Electromechanical Motion Devices

Rotating Magnetic Field-Based Analysis with Online Animations

Third Edition

PAUL KRAUSE
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PREFACE

The list of modern applications requiring high-performance electric machines and drives, already lengthy, is growing rapidly. Originally facilitated by the advent of electronic switching devices in the mid-20th century and accelerated by these devices' ever-increasing performance, this diverse and exciting field has grown into a major industry. Yet, despite the seeming maturity of the field, new applications that require innovative solutions are being developed every day. These include a diverse array of hybrid and electric vehicles – terrestrial- and sea-based crafts; more- and all-electric aircraft, including electrified propulsion systems and unmanned aerial vehicles and drones – large-scale systems for wind energy conversion, and more efficient power generation systems to enable a cleaner grid. Compared to the state-of-the-art, the future machines and drives required to enable these new applications will have higher power, torque, and speed ratings, as well as ever-increasing dynamic performance requirements, all while being subjected to harsher and more demanding environments. The field is no longer limited – nor has it been for decades – to machines and drives that simply sit peacefully on a factory floor driving pumps, fans, or conveyor belts. *Electromechanical Motion Devices Rotating Magnetic Field-Based Analysis and Online Animations Third Edition* has been written specifically to acknowledge this fact and a student who masters the material herein is well on his/her way to contributing to the exciting and dynamic field of high-performance electric machines and drives.

Like its first and second edition predecessors, this introductory-level textbook covers magnetic and magnetically coupled circuits (Chapter 1), electromechanical energy conversion (Chapter 2), and the electric machines in widest use, including dc (Chapter 3), permanent-magnet ac (including the brushless drive) (Chapter 6), synchronous (Chapter 7), induction (Chapter 8), and stepper (Chapter 9), providing sufficient theory, examples, and practice problems to establish a solid base from which to delve deeper into advanced courses and/or texts. In this new edition, Chapters 1, 2, and 9 are unchanged. All of the other chapters have been thoroughly revised and/or rewritten with particular care given to the machine-focused Chapters 6 through 8 to ensure that neither reference frame theory nor control theory is required to understand the material. For each machine, the reference frame derivation which establishes the steady-state phasor equations can be skipped, allowing the instructor to go from the machine variable equations directly to the phasor voltage equations if he or she so chooses. In the case of the drives systems considered, it is assumed that the control is functioning perfectly. Thus, the emphasis is placed on the goal and result of the control, rather than its implementation, and control theory is not needed nor is it recommended as a pre- or co-requisite. Further, the reference frame derivations that establish the phasor voltage equations for the brushless dc and field-orientated drives in Chapters 6 and 8, respectively, can be skipped without loss of cohesiveness. Here, again, the current is assumed to be tightly controlled, whereupon the electric transients are essentially nonexistent and can be neglected. This allows the student to become familiar with the performance of the drives from the steady-state equations without being bogged down with details of the controllers.

Chapter 5 remains devoted to reference frame theory, but it has been completely revised. The subtitle of this third edition, "Rotating Magnetic Field-based Analysis with Online Animations," emphasizes the importance of this revision. It is shown that the basic reference frame transformation is based on and contained in the expression of Tesla's rotating magnetic field and can be obtained using only trigonometry. Applying the transformation to a machine's equations amounts to nothing more than establishing the variables that correctly portray the expression for Tesla's rotating magnetic field as viewed from the given reference frame. This material is not required in order to use the rest of the book; however, it is recommended that at least a part of it be covered since we believe this new way of presenting reference frame theory provides a much clearer and intuitive view of the transformations, allowing the student to gain a deep understanding of the concepts without being bogged down with math and matrix manipulations.

Because this text is intended to serve both the machines and drives areas, there is far more material and detail than can be covered in a three-credit hour course. This is intentional and allows the instructor to pick from a fairly diverse array of material. For example, those interested in the drives area may choose to cover most of Chapter 10 on power electronics while omitting most, if not all, of Chapter 7 on synchronous machines. Alternatively, those interested in the traditional machines area may do the opposite. Moreover, this arrangement of the material allows the text to be used either as a first undergraduate course in machines and drives or as a first semester graduate-level course, where the depth of coverage would be more extensive (or used in both roles at the same school).

Those who are familiar with previous editions will be comfortable using this edition. However, there are important changes, some of which have already been touched upon. The material on reference frame theory has been revised and presented in a new way, and the transformation for rotating circuits has been added. Throughout the text, control theory has been de-emphasized/removed in order to emphasize the goal and result of the controllers, rather than their implementation. The order of Chapters 6 (permanent-magnet ac machine and brushless dc drive) and 8 (induction machines and field orientation) has been switched from the previous editions, and the field-orientation description in Chapter 8 has been revised to follow the new rotating magnetic field-based treatment of reference frame theory in Chapter 5. Chapter 10 on power electronics is new to this edition, and the material on unbalanced operation and single-phase induction machines has been removed.

Finally, as indicated by this edition's new subtitle, detailed online interactive animations that supplement the material in Chapters 3–8 are available. Listed in Appendix B, these animations provide the student and instructor with hands-on visuals and help to portray some of the more difficult concepts. Also, videos and/or PowerPoint presentations of each section of the text are available to the instructor upon request to the publisher.

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CHAPTER 1

MAGNETIC AND MAGNETICALLY COUPLED CIRCUITS

1.1 INTRODUCTION

Before diving into the analysis of electromechanical motion devices, it is helpful to review briefly some of our previous work in physics and in basic electric circuit analysis. In particular, the analysis of magnetic circuits, the basic properties of magnetic materials, and the derivation of equivalent circuits of stationary, magnetically coupled circuits are topics presented in this chapter. Much of this material will be a review for most, since it is covered either in a sophomore physics course for engineers or in introductory electrical engineering courses in circuit theory. Nevertheless, reviewing this material and establishing concepts and terms for later use sets the appropriate stage for our study of electromechanical motion devices.

