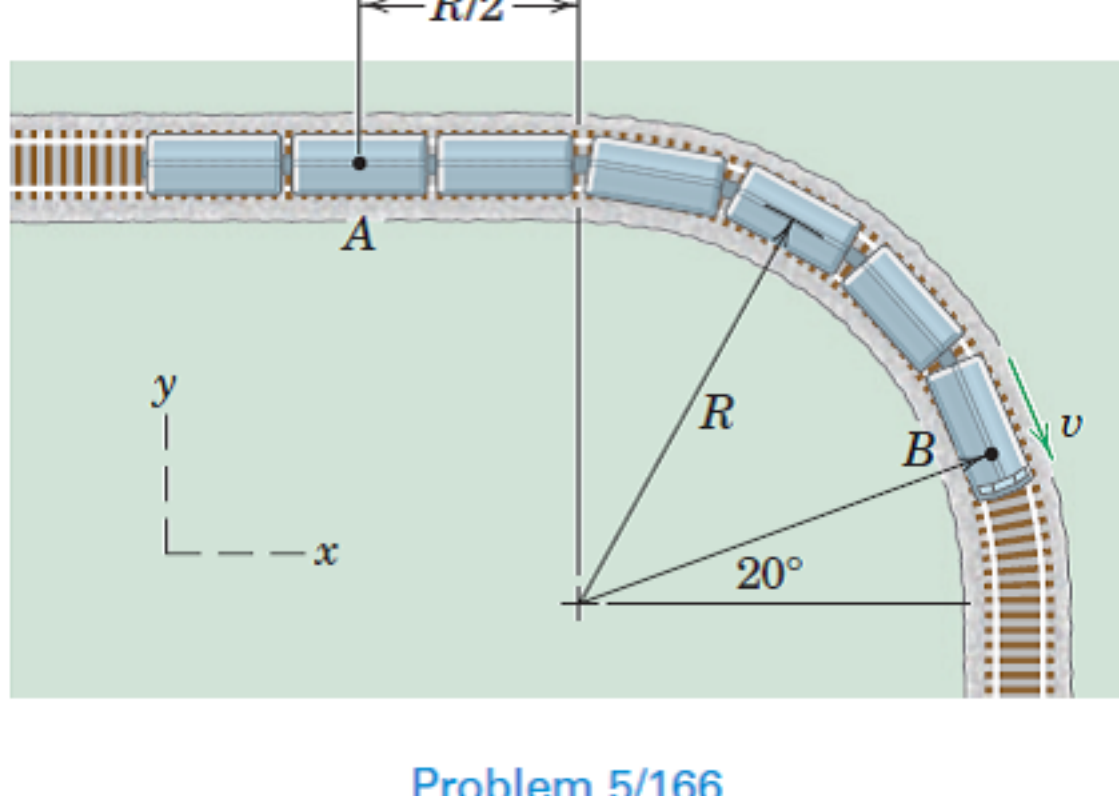


Engineering Mechanics | (8th Edition)

Problem

A train traveling at a constant speed $v = 25$ mi/hr has entered a circular portion of track with a radius $R = 200$ ft. Determine the velocity and acceleration of point A of the train as observed by the engineer B , who is fixed to the locomotive. Use the axes given in the figure.

