = 90.

For a 2-D truss element, the global stiffness matrix is given by (where k_e = the element stiffness matrix , C = cos (θ), and S = sin(θ)):

$$[K]^{g}=k_e\begin{bmatrix}C^2&CS&-C^2&-CS\\CS&S^2&-CS&-S^2\\-C^2&-CS&C^2&CS\\-CS&-S^2&CS&S^2\end{bmatrix}$$

Therefore, the global stiffness matrix and the element characteristic equations are given by:

$$[K_1]^g = k_1 \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix}, \quad k_1 = \left(\frac{E_1 A_1}{L_1}\right) \quad ; \quad \begin{Bmatrix} f_{1x}^1 \\ f_{1y}^1 \\ f_{2x}^1 \\ f_{2y}^1 \end{Bmatrix} = k_1 \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix} \begin{Bmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \end{Bmatrix}$$

For element # 2 (nodes 1-3)
The angle between the element's local coordinates and the global coordinates is θ = 45.
The global stiffness matrix and the element characteristic equations are given by:

$$[K_2]^g = \frac{k_2}{2} \begin{bmatrix} 1 & 1 & -1 & -1 \\ -1 & 1 & -1 & -1 \\ -1 & -1 & 1 & 1 \\ -1 & -1 & 1 & 1 \end{bmatrix}, \quad k_2 = \left(\frac{E_2 A_2}{L_2}\right); \quad \begin{Bmatrix} f_{1x}^2 \\ f_{1y}^2 \\ f_{3x}^2 \\ f_{3y}^2 \end{Bmatrix} = \frac{k_2}{2} \begin{bmatrix} 1 & 1 & -1 & -1 \\ -1 & 1 & -1 & -1 \\ -1 & -1 & 1 & 1 \\ -1 & -1 & 1 & 1 \end{bmatrix} \begin{Bmatrix} u_1 \\ v_1 \\ u_3 \\ v_3 \end{Bmatrix}$$

For element # 3 (nodes 1-4)
The angle between the element's local coordinates and the global coordinates is θ = 0.
The global stiffness matrix and the element characteristic equations are given by:

$$[K_3]^g = k_3 \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad k_3 = \left(\frac{E_3 A_3}{L_3}\right) \quad ; \quad \begin{Bmatrix} f_{1x}^3 \\ f_{1y}^3 \\ f_{4x}^3 \\ f_{4y}^3 \end{Bmatrix} = k_3 \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} u_1 \\ v_1 \\ u_4 \\ v_4 \end{Bmatrix}$$

Step 3 Assembly
The global matrix Assembly $[K]^G$

$$\begin{Bmatrix} f_{1x} \\ f_{1y} \\ f_{2x} \\ f_{2y} \\ f_{3x} \\ f_{3y} \\ f_{4x} \\ f_{4y} \end{Bmatrix} = \begin{bmatrix} \frac{k_2}{2}+k_3 & \frac{k_2}{2} & 0 & 0 & \frac{-k_2}{2} & \frac{-k_2}{2} & -k_3 & 0 \\ \frac{k_2}{2} & \frac{k_2}{2}+k_1 & 0 & -k_1 & \frac{-k_2}{2} & \frac{-k_2}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -k_1 & 0 & k_1 & 0 & 0 & 0 & 0 \\ \frac{-k_2}{2} & \frac{-k_2}{2} & 0 & 0 & \frac{k_2}{2} & \frac{k_2}{2} & 0 & 0 \\ \frac{-k_2}{2} & \frac{-k_2}{2} & 0 & 0 & \frac{k_2}{2} & \frac{k_2}{2} & 0 & 0 \\ -k_3 & 0 & 0 & 0 & 0 & 0 & k_3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \\ u_3 \\ v_3 \\ u_4 \\ v_4 \end{Bmatrix}$$

Where:

$$k_1 = k_3 = \frac{30x10^6 x2}{120} = 5x10^5 \quad Ib/in \qquad k_2 = \frac{30x10^6 x2}{120\sqrt{2}} = \frac{5x10^5}{\sqrt{2}} \quad Ib/in$$