Perhaps the most important new concept presented in this chapter is the fact that in all electromechanical devices, mechanical motion must occur, either translational or rotational, and this motion is reflected into the electric system either as a change of flux linkages in the case of an electromagnetic system or as a change of charge in the case of an electrostatic system. We will deal primarily with electromagnetic systems. If the magnetic system is linear, then the change in flux linkages results, owing to a change in the inductance. In other words, we will find that the inductances of the electric circuits associated with electromechanical motion devices are functions of the mechanical motion. In this chapter, we shall learn to express the self- and mutual inductances for simple translational and rotational electromechanical devices, and to handle these changing inductances in the voltage equations describing the electric circuits associated with the electromechanical system.

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Throughout this text, we will give short problems (SPs) with answers following most sections. If we have done our job, each SP should take less than ten minutes to solve. Also, it may be appropriate to skip or de-emphasize some material in this chapter depending upon the background or interest of the students. At the close of each chapter, we shall take a moment to look back over some of the important aspects of the material that we have just covered and mention what is coming next and how we plan to fit things together as we go along.

1.2 PHASOR ANALYSIS

Phasors are used to analyze steady-state performance of ac circuits and devices. This concept can be readily established by expressing a steady-state sinusoidal variable as

$$F_a = F_p \cos \theta_{ef} \tag{1.2-1}$$

where capital letters are used to denote steady-state quantities and F_p is the peak value of the sinusoidal variation, which is generally voltage or current but could be any electrical or mechanical sinusoidal variable. For steady-state conditions, θ_{ef} may be written as

$$\theta_{ef} = \omega_e t + \theta_{ef}(0) \tag{1.2-2}$$

where ω_e is the electrical angular velocity and $\theta_{ef}(0)$ is the time-zero position of the electrical variable. Substituting (1.2-2) into (1.2-1) yields

$$F_a = F_p \cos\left[\omega_e t + \theta_{ef}(0)\right] \tag{1.2-3}$$

Since

$$e^{j\alpha} = \cos\alpha + j\sin\alpha \tag{1.2-4}$$

equation (1.2-3) may also be written as

$$F_a = Re\left\{F_p e^{j\left[\omega_e t + \theta_{ef}(0)\right]}\right\} \tag{1.2-5}$$

where Re is shorthand for the "real part of." Equations (1.2-3) and (1.2-5) are equivalent. Let us rewrite (1.2-5) as

$$F_a = Re\left\{F_p e^{j\theta_{ef}(0)} e^{j\omega_e t}\right\} \tag{1.2-6}$$

Thus, we need to take a moment to define what is referred to as the root mean square (rms) of a sinusoidal variation. In particular, the rms value is defined as

$$F = \left(\frac{1}{T}\int_0^T F_a^2(t)\,dt\right)^{\frac{1}{2}} \tag{1.2-7}$$

where F is the rms value of $F_a(t)$ and T is the period of the sinusoidal variation. It is left to the reader to show that the rms value of (1.2-3) is $F_p/\sqrt{2}$. Therefore, we can express (1.2-6) as

$$F_a = Re\left[\sqrt{2}Fe^{j\theta_{ef}(0)}e^{j\omega_e t}\right] \tag{1.2-8}$$

By definition, the phasor representing F_a, which is denoted with a raised tilde, is

$$\tilde{F}_a = Fe^{j\theta_{ef}(0)} \tag{1.2-9}$$

which is a complex number. The reason for using the rms value as the magnitude of the phasor will be addressed later in this section. Equation (1.2-6) may now be written as

$$F_a = Re\left[\sqrt{2}\tilde{F}_a e^{j\omega_e t}\right] \tag{1.2-10}$$

A shorthand notation for (1.2-9) is

$$\tilde{F}_a = F\underline{/\theta_{ef}(0)} \tag{1.2-11}$$

Equation (1.2-11) is commonly referred to as the polar form of the phasor. The Cartesian form is

$$\tilde{F}_a = F\cos\theta_{ef}(0) + jF\sin\theta_{ef}(0) \tag{1.2-12}$$

When using phasors to calculate steady-state voltages and currents, we think of the phasors as being stationary at $t = 0$. On the other hand, a phasor is related to the instantaneous value of the sinusoidal quantity it represents. Let us take a moment to consider this aspect of the phasor and, thereby, give some physical meaning to it. From (1.2-4), we realize that $e^{j\omega_e t}$ is a constant-amplitude line of unity length rotating counterclockwise at an angular velocity of ω_e. Therefore,

$$\sqrt{2}\tilde{F}_a e^{j\omega_e t} = \sqrt{2}F\left\{\cos\left[\omega_e t + \theta_{ef}(0)\right] + j\sin\left[\omega_e t + \theta_{ef}(0)\right]\right\} \tag{1.2-13}$$

is a constant-amplitude line $\sqrt{2}F$ in length rotating counterclockwise at an angular velocity of ω_e with a time-zero displacement from the positive real axis of $\theta_{ef}(0)$. Since $\sqrt{2}F$ is the peak value of the sinusoidal variation, the instantaneous value of F_a is the real part of (1.2-13). In other words, the real projection of the phasor $\tilde{F}_a$ is the instantaneous value of $F_a/\sqrt{2}$ at time zero. As time progresses, $\tilde{F}_a e^{j\omega_e t}$ rotates at ω_e in the counterclockwise direction, and its real projection, in accordance with (1.2-10), is the instantaneous value of $F_a/\sqrt{2}$. Thus, for

$$F_a = \sqrt{2}F\cos\omega_e t \tag{1.2-14}$$

the phasor representing F_a is

$$\tilde{F}_a = Fe^{j0} = F\underline{/0^\circ} = F + j0 \tag{1.2-15}$$

For

$$F_a = \sqrt{2}F \sin \omega_e t \tag{1.2-16}$$

the phasor is

$$\tilde{F}_a = Fe^{-j\pi/2} = F\underline{/-90^\circ} = 0 - jF \tag{1.2-17}$$

Although there are several ways to arrive at (1.2-17) from (1.2-16), it is helpful to ask yourself where must the rotating phasor be positioned at time zero so that, when it rotates counterclockwise at ω_e, its real projection is $(1/\sqrt{2})F_p \sin \omega_e t$? Is it clear that a phasor of amplitude F positioned at $\frac{\pi}{2}$ represents $-\sqrt{2}F \sin \omega_e t$?