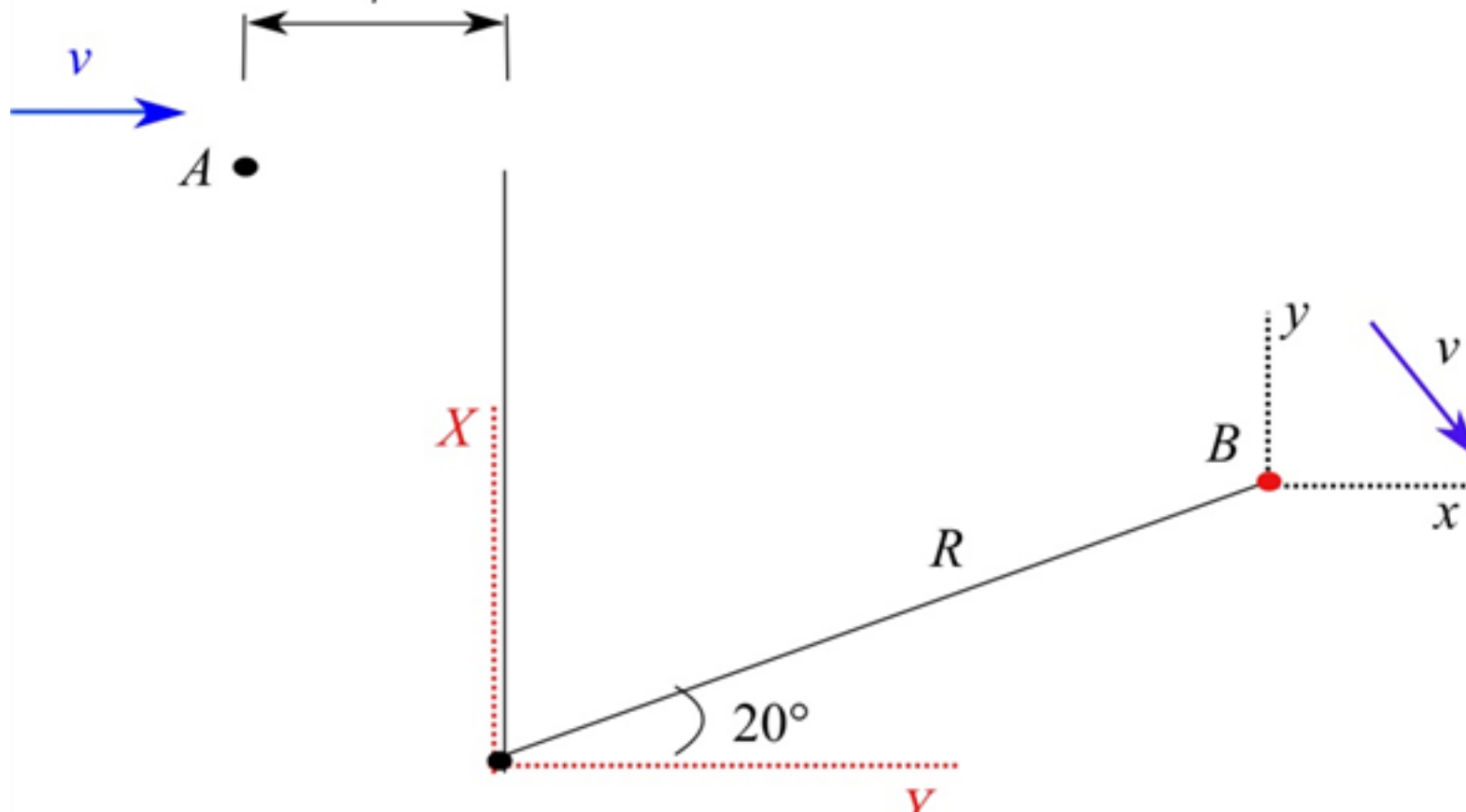


Problem 5/166

Step-by-step solution

Step 1 of 7

Draw the given schematic and label the fixed and rotating coordinate axes as follows.



Here, XY is the fixed coordinate axis, and xy is the rotating coordinate axis.

Comment

Step 2 of 7

Determine the velocity of the point A observed by the engineer at point B using the following formula:

$$\mathbf{v}_A = \mathbf{v}_B + \boldsymbol{\omega} \times \mathbf{r} + \mathbf{v}_{rel} \quad (1)$$

Here, \mathbf{v}_A is the absolute velocity of the point A with respect to origin O . \mathbf{v}_B is the absolute velocity of the point B with respect to origin O . $\boldsymbol{\omega}$ is the angular velocity of the locomotive, \mathbf{r} is the position vector of point A with respect to point B , and \mathbf{v}_{rel} is the velocity of the point A observed by the person at point B .

Convert the uniform velocity of the train from mi/hr to ft/s.

$$\begin{aligned} v &= 25 \text{ mi/hr} \\ &= 25 \frac{\text{mi}}{\text{hr}} \times \left| \frac{1609.344 \text{ m}}{1 \text{ mi}} \times \frac{3.280839 \text{ ft}}{1 \text{ m}} \times \frac{1 \text{ hr}}{3600 \text{ s}} \right| \\ &= 36.66 \text{ ft/s} \end{aligned}$$

Write the vector notation of the velocity of point A .

$$\begin{aligned} \mathbf{v}_A &= v \mathbf{i} + 0 \mathbf{j} \\ &= 36.66 \mathbf{i} \text{ ft/s} \end{aligned}$$

Write the vector notation of the velocity of point B .

$$\begin{aligned} \mathbf{v}_B &= v \sin 20^\circ \mathbf{i} - v \cos 20^\circ \mathbf{j} \\ &= 36.66 \sin 20^\circ \mathbf{i} - 36.66 \cos 20^\circ \mathbf{j} \\ &= 12.5384 \mathbf{i} - 34.449 \mathbf{j} \end{aligned}$$