Step 4 Boundary Conditions and Solution
From the geometry model, the boundary conditions are:

$$u_2 = v_2 = u_3 = v_3 = u_4 = v_4 = 0.0$$

$$f_{1x} = 0.0, \quad and \quad f_{1y} = -10{,}000 \; Ib$$

$$\begin{Bmatrix} 0.0 \\ -10{,}000 \\ f_{2x} \\ f_{2y} \\ f_{3x} \\ f_{3y} \\ f_{4x} \\ f_{4y} \end{Bmatrix} = \begin{bmatrix} \frac{k_2}{2}+k_3 & \frac{k_2}{2} & 0 & 0 & \frac{-k_2}{2} & \frac{-k_2}{2} & -k_3 & 0 \\ \frac{k_2}{2} & \frac{k_2}{2}+k_1 & 0 & -k_1 & \frac{-k_2}{2} & \frac{-k_2}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -k_1 & 0 & k_1 & 0 & 0 & 0 & 0 \\ \frac{-k_2}{2} & \frac{-k_2}{2} & 0 & 0 & \frac{k_2}{2} & \frac{k_2}{2} & 0 & 0 \\ \frac{-k_2}{2} & \frac{-k_2}{2} & 0 & 0 & \frac{k_2}{2} & \frac{k_2}{2} & 0 & 0 \\ -k_3 & 0 & 0 & 0 & 0 & 0 & k_3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} u_1 \\ v_1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix}$$

By eliminating the corresponding rows and columns in the global characteristic equation, one gets:

$$\begin{Bmatrix} 0.0 \\ -10{,}000 \end{Bmatrix} = \begin{bmatrix} \frac{k_2}{2}+k_3 & \frac{k_2}{2} \\ \frac{k_2}{2} & \frac{k_2}{2}+k_1 \end{bmatrix} \begin{Bmatrix} u_1 \\ v_1 \end{Bmatrix} \quad \text{or} \quad \begin{Bmatrix} 0.0 \\ -10{,}000 \end{Bmatrix} = 5x10^5 \begin{bmatrix} \frac{1}{2\sqrt{2}}+1 & \frac{1}{2\sqrt{2}} \\ \frac{1}{2\sqrt{2}} & \frac{1}{2\sqrt{2}}+1 \end{bmatrix} \begin{Bmatrix} u_1 \\ v_1 \end{Bmatrix}$$

$$gives \qquad u_1 = 4.142x10^{-3}\ in \qquad and \qquad v_1 = -0.01586\ in$$

Step 5 Post processing

To get the forces at nodes 2,3 and 4

$$\begin{Bmatrix} 0.0 \\ -10{,}000 \\ f_{2x} \\ f_{2y} \\ f_{3x} \\ f_{3y} \\ f_{4x} \\ f_{4y} \end{Bmatrix} = \begin{bmatrix} \frac{k_2}{2}+k_3 & \frac{k_2}{2} & 0 & 0 & \frac{-k_2}{2} & \frac{-k_2}{2} & -k_3 & 0 \\ \frac{k_2}{2} & \frac{k_2}{2}+k_1 & 0 & -k_1 & \frac{-k_2}{2} & \frac{-k_2}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -k_1 & 0 & k_1 & 0 & 0 & 0 & 0 \\ \frac{-k_2}{2} & \frac{-k_2}{2} & 0 & 0 & \frac{k_2}{2} & \frac{k_2}{2} & 0 & 0 \\ \frac{-k_2}{2} & \frac{-k_2}{2} & 0 & 0 & \frac{k_2}{2} & \frac{k_2}{2} & 0 & 0 \\ -k_3 & 0 & 0 & 0 & 0 & 0 & k_3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} 4.142x10^{-3} \\ -0.01586 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix}$$

$$\begin{Bmatrix} f_{2x} \\ f_{2y} \\ f_{3x} \\ f_{3y} \\ f_{4x} \\ f_{4y} \end{Bmatrix} = \begin{bmatrix} 0 & 0 & \frac{-k_2}{2} & \frac{-k_2}{2} & -k_3 & 0 \\ 0 & -k_1 & \frac{-k_2}{2} & \frac{-k_2}{2} & 0 & 0 \end{bmatrix}^T \begin{Bmatrix} u_1 \\ v_1 \end{Bmatrix}$$