In order to show the facility of the phasor in the analysis of steady-state performance of ac circuits and devices, it is useful to consider a series circuit consisting of a resistance, an inductance, and a capacitance. Thus,

$$v_a = Ri_a + L\frac{di_a}{dt} + \frac{1}{C}\int i_a dt \tag{1.2-18}$$

For steady-state operation, let

$$V_a = \sqrt{2}V \cos [\omega_e t + \theta_{ev}(0)] \tag{1.2-19}$$

$$I_a = \sqrt{2}I \cos [\omega_e t + \theta_{ei}(0)] \tag{1.2-20}$$

where the subscript a is used to distinguish the instantaneous value from the rms value of the steady-state variable. The steady-state voltage equation may be obtained by substituting (1.2-19) and (1.2-20) into (1.2-18), whereupon we can write

$$\sqrt{2}V \cos [\omega_e t + \theta_{ev}(0)] = R\sqrt{2}I \cos [\omega_e t + \theta_{ei}(0)] + \omega_e L\sqrt{2}I \cos \left[\omega_e t + \frac{1}{2}\pi + \theta_{ei}(0)\right] + \frac{1}{\omega_e C}\sqrt{2}I \cos \left[\omega_e t - \frac{1}{2}\pi + \theta_{ei}(0)\right] \tag{1.2-21}$$

The second term on the right-hand side of (1.2-21), which is $L\frac{dI_a}{dt}$, can be written as

$$\omega_e L\sqrt{2}I \cos \left[\omega_e t + \frac{1}{2}\pi + \theta_{ei}(0)\right] = \omega_e L Re\left[\sqrt{2}Ie^{j\frac{1}{2}\pi}e^{j\theta_{ei}(0)}e^{j\omega_e t}\right] \tag{1.2-22}$$

Since $\tilde{I}_a = Ie^{j\theta_{ei}(0)}$, we can write

$$L\widetilde{\frac{dI_a}{dt}} = \omega_e L e^{j\frac{1}{2}\pi}\tilde{I}_a \tag{1.2-23}$$

Since $e^{j\frac{1}{2}\pi} = j$, (1.2-23) may be written as

$$L\widetilde{\frac{dI_a}{dt}} = j\omega_e L\widetilde{I}_a \tag{1.2-24}$$

If we follow a similar procedure, we can show that

$$\frac{1}{C}\widetilde{\int I_a dt} = -j\frac{1}{\omega_e C}\widetilde{I}_a \tag{1.2-25}$$

It is interesting that differentiation of a steady-state sinusoidal variable rotates the phasor counterclockwise by $\frac{1}{2}\pi$, whereas integration rotates the phasor clockwise by $\frac{1}{2}\pi$.

The steady-state voltage equation given by (1.2-21) can be written in phasor form as

$$\widetilde{V}_a = \left[R + j\left(\omega_e L - \frac{1}{\omega_e C}\right)\right]\ \widetilde{I}_a \tag{1.2-26}$$

We can express (1.2-26) compactly as

$$\tilde{V}_a = Z\tilde{I}_a \tag{1.2-27}$$

where Z, the impedance, is a complex number; it is not a phasor. It is often expressed as

$$Z = R + j(X_L - X_C) \tag{1.2-28}$$

where $X_L = \omega_e L$ is the inductive reactance and $X_C = \frac{1}{\omega_e C}$ is the capacitive reactance.

The instantaneous power is

$$\begin{aligned} P &= V_a I_a \\ &= \sqrt{2}V\cos\left[\omega_e t + \theta_{ev}(0)\right]\sqrt{2}I\cos\left[\omega_e t + \theta_{ei}(0)\right] \end{aligned} \tag{1.2-29}$$

After some manipulation, we can write (1.2-29) as

$$P = VI\cos\left[\theta_{ev}(0) - \theta_{ei}(0)\right] + VI\cos\left[2\omega_e t + \theta_{ev}(0) + \theta_{ei}(0)\right] \tag{1.2-30}$$

Therefore, the average power P_{ave} may be written as

$$P_{\text{ave}} = |\widetilde{V}_a||\widetilde{I}_a|\ \cos\left[\theta_{ev}(0) - \theta_{ei}(0)\right] \tag{1.2-31}$$

where $|\widetilde{V}_a|$ and $|\widetilde{I}_a|$ are the magnitude of the phasors (rms value), $\theta_{ev}(0) - \theta_{ei}(0)$ is the power factor angle φ_{pf}, and $\cos\left[\theta_{ev}(0) - \theta_{ei}(0)\right]$ is referred to as the power factor. If current is positive in the direction of voltage drop then (1.2-31) is positive if power is consumed and negative if power is generated. It is interesting to point out that in going from (1.2-29) to (1.2-30), the coefficient of the two right-hand terms is $\frac{1}{2}\left(\sqrt{2}V\sqrt{2}I\right)$ or one-half the product of the peak values of the sinusoidal variables. Therefore, it was considered more convenient to use the rms values for the phasors, whereupon average power could be calculated by the product of the magnitude of the voltage and current phasors as given by (1.2-31).