Comment

Step 3 of 7

Write the vector notation of the angular velocity of the locomotive.

$$\boldsymbol{\omega} = -\frac{v}{R} \mathbf{k}$$

Write the position vector of point A with respect to point O .

$$\mathbf{r}_A = -\frac{R}{2} \mathbf{i} + R \mathbf{j}$$

Write the position vector of point B with respect to point O .

$$\mathbf{r}_B = R \cos 20^\circ \mathbf{i} + R \sin 20^\circ \mathbf{j}$$

Calculate the position vector of point A with respect to point B .

$$\begin{aligned} \mathbf{r} &= \mathbf{r}_{A/B} \\ &= \mathbf{r}_A - \mathbf{r}_B \\ &= -\frac{R}{2} \mathbf{i} + R \mathbf{j} - (R \cos 20^\circ \mathbf{i} + R \sin 20^\circ \mathbf{j}) \\ &= -\frac{R}{2} \mathbf{i} + R \mathbf{j} - R \cos 20^\circ \mathbf{i} - R \sin 20^\circ \mathbf{j} \end{aligned}$$

Comment

Step 4 of 7

Calculate the value of the $\boldsymbol{\omega} \times \mathbf{r}$.

$$\begin{aligned} \boldsymbol{\omega} \times \mathbf{r} &= \left(-\frac{v}{R} \mathbf{k} \right) \times \left(-\frac{R}{2} \mathbf{i} + R \mathbf{j} - R \cos 20^\circ \mathbf{i} - R \sin 20^\circ \mathbf{j} \right) \\ &= 0.5 v \mathbf{i} + v \mathbf{j} + v \cos 20^\circ \mathbf{j} - v \sin 20^\circ \mathbf{i} \\ &= (1 - \sin 20^\circ) v \mathbf{i} + (0.5 + \cos 20^\circ) v \mathbf{j} \end{aligned}$$

Substitute 36.66 ft/s for v .

$$\begin{aligned} \boldsymbol{\omega} \times \mathbf{r} &= (1 - \sin 20^\circ) v \mathbf{i} + (0.5 + \cos 20^\circ) v \mathbf{j} \\ &= (1 - \sin 20^\circ) \times 36.66 \mathbf{i} + (0.5 + \cos 20^\circ) \times 36.66 \mathbf{j} \\ &= 24.12 \mathbf{i} + 52.78 \mathbf{j} \end{aligned}$$

Substitute 36.66 ft/s for \mathbf{v}_A , $(12.5384 \mathbf{i} - 34.449 \mathbf{j})$ for \mathbf{v}_B , and $(24.12 \mathbf{i} + 52.78 \mathbf{j})$ for $\boldsymbol{\omega} \times \mathbf{r}$ in the equation (1).

$$\begin{aligned} \mathbf{v}_A &= \mathbf{v}_B + \boldsymbol{\omega} \times \mathbf{r} + \mathbf{v}_{rel} \\ 36.66 \mathbf{i} &= 12.5384 \mathbf{i} - 34.449 \mathbf{j} + (24.12 \mathbf{i} + 52.78 \mathbf{j}) + \mathbf{v}_{rel} \end{aligned}$$

$$\mathbf{v}_{rel} = -18.33 \mathbf{j} \text{ ft/s}$$

Therefore, the velocity of the point A observed by the engineer at point B is $-18.33 \mathbf{j} \text{ ft/s}$.

Comment

Step 5 of 7

Determine the acceleration of the point A observed by the engineer at point B using the following formula:

$$\mathbf{a}_A = \mathbf{a}_B + \boldsymbol{\omega} \times \mathbf{r} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}) + 2\boldsymbol{\omega} \times \mathbf{v}_{rel} + \mathbf{a}_{rel} \quad (2)$$

Here, \mathbf{a}_A is the absolute acceleration of the point A with respect to origin O . \mathbf{a}_B is the absolute acceleration of the point B with respect to origin O . $\boldsymbol{\omega}$ is the angular velocity of the locomotive (rotating axis), \mathbf{r} is the position vector of point A with respect to point B , and \mathbf{a}_{rel} is the acceleration of the point A observed by the engineer at point B .