$$f_{2x} = 0.0$$
$$f_{2y} = -k_1 v_1 = -5x10^5 x(-0.01586) = 7929\ Ib$$
$$f_{3x} = \frac{-k_2}{2}(u_1 + v_1) = \frac{-5x10^5}{2\sqrt{2}}(4.142x10^{-3} - 0.01586) = 2071\ Ib$$
$$f_{3y} = \frac{-k_2}{2}(u_1 + v_1) = \frac{-5x10^5}{2\sqrt{2}}(4.142x10^{-3} - 0.01586) = 2071\ Ib$$
$$f_{4x} = -k_3 u_1 = -5x10^5 x(4.142x10^{-3}) = -2071\ Ib$$
$$f_{4y} = 0.0$$

Stresses at each bar

Since the structure in hand is a truss structure, the stresses are axial stresses (tension or compression)

Stresses at bar #1

$$\sigma_1 = \frac{F_1}{A_1} = \frac{\sqrt{f_{2x}^2 + f_{2y}^2}}{A_1} = \frac{\sqrt{0 + 7929^2}}{2} = 3964.5\ psi, \quad Tension$$

Stresses at bar #2

$$\sigma_2 = \frac{F_2}{A_2} = \frac{\sqrt{f_{3x}^2 + f_{3y}^2}}{A_2} = \frac{\sqrt{2071^2 + 2071^2}}{2} = 1464.4\ psi, \quad Tension$$

Stresses at bar #3

$$\sigma_3 = \frac{F_3}{A_3} = \frac{-2071}{2} = -1035.5\ psi, \quad Compression$$

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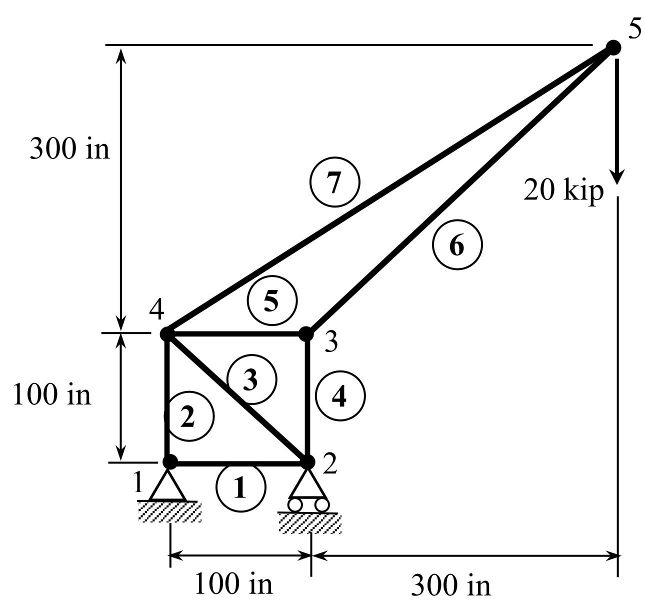
[4] The model has 7-2D truss elements, with 2-DOF/node as shown.



Element 1

$$\begin{bmatrix} K_1 & 0 & -K_1 & 0 \\ 0 & 0 & 0 & 0 \\ -K_1 & 0 & K_1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \end{Bmatrix} = \begin{Bmatrix} F_{1x}^1 \\ F_{1y}^1 \\ F_{2x}^1 \\ F_{2y}^1 \end{Bmatrix}$$

where $K_1 = \dfrac{E_1 A_1}{L_1}$

Element 2

$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & K_2 & 0 & -K_2 \\ 0 & 0 & 0 & 0 \\ 0 & -K_2 & 0 & K_2 \end{bmatrix} \begin{Bmatrix} u_1 \\ v_1 \\ u_4 \\ v_4 \end{Bmatrix} = \begin{Bmatrix} F_{1x}^2 \\ F_{1y}^2 \\ F_{4x}^2 \\ F_{4y}^2 \end{Bmatrix}$$