We see from (1.2-30) that the instantaneous power of a single-phase ac circuit oscillates at $2\omega_e t$ about an average value. Let us take a moment to calculate the steady-state power of a two-phase ac system. Balanced, steady-state, two-phase variables (a and b phase) may be expressed as

$$V_a = \sqrt{2}V\cos[\omega_e t + \theta_{ev}(0)] \tag{1.2-32}$$

$$I_a = \sqrt{2}I\cos[\omega_e t + \theta_{ei}(0)] \tag{1.2-33}$$

$$V_b = \sqrt{2}V\cos\left[\omega_e t - \frac{1}{2}\pi + \theta_{ev}(0)\right] \tag{1.2-34}$$

$$I_b = \sqrt{2}I\cos\left[\omega_e t - \frac{1}{2}\pi + \theta_{ei}(0)\right] \tag{1.2-35}$$

The total instantaneous power is

$$P = V_a I_a + V_b I_b \tag{1.2-36}$$

Substituting (1.2-32) through (1.2-35) into (1.2-36) and after some trigonometric manipulation, the total power for a balanced two-phase system becomes

$$P = 2\,|\,\tilde{V}_a\,||\,\tilde{I}_a\,|\,\cos\varphi_{pf} \tag{1.2-37}$$

It is important to note that the $2\omega_e t$ oscillation is not present. In other words, the total instantaneous steady-state power is constant. In the case of a three-phase balanced system, the phasors of the three voltages or currents are displaced 120° and the instantaneous steady-state power is also constant and three times the average power of one phase. In other words, the 2 in (1.2-37) becomes 3 when considering a three-phase system.

Example 1A Phasor Diagram

It is often instructive to construct a phasor diagram. For example, let us consider a voltage equation of the form

$$\tilde{V} = Z\tilde{I} + \tilde{E} \tag{1A-1}$$

where Z is given by (1.2-28). Let us assume that $\tilde{V}$ and $\tilde{I}$ are known and that we are to calculate $\tilde{E}$. The phasor diagram may be used as a rough check on these calculations. Let us construct this phasor diagram by assuming that $|X_L| > |X_C|$ and $\tilde{V}$ and $\tilde{I}$ are known as shown in Fig. 1A-1. Solving (1A-1) for $\tilde{E}$ yields

$$\tilde{E} = \tilde{V} - [R + j(X_L - X_C)]\tilde{I} \tag{1A-2}$$

To perform this graphically, start at the origin in Fig. 1A-1 and walk to the terminus of $\tilde{V}$. Now, we want to subtract $R\tilde{I}$. To achieve the proper orientation to do this, stand at the terminus of $\tilde{V}$, turn, and look in the $\tilde{I}$ direction which is at the angle

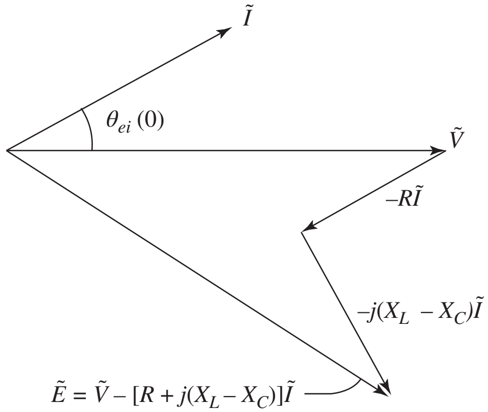

Figure 1A-1 Phasor diagram for (1A-2).

$\theta_{ei}(0)$. But we must subtract $R\tilde{I}$; hence, $-\tilde{I}$ is 180° from $\tilde{I}$, so do an about-face and now we are headed in the $-\tilde{I}$ direction, which is $\theta_{ei}(0) - 180°$. Start walking in the direction of $-\tilde{I}$ for the distance $R\,|\tilde{I}|$ and then stop. While still facing in the $-\tilde{I}$ direction, let us consider the next term. Now, since we have assumed that $|X_L| > |X_C|$, we must subtract $j(X_L - X_C)\tilde{I}$, so let us face in the direction of $-j\tilde{I}$. We are still looking in the $-\tilde{I}$ direction, so we need only to j ourselves. Thus, we must rotate 90° in the counterclockwise direction, whereupon we are standing at the end of $\tilde{V} - R\tilde{I}$ looking in the direction of $\theta_{ei}(0) - 180° + 90°$. Start walking in this direction for the distance of $(X_L - X_C)\,|\tilde{I}|$, whereupon we are at the terminus of $\tilde{V} - [R + j(X_L - X_C)]\tilde{I}$. According to (1A-2), $\tilde{E}$ is the phasor drawn from the origin of the phasor diagram to where we are.

The average steady-state power for a single-phase circuit may be calculated using (1.2-31). We will mention in passing that the reactive power is defined as

$$Q = |\tilde{V}||\tilde{I}| \sin[\theta_{ev}(0) - \theta_{ei}(0)] \tag{1A-3}$$

The units of Q are var (voltampere reactive). An inductance is said to absorb reactive power and thus, by definition, Q is positive for an inductor and negative for a capacitor. Actually, Q is a measure of the interchange of energy stored in the electric (capacitor) and magnetic (inductance) fields.

SP1.2-1 If $\tilde{V} = 1\underline{/0°}$ and $\tilde{I} = 1\underline{/180°}$ in the direction of the voltage drop, calculate Z and P_{ave}. Is power generated or consumed? [$(-1 + j0)$ ohms, 1 watt, generated]

SP1.2-2 For SP1.2-1, express instantaneous voltage, current, and power if the frequency is 60 Hz. [$V = \sqrt{2}\cos 377t$, $I = \sqrt{2}\cos(377t + \pi)$, $P = -1 + 1\cos(754t + \pi)$]

SP1.2-3 $A = \sqrt{2}\underline{/0°}$, $B = \sqrt{2}\underline{/90°}$. Calculate $A + B$ and $A \times B$. [$1\underline{/45°}$, $2\underline{/90°}$]

SP1.2-4 In Example 1A, $X_L > X_C$ and yet $\tilde{I}$ was given as leading $\tilde{V}$. How can this be? [$\tilde{E}$]