Write the vector notation of the acceleration of the point A .

$$\begin{aligned} \mathbf{a}_A &= 0 \mathbf{i} + 0 \mathbf{j} \\ &= 0 \text{ ft/s}^2 \end{aligned}$$

Write the vector notation of the acceleration of the point B .

$$\begin{aligned} \mathbf{a}_B &= \mathbf{a}_A + \mathbf{a}_1 \\ &= -\frac{v^2}{R} \cos 20^\circ \mathbf{i} - \frac{v^2}{R} \sin 20^\circ \mathbf{j} + \frac{dv}{dt} \sin 20^\circ \mathbf{i} - \frac{dv}{dt} \cos 20^\circ \mathbf{j} \end{aligned}$$

Substitute 36.66 ft/s for v , and 200 ft for R .

$$\begin{aligned} \mathbf{a}_B &= -\frac{v^2}{R} \cos 20^\circ \mathbf{i} - \frac{v^2}{R} \sin 20^\circ \mathbf{j} + \frac{dv}{dt} \sin 20^\circ \mathbf{i} - \frac{dv}{dt} \cos 20^\circ \mathbf{j} \\ &= -\frac{(36.66)^2}{200} \cos 20^\circ \mathbf{i} - \frac{(36.66)^2}{200} \sin 20^\circ \mathbf{j} + \frac{d36.66}{dt} \sin 20^\circ \mathbf{i} - \frac{d36.66}{dt} \cos 20^\circ \mathbf{j} \\ &= -6.314 \mathbf{i} - 2.298 \mathbf{j} \text{ ft/s}^2 \end{aligned}$$

Comment

Step 6 of 7

Calculate the value of the $\boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r})$.

$$\begin{aligned} \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}) &= -\frac{v}{R} \mathbf{k} \times (24.12 \mathbf{i} + 52.78 \mathbf{j}) \\ &= -\frac{36.66}{200} \mathbf{k} \times (24.12 \mathbf{i} + 52.78 \mathbf{j}) \\ &= -4.42 \mathbf{j} + 9.67 \mathbf{i} \\ &= 9.67 \mathbf{i} - 4.42 \mathbf{j} \end{aligned}$$

Calculate the value of $2\boldsymbol{\omega} \times \mathbf{v}_{rel}$.

$$\begin{aligned} 2\boldsymbol{\omega} \times \mathbf{v}_{rel} &= 2 \left(-\frac{v}{R} \mathbf{k} \right) \times -18.33 \mathbf{j} \\ &= 2 \left(-\frac{36.66}{200} \mathbf{k} \right) \times -18.33 \mathbf{j} \\ &= -6.72 \mathbf{i} \text{ ft/s}^2 \end{aligned}$$

Comment

Step 7 of 7

Calculate the value of the angular velocity of the locomotive, $\boldsymbol{\omega}$.

$$\boldsymbol{\omega} = \frac{d\boldsymbol{\omega}}{dt}$$

Substitute $\left(-\frac{v}{R} \mathbf{k} \right)$ 36.66 ft/s for v .

$$\begin{aligned} \boldsymbol{\omega} &= \frac{d \left(-\frac{v}{R} \mathbf{k} \right)}{dt} \\ &= \frac{d \left(-\frac{36.66}{200} \mathbf{k} \right)}{dt} \\ &= 0 \text{ rad/s}^2 \end{aligned}$$

Substitute 0 ft/s for \mathbf{a}_A , $(-6.314 \mathbf{i} - 2.298 \mathbf{j})$ for \mathbf{a}_B , 0 rad/s² for $\boldsymbol{\omega}$, $(9.67 \mathbf{i} - 4.42 \mathbf{j})$ for $\boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r})$, and $-6.72 \mathbf{i} \text{ ft/s}^2$ for $2\boldsymbol{\omega} \times \mathbf{v}_{rel}$ in the equation (2).

$$\begin{aligned} \mathbf{a}_A &= \mathbf{a}_B + \boldsymbol{\omega} \times \mathbf{r} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}) + 2\boldsymbol{\omega} \times \mathbf{v}_{rel} + \mathbf{a}_{rel} \\ 0 &= (-6.314 \mathbf{i} - 2.298 \mathbf{j}) + 0 + (9.67 \mathbf{i} - 4.42 \mathbf{j}) + (-6.72 \mathbf{i}) + \mathbf{a}_{rel} \end{aligned}$$

$$\mathbf{a}_{rel} = 3.36 \mathbf{i} + 6.72 \mathbf{j} \text{ ft/s}^2$$

Therefore, the acceleration of the point A observed by the engineer at point B is $3.36 \mathbf{i} + 6.72 \mathbf{j} \text{ ft/s}^2$.