where $K_2 = \dfrac{E_2 A_2}{L_2}$

Element 3

α =135º; $\cos 135° = -0.7071$, $\sin 135° = 0.7071$, $\cos^2 135° = 0.5$, $\sin^2 135° = 0.5$

$$\begin{bmatrix} 0.5K_3 & -0.5K_3 & -0.5K_3 & 0.5K_3 \\ -0.5K_3 & 0.5K_3 & 0.5K_3 & -0.5K_3 \\ -0.5K_3 & 0.5K_3 & 0.5K_3 & -0.5K_3 \\ 0.5K_3 & -0.5K_3 & -0.5K_3 & 0.5K_3 \end{bmatrix} \begin{Bmatrix} u_2 \\ v_2 \\ u_4 \\ v_4 \end{Bmatrix} = \begin{Bmatrix} F_{2x}^3 \\ F_{2y}^3 \\ F_{4x}^3 \\ F_{4y}^3 \end{Bmatrix} \quad \text{where } K_3 = \frac{E_3 A_3}{L_3}$$

Element 4

$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & K_4 & 0 & -K_4 \\ 0 & 0 & 0 & 0 \\ 0 & -K_4 & 0 & K_4 \end{bmatrix} \begin{Bmatrix} u_2 \\ v_2 \\ u_3 \\ v_3 \end{Bmatrix} = \begin{Bmatrix} F_{2x}^4 \\ F_{2y}^4 \\ F_{3x}^4 \\ F_{3y}^4 \end{Bmatrix}$$

where $K_4 = \dfrac{E_4 A_4}{L_4}$

Element 5

$$\begin{bmatrix} K_5 & 0 & -K_5 & 0 \\ 0 & 0 & 0 & 0 \\ -K_5 & 0 & K_5 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} u_3 \\ v_3 \\ u_4 \\ v_4 \end{Bmatrix} = \begin{Bmatrix} F_{3x}^5 \\ F_{3y}^5 \\ F_{4x}^5 \\ F_{4y}^5 \end{Bmatrix}$$

where $K_5 = \dfrac{E_5 A_5}{L_5}$

Element 6

α =45º; $\cos 45° = 0.7071$ $\sin 45° = 0.7071$ $\cos^2 45° = 0.5$ $\sin^2 45° = 0.5$

$$\begin{bmatrix} 0.5K_6 & 0.5K_6 & -0.5K_6 & -0.5K_6 \\ 0.5K_6 & 0.5K_6 & -0.5K_6 & -0.5K_6 \\ -0.5K_6 & -0.5K_6 & 0.5K_6 & 0.5K_6 \\ -0.5K_6 & -0.5K_6 & 0.5K_6 & 0.5K_6 \end{bmatrix} \begin{Bmatrix} u_3 \\ v_3 \\ u_5 \\ v_5 \end{Bmatrix} = \begin{Bmatrix} F_{3x}^6 \\ F_{3y}^6 \\ F_{5x}^6 \\ F_{5y}^6 \end{Bmatrix} \quad \text{where } K_6 = \frac{E_6 A_6}{L_6}$$

Element 7

$\theta = \text{atan}\left(\frac{3}{4}\right) = 36.9^o$; $\cos\theta = 0.8$ $\sin\theta = 0.6$ $\cos^2\theta = 0.64$ $\sin^2\theta = 0.36$

$$\begin{bmatrix} 0.64K_6 & 0.48K_6 & -0.64K_6 & -0.48K_6 \\ 0.48K_6 & 0.36K_6 & -0.48K_6 & -0.36K_6 \\ -0.64K_6 & -0.48K_6 & 0.64K_6 & 0.48K_6 \\ -0.48K_6 & -0.36K_6 & 0.48K_6 & 0.36K_6 \end{bmatrix} \begin{Bmatrix} u_4 \\ v_4 \\ u_5 \\ v_5 \end{Bmatrix} = \begin{Bmatrix} F_{4x}^7 \\ F_{4y}^7 \\ F_{5x}^7 \\ F_{5y}^7 \end{Bmatrix} \quad \text{where } K_7 = \frac{E_7 A_7}{L_7}$$