1.3 MAGNETIC CIRCUITS

An elementary magnetic circuit is shown in Fig. 1.3-1. This system consists of an electric conductor wound N times about the magnetic member, which is generally some type of ferromagnetic material. In this example system, the magnetic member contains an air gap of uniform length between points a and b. We will assume that the magnetic system (circuit) consists only of the magnetic member and the air gap. Recall that Ampere's law states that the line integral of the field intensity **H** about a closed path is equal to the net current enclosed within this closed path of integration. That is,

$$\oint \mathbf{H} \cdot d\mathbf{L} = i_n \tag{1.3-1}$$

where i_n is the net current enclosed. Let us apply Ampere's law to the closed path depicted as a dashed line in Fig. 1.3-1. In particular,

$$\int_a^b H_i dL + \int_b^a H_g dL = Ni \tag{1.3-2}$$

where the path of integration is assumed to be in the clockwise direction. This equation requires some explanation. First, we are assuming that the field intensity exists only in the direction of the given path, hence we have dropped the vector notation. The subscript i denotes the field intensity (H_i) in the ferromagnetic material (iron or steel) and g denotes the field intensity (H_g) in the air gap. The path of integration is taken as the mean length about the magnetic member, for purposes we shall explain later. The right-hand side of (1.3-2) represents the net current enclosed. In particular, we have enclosed the current i, N times. This has the units of amperes but is commonly referred to as ampere-turns (A·t) or magnetomotive force (mmf). We will find that the mmf in magnetic circuits is analogous to the electromotive force (emf) in electric circuits. Note that the current enclosed is positive in (1.3-2) if the current i is positive. The sign of the right-hand side of (1.3-2)

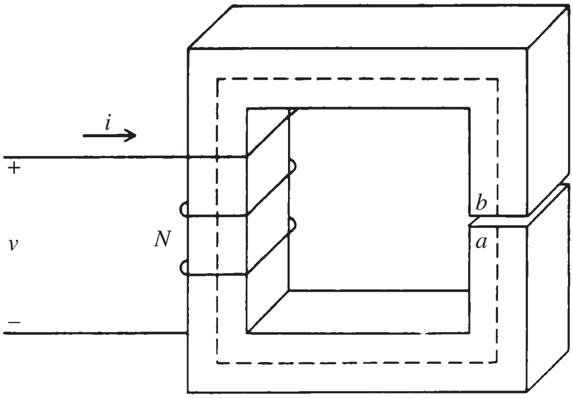

Figure 1.3-1 Elementary magnetic circuit.

may be determined by the so-called "corkscrew" rule. That is, the current enclosed is positive if its assumed positive direction is in the same direction as the advance of a right-hand screw if it were turned in the direction of the path of integration, which in Fig. 1.3-1 is clockwise. Before continuing, it should be mentioned that we refer to **H** as the field intensity; however, some authors prefer to call **H** the field strength.

If we carry out the line integration, (1.3-2) can be written as

$$H_i l_i + H_g l_g = Ni \tag{1.3-3}$$

where l_i is the mean length of the magnetic material and l_g is the length across the air gap. Now, we have some explaining to do. We have assumed that the magnetic circuit consists only of the ferromagnetic material and the air gap, and that the magnetic field intensity is always in the direction of the path of integration or, in other words, perpendicular to a cross section of the magnetic material taken in the same sense as the air gap is cut through the material. The assumed direction of the magnetic field intensity is valid except in the vicinity of the corners. The direction of the field intensity changes gradually rather than abruptly at the corners. Nevertheless, the "mean length approximation" is widely used as an adequate means of analyzing this type of magnetic circuit.

Let us now take a cross section of the magnetic material as shown in Fig. 1.3-2. From our study of physics, we know that for linear, isotropic magnetic materials the flux density **B** is related to the field intensity as

$$\mathbf{B} = \mu \mathbf{H} \tag{1.3-4}$$

where μ is the permeability of the medium. Hence, we can write (1.3-3) in terms of flux density as

$$\frac{B_i}{\mu_i} l_i + \frac{B_g}{\mu_g} l_g = Ni \tag{1.3-5}$$

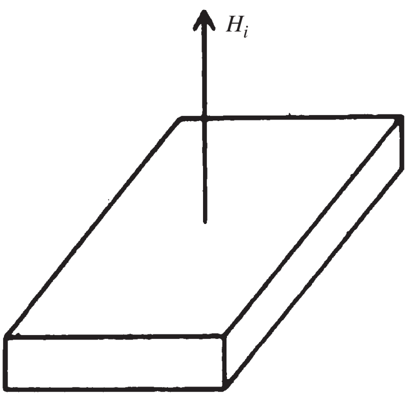

Figure 1.3-2 Cross section of magnetic material.

The surface integral of the flux density is equal to the flux Φ, thus

$$\Phi = \int_A \mathbf{B} \cdot d\mathbf{S} \tag{1.3-6}$$

If we assume that the flux density is uniform over the cross-sectional area, then

$$\Phi_i = B_i A_i \tag{1.3-7}$$

where Φ_i is the total flux in the magnetic material and A_i is the associated cross-sectional area. In the air gap,

$$\Phi_g = B_g A_g \tag{1.3-8}$$

where A_g is the cross-sectional area of the gap. From physics, it is known that the streamlines of flux density **B** are closed; hence, the flux in the air gap is equal to the flux in the core. That is, $\Phi_i = \Phi_g$, and, if the air gap is small, $A_i \cong A_g$, and, therefore, $B_i \cong B_g$. However, the effective area of the air gap is larger than that of the magnetic material, since the flux will tend to balloon or spread out (fringing effect), covering a maximum area midway across the air gap. Generally, this is taken into account by assuming that $A_g = kA_i$, where k, which is greater than unity, is determined primarily by the length of the air gap. Although we shall keep this in mind, it is sufficient for our purposes to assume $A_g = A_i$. If we let $\Phi_i = \Phi_g = \Phi$ and substitute (1.3-7) and (1.3-8) into (1.3-5), we obtain

$$\frac{l_i}{\mu_i A_i}\Phi + \frac{l_g}{\mu_g A_g}\Phi = Ni \tag{1.3-9}$$

The analogy to Ohm's law is at hand. Ni (mmf) is analogous to the voltage (emf), and the flux Φ is analogous to the current. We can complete this analogy if we recall that the resistance of a conductor is proportional to its length and inversely proportional to its conductivity and cross-sectional area. Similarly, $l_i/\mu_i A_i$ and $l_g/\mu_g A_g$ are the reluctances of the magnetic material and air gap, respectively. Generally, the permeability is expressed in terms of relative permeability as