Comment

Related videos

Related examples

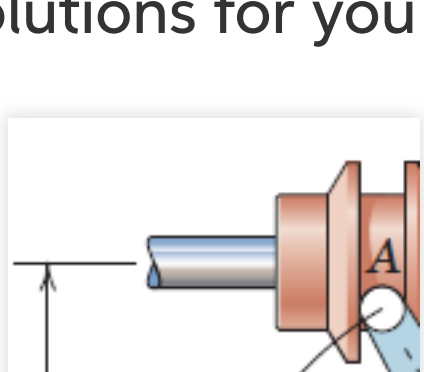
Was this solution helpful?

Recommended solutions for you in Chapter 5

Chapter 5, Problem 193P

The pin A in the bell crank AOD is guided by the flanges of the collar B, which slides with a constant velocity v_B of 3 ft/sec...

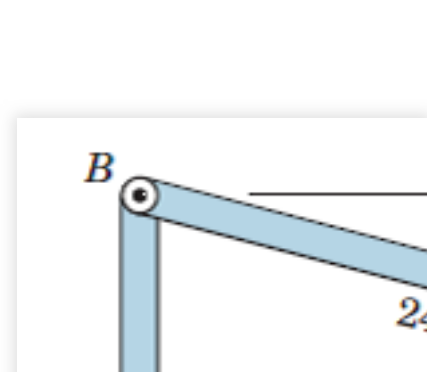
See solution



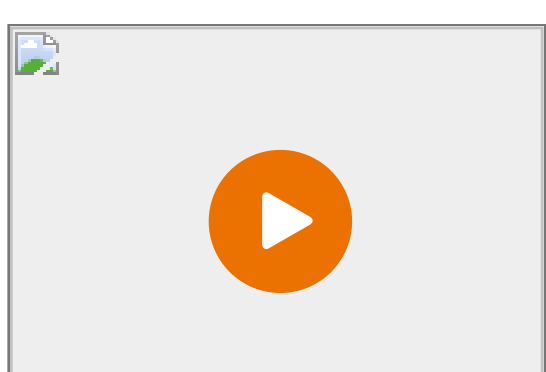
Chapter 5, Problem 86P

A four-bar linkage is shown in the figure (the ground link OC is considered the fourth bar). If the drive link OA has a...

See solution



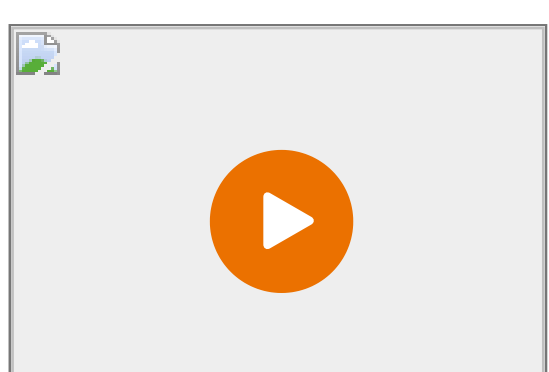
Videos related to Chapter 5



TEXTBOOK SOLUTION

Chain Rule

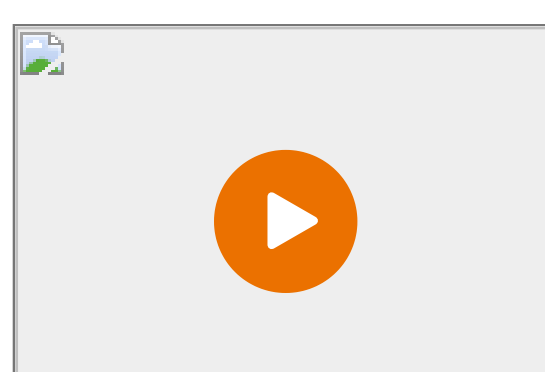
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TEXTBOOK SOLUTION

Matrix Multiplication

👍 0 👎 1



TEXTBOOK SOLUTION

Linear Algebra Checklist

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Examples related to Chapter 5

Example 1

A flywheel rotating freely at 1800 rev/min clockwise is subjected to a variable counterclockwise torque which is first applied at time $t = 0$. The torque produces a counterclockwise angular acceleration $\alpha = 4 \text{ rad/s}^2$, where t is the time in seconds during which the torque is applied. Determine (a) the time required for the flywheel to reduce its clockwise angular speed to 900 rev/min, (b) the time required for the flywheel to reverse its direction of rotation, and (c) the total number of revolutions, clockwise plus counterclockwise, turned by the flywheel during the first 14 seconds of torque application.

Show more

Example 2

Example 3

Example 4

Example 5

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