Solutions to Exercises in Chapter 3

3. For the Black-Scholes Equation (RE-3.1),
Let

$$C = E f(S/E) \quad \text{(RE-3.5)}$$

for some function *f, then*

$$\mathrm{C/E} = f(S/E) \quad \text{(RE-3.6)}$$

Apply the change of variables stated in (RE-3.5) to reduce the Black-Scholes Equation (RE-3.1) to its simpler form:

$$\partial C/\partial t + {}^1\!/_2\, \sigma^2 E^2 \partial^2 C/\partial E^2 - rE\partial C/\partial E = 0 \quad \text{(RE-3.7)}$$

which is (RE-.2).

Proof:

Since $C = E\, f(S/E)$, from (RE-3.5),

$$\begin{aligned} \partial C/\partial E &= \partial/\partial E\{Ef(S/E)\} \\ &= f(S/E)\}\partial\{E\}/\partial E + E\partial\{f(S/E)\}/\partial E \\ &= \{f(S/E)\}.1 + E.S\{(-1/E^2)\partial F/\partial E\} \\ &= f(S/E) - (S/E)F'(S/E) \end{aligned}$$

therefore, upon substituting for $f(S/E)$ from (RE-3.6):

$$\partial C/\partial E = C/E - (S/E)F'(S/E) \quad \text{(RE-3.8)}$$

Again, since $C = E\,f(S/E)$, from (RE-3.4),

$$\begin{aligned}\partial C/\partial S &= (\partial/\partial S)[C]\\ &= (\partial/\partial S)[Ef(S/E)]\\ &= E(\partial/\partial S)[f(S/E)]\\ &= E(1/E)(\partial/\partial S)[f(S/E)]\\ &= (1)f'(S/E)]\\ &= f'(S/E)]\end{aligned}$$

therefore,

$$\partial C/\partial S = f'(S/E)] \qquad \text{(RE-3.9)}$$

or,

$$\partial C/\partial S = f'(S/E)] = f'$$

hence,

$$S\,\partial C/\partial S = S\,f'(S/E)] = S\,f'$$

Again, from (RE-3.5):

$$C = E\,f(S/E)$$

therefore

$$\begin{aligned}\partial C/\partial S &= (\partial/\partial \mathrm{S})\,[E\,f(S/E)]\\ &= (1/E)\partial/\partial(S/E)\,[E\,f(S/E)]\\ &= \partial/\partial(S/E)[f(S/E)]\\ &= f\,'\,(S/E) \qquad \text{(RE-3.10)}\end{aligned}$$

Therefore

$$E\,\partial/\partial E(C) = \partial/\partial E\,[E\,f(S/E)],$$

upon substituting for C from (RE-3.5).

Also, from (RE-3.8):

$$\partial C/\partial E = C/E - (S/E)\,F'(S/E) \qquad \text{(RE-3.11)}$$

Hence,

$$\begin{aligned}\partial C/\partial E &= \partial/\partial E[E\,f(S/E)]\\ &= f(S/E) \qquad \text{(RE-3.12)}\end{aligned}$$

Now,

$$\begin{aligned} E\,\partial C/\partial E &= \mathrm{E}[C/E - (S/E)\,F'(S/E)], \text{ from (RE} - 3.11) \\ &= C - SF'(S/E) \\ &= C - S\,\partial C/\partial S, \text{ from (RE} - 3.10) \end{aligned}$$

Hence,

$$\boldsymbol{S\,\partial C/\partial S = C - E\,\partial C/\partial E} \qquad \text{(RE-3.13)}$$

which is an important ***intermediate*** result.

Similarly, it may be shown that:

$$\boldsymbol{S^2\,\partial^2 C/\partial S^2 = E^2\,\partial^2 C/\partial E^2} \qquad \text{(RE-3.14)}$$

which is a ***final*** result which may be readily obtained as follows:

Since

$$S\,\partial C/\partial S = C - E\,\partial C/\partial E\ ,\ from\ \text{(RE-3.13)}$$

then

$$\partial C/\partial S = C/S - (E/S)\,\partial C/\partial E$$

and

$$\begin{aligned} \partial^2 C/\partial S^2 &= \partial/\partial S\{C/S - (E/S)\,\partial C/\partial E\} \\ &= \partial/\partial S\{C/S\} - \partial/\partial S\{(E/S)\,(\partial C/\partial E)\} \\ &= -(C/S^2) + (E/S^2)(\partial C/\partial E) \end{aligned}$$

viz.,

$$S^2\,\partial^2 C/\partial S^2 = -C + E(\partial C/\partial E) \qquad \text{(RE-3.15)}$$

Moreover, since

$$\begin{aligned} \partial C/\partial E &= F(S/E), \text{ from (RE-3.12)} \\ E\,\partial C/\partial E &= E\,F(S/E) \\ &= C, \text{ from (RE-3.5)} \end{aligned}$$

Therefore,

$$\partial C/\partial E = C/E \qquad \text{(RE-3.16)}$$

and

$$(\partial^2 C/\partial E^2) = \partial/\partial E[\partial C/\partial E], \text{ by definition}$$
$$= \partial/\partial E[C/E], \text{ from (RE-3.16)}$$
$$= -(C/E^2) + (1/E)(\partial C/\partial E),$$

by the differentiation of a quotient ruleor,

$$E^2(\partial^2 C/\partial E^2) = -C + E(\partial C/\partial E) \quad \text{(RE-3.17)}$$

Combining (RE-3.15) and (RE-3.17), one finally obtains (RE-3.14), as required.

And now, upon substituting for the terms $S^2\partial^2 C/\partial S^2$ and $S\partial C/\partial S$, respectively from (RE-3.14) and (RE-3.13), respectively, into the Black-Scholes equation, (RE-3.1), the results is:

$$\partial C/\partial t + ½\,\sigma^2 S^2\,\partial^2 C/\partial S^2 + rS\partial C/\partial S - rC = 0 \quad \text{(RE-3.1)}$$

viz.,

$$\partial C/\partial t + ½\,\sigma^2(S^2\,\partial^2 C/\partial S^2) + r(S\partial C/\partial S) - rC = 0$$

viz.,

$$\partial C/\partial t + ½\,\sigma^2(E^2\,\partial^2 C/\partial E^2) + r(C - E\partial C/\partial E) - rC = 0$$

viz.,

$$\partial C/\partial t + ½\,\sigma^2(E^2\,\partial^2 C/\partial E^2) - rE\,\partial C/\partial E = 0 \quad \text{(RE-3.2)}$$

which is (RE-.7), as required.

4. Simulation Results Based on the Model BLCOP: **Black-Litterman and Copula Opinion Pooling Frameworks**

In the R domain:

```
>
> install.packages("BLCOP")
The downloaded binary packages are in

C:\Users\Bert\AppData\Local\Temp\Rtmp2jzxqk\
   downloaded_packages
> library(BLCOP)
Loading required package: MASS
Loading required package: quadprog
In addition: Warning messages:
1: package 'BLCOP' was built under R version 3.2.5
> ls("package:BLCOP")
 [1] "addBLViews"                "addCOPViews"
 [3] "assetSet"                  "BLCOPOptions"
```