$$\mu_i = \mu_{ri}\mu_0 \tag{1.3-10}$$

$$\mu_g = \mu_{rg}\mu_0 \tag{1.3-11}$$

where μ_0 is the permeability of free space ($4\pi \times 10^{-7}$ Wb/A · m or $4\pi \times 10^{-7}$ H/m, since Wb/A is a henry) and μ_{ri} and μ_{rg} are the relative permeability of the magnetic material and the air gap, respectively. For all practical purposes, $\mu_{rg} = 1$; however, μ_{ri} may be as large as 500–4000 depending upon the type of ferromagnetic material. We will use $\mathcal{R}$ to denote reluctance so as to distinguish reluctance from resistance, which will be denoted by r or R. We can now write (1.3-9) as

$$(\mathcal{R}_i + \mathcal{R}_g)\Phi = Ni \tag{1.3-12}$$

where $\mathcal{R}_i$ and $\mathcal{R}_g$ are the reluctance of the iron and air gap, respectively.

Example 1B Magnetic Circuit – dc Source

A magnetic system is shown in Fig. 1B-1. The total number of turns is 100, the relative permeability of the iron is 1000, and the current is 10 A. Calculate the total flux in the center leg.

Let us draw the electric circuit analog of this magnetic system for which we will need to calculate the reluctance of the various paths:

$$\begin{aligned}\mathcal{R}_{ab} &= \frac{l_{ab}}{\mu_{ri}\mu_0 A_i} \\ &= \frac{0.22}{1000\left(4\pi\times10^{-7}\right)(0.04)^2} = 109{,}419\ \mathrm{H}^{-1}\end{aligned} \tag{1B-1}$$

Similarly,

$$\mathcal{R}_{bcda} = \frac{0.25+0.22+0.25}{(1000)\left(4\pi\times10^{-7}\right)(0.04)^2} = 358{,}099\ \mathrm{H}^{-1} \tag{1B-2}$$

Neglecting the air gap length,

$$\mathcal{R}_{bef} = \mathcal{R}_{gha} = \frac{1}{2}\mathcal{R}_{bcda} = 179{,}049\ \mathrm{H}^{-1} \tag{1B-3}$$

The reluctance of the air gap is

$$\mathcal{R}_{fg} = \frac{0.002}{\left(4\pi\times10^{-7}\right)(0.04)^2} = 994{,}718\ \mathrm{H}^{-1} \tag{1B-4}$$

The electric circuit analog is given in Fig. 1B-2. The polarity of the mmf is determined by the right-hand rule. That is, if we grasp one of the turns of the

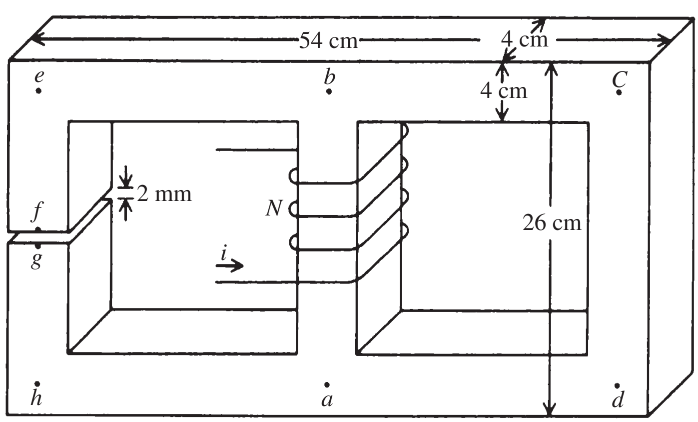

Figure 1B-1 Single-winding magnetic system.

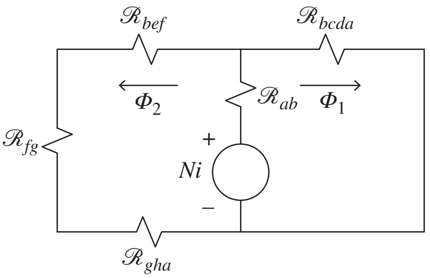

Figure 1B-2 Electric-circuit analog of Fig. 1B-1.

winding with our right hand with the thumb pointed in the direction of positive current, then our fingers will point in the direction of positive flux which flows in the direction of an mmf rise. Or, if we grasp the winding (center leg) with the fingers of our right hand in the direction of positive current, then our thumb will be in the direction of positive flux and in the direction of a rise in mmf.

We can now apply dc circuit theory to solve for the total flux, $\Phi_1 + \Phi_2$, flowing in the center leg. For example, we can use loop equations or, as we will do here, reduce the series–parallel circuit to an equivalent reluctance. The equivalent reluctance of the parallel combination is

$$\begin{aligned}\mathcal{R}_{eq} &= \frac{(\mathcal{R}_{bcda})(\mathcal{R}_{bef} + \mathcal{R}_{fg} + \mathcal{R}_{gha})}{\mathcal{R}_{bcda} + \mathcal{R}_{bef} + \mathcal{R}_{fg} + \mathcal{R}_{gha}} \\ &= \frac{(358,099)(179,049 + 994,718 + 179,049)}{358,099 + 179,049 + 994,718 + 179,049} \\ &= \frac{(358,099)(1,352,816)}{1,710,915} = 283,148\ \text{H}^{-1}\end{aligned} \tag{1B-5}$$

$$\begin{aligned}\Phi_1 + \Phi_2 &= \frac{Ni}{\mathcal{R}_{ab} + \mathcal{R}_{eq}} \\ &= \frac{(100)(10)}{109,419 + 283,148} = 2.547 \times 10^{-3}\ \text{Wb}\end{aligned} \tag{1B-6}$$

Example 1C Magnetic Circuit – ac Source

Consider the magnetic system shown in Fig. 1C-1. The windings are supplied from ac sources and, in the steady state, $I_1 = \sqrt{2}\cos\omega_e t$ and $I_2 = \sqrt{2}\,0.3\cos(\omega_e t + 45°)$, where capital letters are used to denote steady-state conditions. $N_1 = 150$ turns, $N_2 = 90$ turns, and $\mu_r = 3000$. Calculate the flux in the center leg.