Assembly of Global Equation

[K]{U} = {F} where

$$K =$$

$$\begin{bmatrix} k & 0 & -k & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & k & 0 & 0 & 0 & 0 & 0 & -k & 0 & 0 \\ -k & 0 & k+0.5K_3 & -0.5K_3 & 0 & 0 & -0.5K_3 & 0.5K_3 & 0 & 0 \\ 0 & 0 & -0.5K_3 & k+0.5K_3 & 0 & -k & 0.5K_3 & -0.5K_3 & 0 & 0 \\ 0 & 0 & 0 & 0 & k+0.5K_6 & 0.5K_6 & -k & 0 & 0.5K_6 & -0.5K_6 \\ 0 & 0 & 0 & -k & 0.5K_6 & 0 & 0 & 0 & 0.5K_6 & -0.5K_6 \\ 0 & -k & -0.5K_3 & 0.5k_3 & -k & 0 & k+0.5K_3+0.64K_7 & -0.5K_3+0.48K_7 & -0.64K_7 & -0.48K_7 \\ 0 & -k & 0.5K_3 & -0.5k_3 & 0 & 0 & -0.5K_3+0.48K_7 & 0.5K_3+k+0.36K_7 & -0.48K_7 & -0.36K_7 \\ 0 & 0 & 0 & 0 & -0.5K_6 & -0.5K_6 & -0.64K_7 & -0.48K_7 & 0.5K_6+0.64K_7 & 0.5K_6+0.48K_7 \\ 0 & 0 & 0 & 0 & -0.5K_6 & -0.5K_6 & -0.48K_7 & -0.36K_7 & 0.5K_6+0.48K_7 & 0.5K_6+0.36K_7 \end{bmatrix}$$

k$=K_1=K_2=K_4=K_5$

$$\{U\}=\{U_1 \quad V_1 \quad U_2 \quad V_2 \quad U_3 \quad V_3 \quad U_4 \quad V_4 \quad U_5 \quad V_5\}^T$$

$$\{F\}=\{F_{1x} \quad F_{1y} \quad F_{2x} \quad F_{2y} \quad F_{3x} \quad F_{3y} \quad F_{4x} \quad F_{4y} \quad F_{5x} \quad F_{5y}\}^T$$

Boundary Conditions

$U_1 = V_1 = V_2 = 0$, therefore we can eliminate 1st, 2nd, 4th rows and columns from the unreduced global stiffness matrix above, we get:

$$\begin{bmatrix} k+0.5K_3 & 0 & 0 & -0.5K_6 & 0.5K_6 & 0 & 0 \\ 0 & k+0.5K_6 & 0.5K_6 & -k & 0 & -0.5K_6 & -0.5K_6 \\ 0 & 0.5K_6 & k+0.5K_6 & 0 & 0 & -0.5K_6 & -0.5K_6 \\ -0.5K_3 & -k & 0 & k+0.5K_3+0.64K_7 & -0.5K_6+0.36K_7 & -0.64K_7 & -0.48K_7 \\ 0.5K_3 & 0 & 0 & -0.5K_3+0.48K_7 & k+0.5K_3+0.36K_7 & -0.48K_7 & -0.36K_7 \\ 0 & -0.5K_6 & -0.5K_6 & -0.64K_7 & -0.48K_7 & 0.5K_6+0.64K_7 & 0.5K_6+0.48K_7 \\ 0 & -0.5K_6 & 0.5K_6 & -0.48K_7 & -0.36K_7 & 0.5K_6+0.48K_7 & 0.5K_6+0.36K_7 \end{bmatrix}$$

$$\{U\}=\{U_2 \quad U_3 \quad V_3 \quad U_4 \quad V_4 \quad U_5 \quad V_5\}^T$$

$$\{F\}=\{0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad -20000\}^T$$

Using:

E = 30e6 psi, $L_1 = L_2 = L_4 = L_5$ = 100 in, L_3 = 141.42 in

L_6 = 424.264 in, L_7 = 500 in, Area, A = 1 in^2

We get:

k$=K_1=K_2=K_4=K_5$ = 300e3 lb/in, K_3 = 212.132 e3 lb/in

K_6 = 70.7107 e3 lb/in, K_7 = 60 e3 lb/in

By substituting the K values back into the reduced global matrix and solving for unknowns, we get:

