Engineering Mechanics | (8th Edition)

Chapter 6, Problem 63P | (4 Bookmarks) | Show all steps: On

Problem

The gear train shown operates in a horizontal plane and is used to transmit motion to the rack D of mass m_D. If an input torque M is applied to gear A, what will be the resulting acceleration a of the unloaded rack? (The mechanism which it normally drives has been disengaged.) Gear C is keyed to the same shaft as gear B. Gears A, B, and C have pitch diameters d_A, d_B, and d_C, and centroidal mass moments of inertia I_A, I_B, and I_C, respectively. All friction is negligible.

Problem 6/63

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Step-by-step solution

Step 1 of 9

Draw the free body diagram of the gear A.

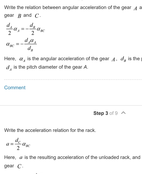

Figure (1)

Comment

Step 2 of 9

Since the gear B and gear C is keyed to the same shaft, the angular acceleration of the gear B and gear C is equal.

$$\alpha_B = \alpha_C$$
$$= \alpha_{BC}$$

Here, α_{BC} is the angular acceleration of the gear B and gear C, α_B is the angular acceleration of the gear B, and α_C is the angular acceleration of the gear C.

Write the relation between angular acceleration of the gear A and angular acceleration of the gear B and C.

$$\frac{d_A}{2}\alpha_A = -\frac{d_B}{2}\alpha_{BC}$$
$$\alpha_{BC} = -\frac{d_A}{d_B}\alpha_A$$

Here, α_A is the angular acceleration of the gear A, d_A is the pitch diameter of the gear B, and d_B is the pitch diameter of the gear A.

Comment

Step 3 of 9

Write the acceleration relation for the rack.

$$a = \frac{d_C}{2}\alpha_{BC}$$

Here, a is the resulting acceleration of the unloaded rack, and d_C is the pitch diameter of the gear C.

Substitute $-\frac{d_A}{d_B}\alpha_A$ for α_{BC}.

$$a = \frac{d_C}{2}\left(-\frac{d_A}{d_B}\alpha_A\right)$$
$$\alpha_A = -\frac{2a}{d_A d_B}$$

Comment

Step 4 of 9

From the figure (1), taking moments about point A.

$$\sum M_A = I_A\alpha_A$$
$$M - \frac{d_A}{2}F_{AB} = I_A\alpha_A$$
$$M - I_A\alpha_A = \frac{d_A}{2}F_{AB}$$
$$F_{AB} = \frac{2}{d_A}\left(M - I_A\alpha_A\right)$$

Here, M is the input torque, I_A is the centroidal mass moment of inertia of the gear A, and F_{AB} is the force acting on the gear A and gear B.

Comment

Step 5 of 9

Draw the free body diagram of the gear B and C.

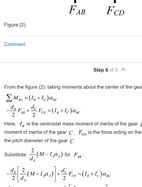

Figure (2)

Comment

Step 6 of 9

From the figure (2), taking moments about the center of the gear.

$$\sum M_{BC} = \left(I_B + I_C\right)\alpha_{BC}$$
$$-\frac{d_B}{2}F_{AB} + \frac{d_C}{2}F_{CD} = \left(I_B + I_C\right)\alpha_{BC}$$

Here, I_B is the centroidal mass moment of inertia of the gear B, I_C is the centroidal mass moment of inertia of the gear C, F_{CD} is the force acting on the gear C and rack D, and d_C is the pitch diameter of the gear C.

Substitute $\frac{2}{d_A}\left(M - I_A\alpha_A\right)$ for F_{AB}.

$$-\frac{d_B}{2}\left[\frac{2}{d_A}\left(M - I_A\alpha_A\right)\right] + \frac{d_C}{2}F_{CD} = \left(I_B + I_C\right)\alpha_{BC}$$
$$\frac{d_C}{2}F_{CD} = \left(I_B + I_C\right)\alpha_{BC} + \frac{d_B}{2}\left[\frac{2}{d_A}\left(M - I_A\alpha_A\right)\right]$$
$$F_{CD} = \frac{2}{d_C}\left[\left(I_B + I_C\right)\alpha_{BC} + \frac{d_B}{2}\left[\frac{2}{d_A}\left(M - I_A\alpha_A\right)\right]\right]$$

Comment

Step 7 of 9

Draw the free body diagram of the rack D.