The electric circuit analog is given in Fig. 1C-2. The reluctance $\mathcal{R}_x$ is the reluctance of the center leg and $\mathcal{R}_y$ is the reluctance of one of the two parallel paths

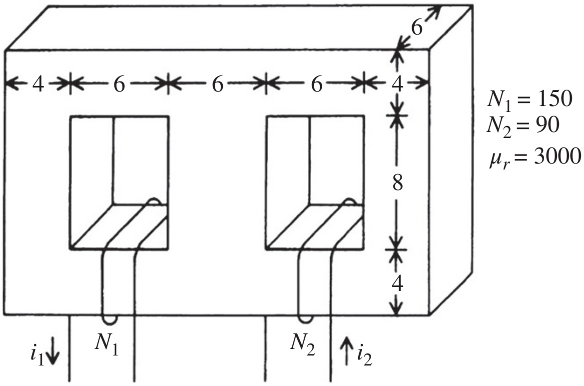

Figure 1C-1 A two-winding magnetic system with dimensions in centimeters.

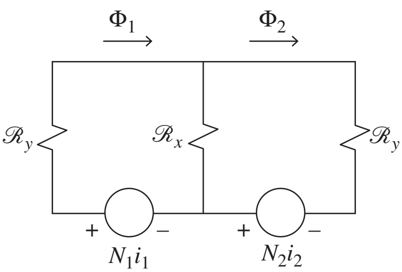

Figure 1C-2 Electric-circuit analog of Fig. 1C-1.

from the top of the center leg through an outside leg to the bottom of the center leg. In particular,

$$\mathcal{R}_y = \frac{2(0.03 + 0.06 + 0.02) + 0.12}{3000(4\pi \times 10^{-7})(0.06)(0.04)} = 37,578\ \mathrm{H}^{-1} \tag{1C-1}$$

$$\mathcal{R}_x = \frac{0.12}{3000(4\pi \times 10^{-7})0.06^2} = 8842\ \mathrm{H}^{-1} \tag{1C-2}$$

Since the currents are sinusoidal, the mmfs will be sinusoidal. Thus, it is convenient to use phasors to solve for Φ_1 and Φ_2. The loop equations are

$$\widetilde{\mathrm{mmf}}_1 = \mathcal{R}_y\tilde{\Phi}_1 + \mathcal{R}_x\left(\tilde{\Phi}_1 - \tilde{\Phi}_2\right) \tag{1C-3}$$

$$\widetilde{\mathrm{mmf}}_2 = \mathcal{R}_x\left(\tilde{\Phi}_2 - \tilde{\Phi}_1\right) + \mathcal{R}_y\tilde{\Phi}_2 \tag{1C-4}$$

which may be written in matrix form as

$$\begin{bmatrix} \widetilde{\mathrm{mmf}}_1 \\ \widetilde{\mathrm{mmf}}_2 \end{bmatrix} = \begin{bmatrix} \mathcal{R}_x + \mathcal{R}_y & -\mathcal{R}_x \\ -\mathcal{R}_x & \mathcal{R}_x + \mathcal{R}_y \end{bmatrix} \begin{bmatrix} \tilde{\Phi}_1 \\ \tilde{\Phi}_2 \end{bmatrix} \tag{1C-5}$$

Now,

$$\tilde{\text{mmf}}_1 = N_1\tilde{I}_1 = (150)(1\underline{/0^\circ}) = 150\underline{/0^\circ}\text{At} \tag{1C-6}$$

$$\tilde{\text{mmf}}_2 = N_2\tilde{I}_2 = (90)(0.3\underline{/45^\circ}) = 27\underline{/45^\circ}\text{At} \tag{1C-7}$$

Solving (1C-5) yields

$$\tilde{\Phi}_1 = (3.434 + j0.081) \times 10^{-3} \text{ Wb} \tag{1C-8}$$

$$\tilde{\Phi}_2 = (1.065 + j0.427) \times 10^{-3} \text{ Wb} \tag{1C-9}$$

The flux flowing down through the center leg is

$$\begin{aligned}\tilde{\Phi}_1 - \tilde{\Phi}_2 &= (2.369 - j0.346) \times 10^{-3} \\ &= 2.39 \times 10^{-3}\underline{/-8.3^\circ} \text{ Wb}\end{aligned} \tag{1C-10}$$

SP1.3-1 Calculate Φ_1 in Example 1B. $[\Phi_1 = 2.014 \times 10^{-3}$ Wb$]$

SP1.3-2 Calculate $\tilde{\Phi}_1 + \tilde{\Phi}_2$ in Example 1B when $I = \sqrt{2}\ 10\cos(\omega_e t - 30^\circ)$. $[\tilde{\Phi}_1 + \tilde{\Phi}_2 = 2.547 \times 10^{-3}\underline{/-30^\circ}$ Wb, rms$]$

SP1.3-3 Remove the center leg of the magnetic system shown in Fig. 1C-1. Calculate the total flux when $I_1 = 9$ A and $I_2 = -15$ A. [Zero]

SP1.3-4 Express the sinusoidal variation represented by $\tilde{\Phi}_2$ given by (1C-9). $[\sqrt{2}\, 1.147 \times 10^{-3}\cos(\omega_e t + 21.8^\circ)]$

1.4 PROPERTIES OF MAGNETIC MATERIALS

We may be aware from our study of physics that, when ferromagnetic materials such as iron, nickel, cobalt, or alloys of these elements, such as various types of steels, are placed in a magnetic field, the flux produced is markedly larger (500–4000 times, for example) than that which would be produced when a non-magnetic material is subjected to the same magnetic field. We must take some time to review briefly the basic properties of ferromagnetic materials and to establish terminology for later use.

Let us begin by considering the relationship between B and H shown in Fig. 1.4-1 which is typical of silicon steel used in transformers. We will assume that the ferromagnetic core is initially completely demagnetized (both B and H are zero). As we apply an external H field by increasing the current in a winding wound around the core, the flux density B also increases, but nonlinearly, as shown in Fig. 1.4-1. After H reaches a value of approximately 150 A·t/m, the flux density rises more slowly and the material begins to saturate.