Node #	2		3		4		5	
	X disp.	*Y disp.*	*X disp.*	*Y disp.*	*X disp.*	*Y disp.*	*X disp.*	*Y disp*
	0.00	0.00	-0.667	-0.2667	0.2	0.2	17.5215	-20.1176

And substituting these values back into unreduced global matrix, we get reactions at nodes 1 & 2:

$F_{1x} = 0$, $F_{1y} = -60000$ lb, $F_{2y} = 80000$ lb

Hand calculation Checks using Mechanics of Materials:

External support forces

$\sum M_1 = 0:$ (20000)(400) - $F_{2y}(100) = 0$ gives: $F_{2y} = 80000$ lbs

$\sum F_y = 0:$ $F_{1y} + 80000 = 20000$ gives: $F_{1y} = -60000$ lbs

$\sum F_x = 0:$ $F_{1x} = 0$

Internal Members' Forces

Using the method of joints:

At Joint 1

$\sum F_x = 0 \Rightarrow F_1 = 0$, $\sum F_y = 0 \Rightarrow F_2 = 60000$ lbs (tension)

At Joint 2

$\sum F_x = 0 \Rightarrow F_3 = 0$, $\sum F_y = 0 \Rightarrow F_4 = 80000$ lbs (compression)

At Joint 3

$\sum F_y = 0 \Rightarrow F_6(\frac{1}{\sqrt{2}}) = 80000$ $F_6 = 113137.1$ lbs (compression)

$\sum F_x = 0 \Rightarrow F_5 = F_6(\frac{1}{\sqrt{2}}) = 80000$ lbs (compression)

At Joint 4

$\sum F_x = 0 \Rightarrow F_7(\frac{4}{5}) - 80000 = 0$ $F_7 = 100000$ lbs (tension)