Figure (3)

Comment

Step 8 of 9

From the figure (3), resolving the forces in the vertical direction.

$$\sum F_x = m_D a$$
$$-F_{CD} = m_D a$$

Here, m_D is the mass of rack D.

Substitute $\frac{2}{d_C}\left[\left(I_B + I_C\right)\alpha_{BC} + \frac{d_B}{2}\left[\frac{2}{d_A}\left(M - I_A\alpha_A\right)\right]\right]$ for F_{CD}, $-\frac{d_A}{2}\alpha_A$ for α_{BC}, and $-\frac{2a}{d_A d_B}$ for α_A.

$$-\frac{2}{d_C}\left[\left(I_B + I_C\right)\left(-\frac{d_A}{d_B}\alpha_A\right) + \frac{d_B}{2}\left[\frac{2}{d_A}\left(M - I_A\left[-\frac{2a}{d_A d_B}\right]\right)\right]\right] = m_D a$$
$$-\frac{2}{d_C}\left[\left(I_B + I_C\right)\left[-\frac{d_A}{d_B}\left(-\frac{2a}{d_A d_B}\right)\right] + \frac{d_B}{2}\left[\frac{2}{d_A}\left(M - I_A\left[-\frac{2a}{d_A d_B}\right]\right)\right]\right] = m_D a$$
$$-\frac{2}{d_C}\left[\left(I_B + I_C\right)\left[\frac{2a}{d_B^2}\right] + \frac{d_B}{2}\left[\frac{2}{d_A}\left(M + \frac{2aI_A}{d_A d_B}\right)\right]\right] = m_D a$$
$$-\frac{4a\left(I_B + I_C\right)}{d_B^2} - \frac{2d_B M}{d_A d_C} - \frac{4aI_A d_B}{d_A^2 d_C^2} = m_D a$$

Comment

Step 9 of 9

By solving the preceding equation, the following relation is obtained.

$$-\frac{4a\left(I_B + I_C\right)}{d_B^2} - \frac{4aI_A d_B}{d_A^2 d_C^2} - m_D a = \frac{2d_B M}{d_A d_C}$$
$$\frac{4a\left(I_B + I_C\right) + 4aI_A d_B^2 + m_D a d_B^2 d_C^2}{d_B^2 d_C^2} = -\frac{2d_B M}{d_A d_C}$$
$$a\left[4a\left(I_B + I_C\right) + 4aI_A + m_D a d_B^2 d_C^2\right] = -2d_B^2 d_C^2 M$$
$$a = \frac{2d_B d_C d_A M}{\left[4a\left(I_B + 4aI_A\left(I_B + I_C\right) + m_D a d_B^2 d_C^2\right]}$$

Therefore, the resulting acceleration of the unloaded rack is

$$\boxed{\frac{2d_B d_C d_A M}{4a\left(I_B + 4aI_A\left(I_B + I_C\right) + m_D a d_B^2 d_C^2\right]}\text{ Answer}}$$

Comment

Was this solution helpful? 0 0

Recommended solutions for you in Chapter 6

Chapter 6, Problem 12P

If the collar P is given a constant acceleration $a = 3g$ to the right, the pendulum will assume a steadystate deflection $\theta = \ldots$

See solution

Chapter 6, Problem 140P

The 8-kg crank OA, with mass center at G and radius of gyration about O of 0.22 m, is connected to the 12-kg uniform slender...

See solution

Videos related to Chapter 6

TEXTBOOK SOLUTION
Free Body Diagrams: Connected Rigid Bodies

TEXTBOOK SOLUTION
Center of Mass Calculation (Single Body)

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TEXTBOOK SOLUTION
Linear Algebra Checklist

Examples related to Chapter 6

Example 1

The pickup truck weighs 3220 lb and reaches a speed of 30 mi/hr from rest in a distance of 200 ft up the 10-percent incline with constant acceleration. Calculate the normal force under each pair of wheels and the friction force under the rear driving wheels. The effective coefficient of friction between the tires and the road is known to be at least 0.80.

Show more

Example 2

Example 3

Example 4

Example 5