In ferromagnetic materials, the combination of the magnetic moments produced by the electrons orbiting the nucleus of an atom and the electron itself spinning on its axis produce a net magnetic moment of the atom that is not canceled

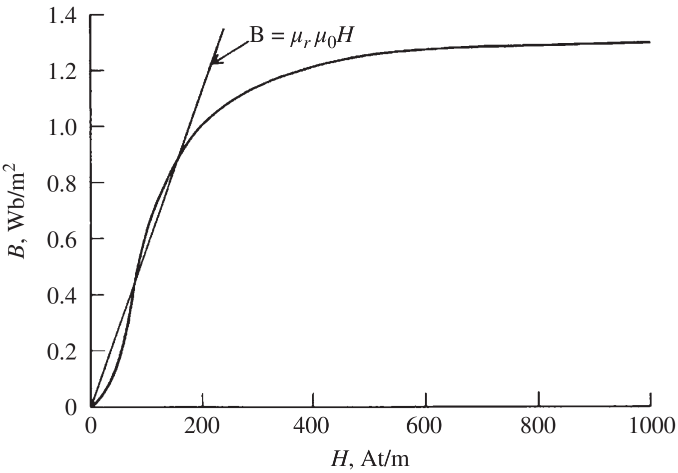

Figure 1.4-1 *B–H* curve for typical silicon steel used in transformers.

by an opposing magnetic moment of a neighboring atom. Ferromagnetic materials have been found to be divided into magnetic domains wherein all magnetic moments (dipoles) are aligned. Although the magnetic moments are all aligned within a magnetic domain, the direction of this alignment will differ from one domain to another.

When a ferromagnetic material is subjected to an external magnetic field, those domains, which originally tend to be aligned with the applied magnetic field, grow at the expense of those domains with magnetic moments that are less aligned. Thereby, the flux is increased from that which would occur with a nonmagnetic material. This is known as *domain-wall motion* [1]. As the strength of the magnetic field increases, the aligned domains continue to grow in nearly a linear fashion. Thus, a nearly linear *B–H* curve results $(B \cong \mu_r \mu_0 H)$ until the ability of the aligned domains to take from the unaligned domains starts to slow. This gives rise to the knee of the *B–H* curve and the beginning of saturation. At this point, the displacements of the domain walls are complete. That is, there are no longer unaligned domains from which to take. However, the remaining domains may still not be in perfect alignment with the external H field. A further increase in H will cause a rotation of the atomic dipole moments within the remaining domains toward a more perfect alignment. However, the marginal increase in B due to rotation is less than the original increase in B due to domain-wall motion, resulting in a decrease in slope of the *B–H* curve. The magnetic material is said to be completely saturated when the remaining domains are perfectly aligned. In this case, the slope of the *B–H* curve becomes μ_0 [1]. If it is assumed that the magnetic flux is uniform through most of the magnetic material, then B is proportional to Φ and H is proportional to mmf. Hence, a plot of flux versus current is of the same shape as the *B–H* curve.

A transformer is generally designed so that some saturation occurs during normal operation. Electric machines are also designed similarly in that a machine generally operates slightly in the saturated region during normal, rated operating conditions. Since saturation causes the coefficients of the differential equations describing the behavior of an electromagnetic device to be functions of the winding currents, a transient analysis is difficult without the aid of a computer. However, it is not our purpose to set forth methods of analyzing nonlinear magnetic systems.

In the previous discussion, we have assumed that the ferromagnetic material is initially demagnetized and that the applied field intensity is gradually increased from zero. However, if a ferromagnetic material is subjected to an alternating field intensity, the resulting *B–H* curve exhibits hysteresis. For example, let us assume that a ferromagnetic material is subjected to an alternating field intensity (alternating current flowing in the winding) and initially the flux density and field intensity are both zero. As *H* increases from zero, *B* increases along the initial *B–H* curve, as shown in Fig. 1.4-2. However, the field intensity varies sinusoidally and, when *H* decreases from a maximum, *B* does not follow back down the original *B–H* curve. After several cycles, the magnetic system will reach a steady-state condition and the plot of *B* versus *H* will form a hysteresis loop or a double-valued function, as shown in Fig. 1.4-2. What is happening is very complex. In simple terms, the growth of aligned domains for an incremental change in *H* in one direction is not equal to the growth of oppositely aligned domains if this change in *H* were suddenly reversed. We could become quite involved by discussing minor hysteresis loops which would occur if, during the sinusoidal variation of *H*, it were suddenly stopped at some nonzero value then reversed, stopped, and reversed again [1]. We shall only mention this phenomenon in passing.

A family of hysteresis loops is shown in Fig. 1.4-3. In each case, the applied *H* is sinusoidal; however, the amplitude of the *H* field is varied to give the family of

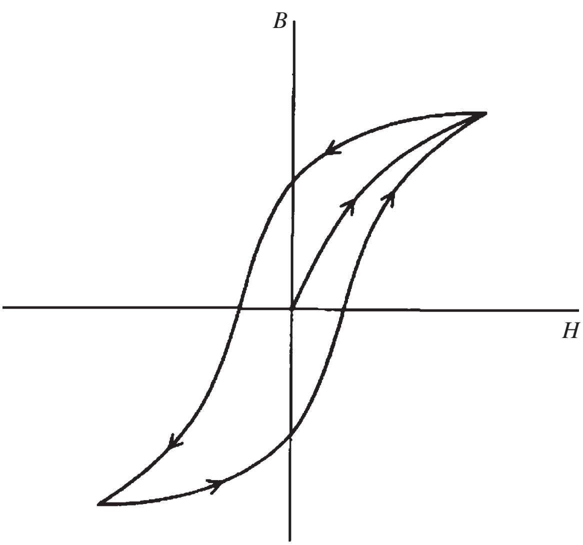

Figure 1.4-2 Hysteresis loop.