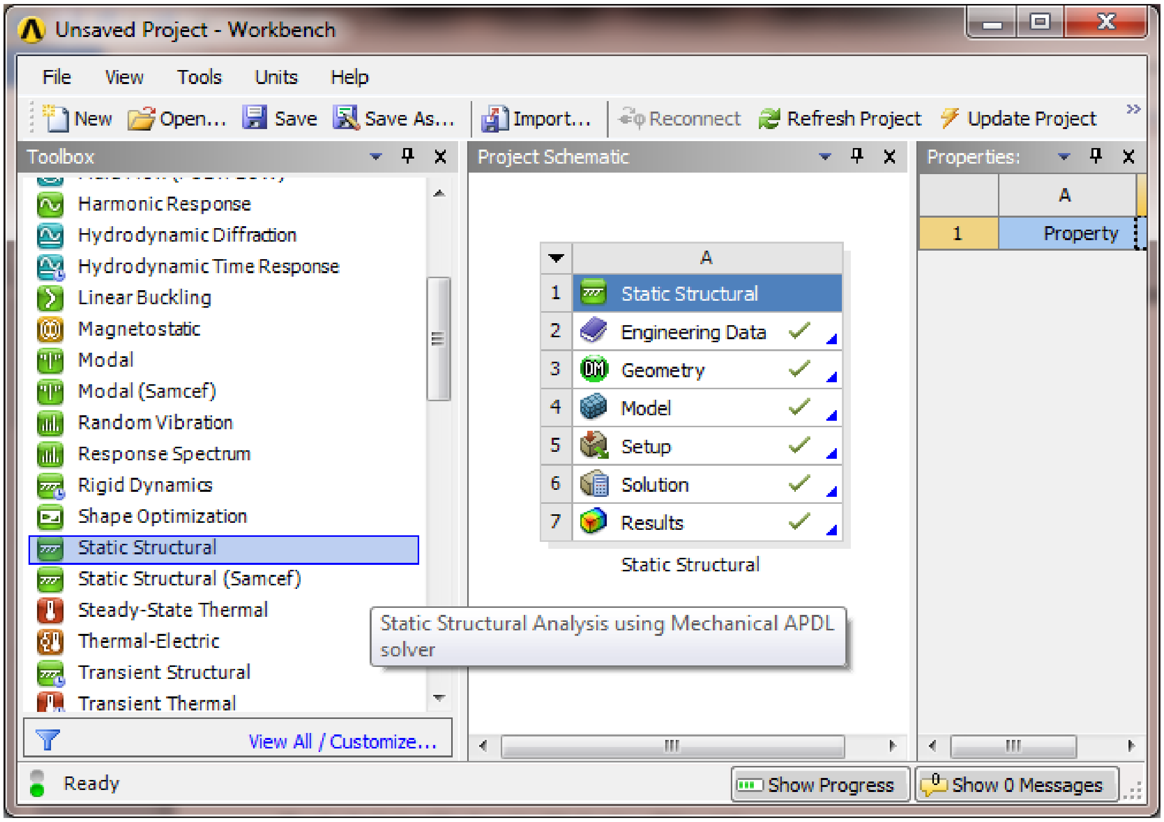
To Check Buckling

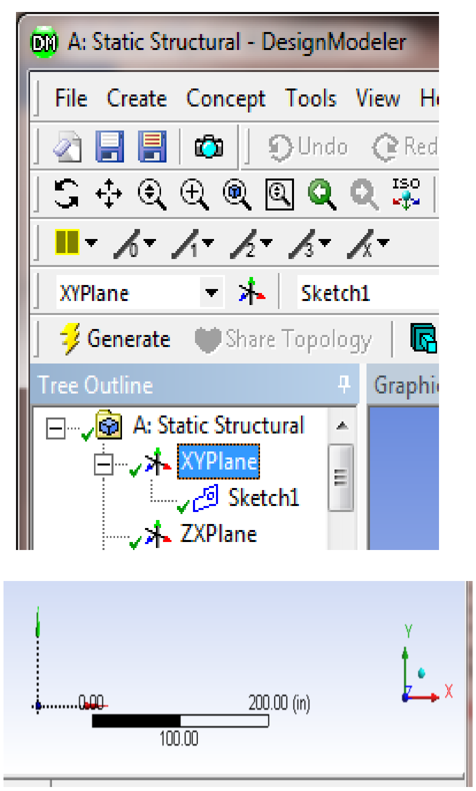
Critical Load is: $P_{CR} = \dfrac{\pi^2 EI}{L^2}$

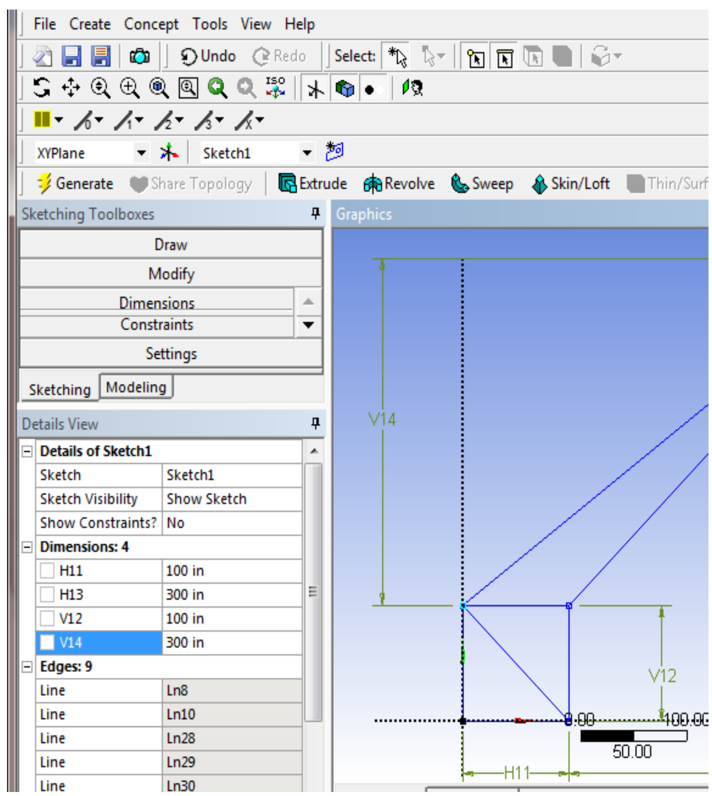
Member	**L (in)**	**Compression Force (lb)**	**P_{CR} (lb)**
4	100	80,000	29,609 *I*
5	100	80,000	29,609 *I*
6	424.26	113,137.	1645 *I*

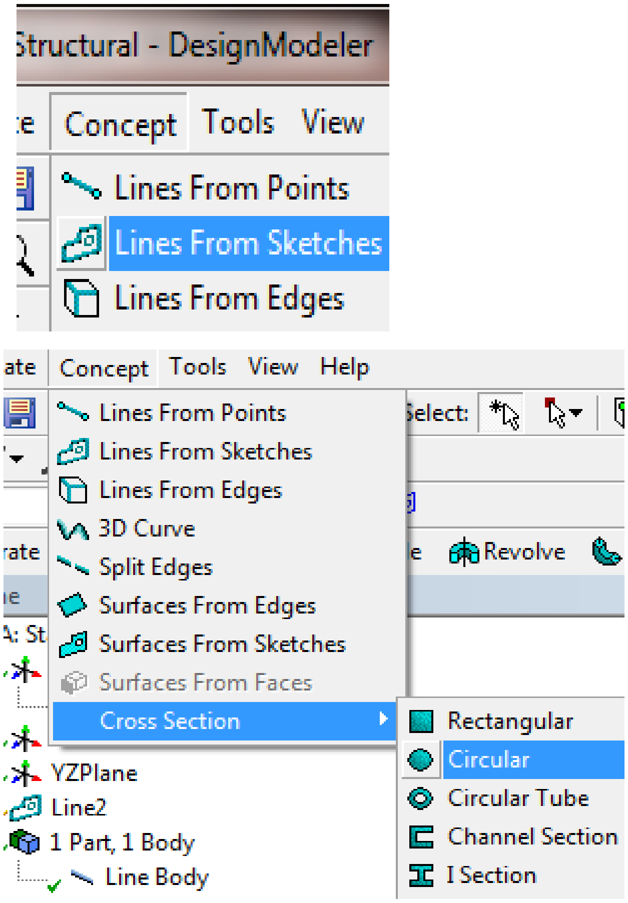
Depending on the value of *I*, the criticality of buckling may be determined from the above table.

ANSYS/Workbench Solution Synopsis





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Lines From Edges
Concept
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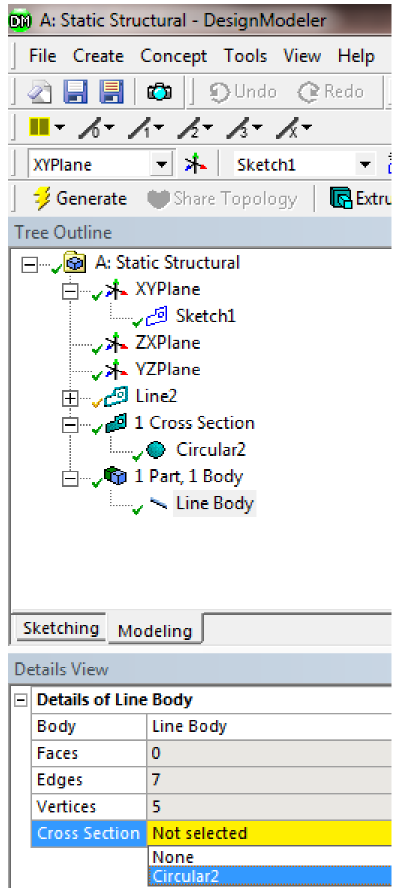
Lines From Sketches
Lines From Edges
3D Curve
Split Edges
Surfaces From Edges
Surfaces From Sketches
Surfaces From Faces
Cross Section
Rectangular
Circular
Circular Tube
Channel Section
I Section
Revolve
YZPlane
Line2
1 Part, 1 Body
Line Body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ZXPlane
YZPlane
Line2
1 Cross Section
Circular2
1 Part, 1 Body
Line Body
Sketching
Modeling
Details View
Details of Line Body
Body
Line Body
Faces
0
Edges
7
Vertices
5
Cross Section
Not selected
None
Circular2