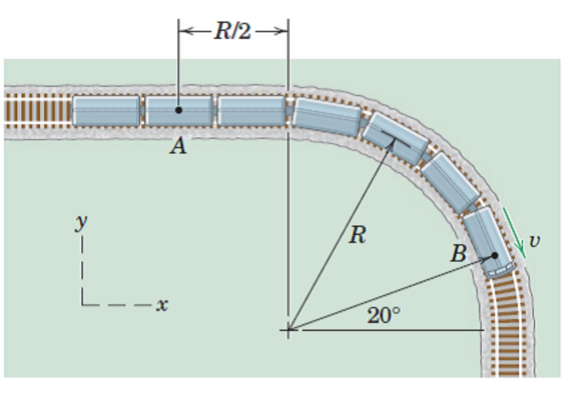
Engineering Mechanics | (8th Edition)

Problem

A train traveling at a constant speed v = 25 mi /hr has entered a circular portion of track with a radius R = 200 ft. Determine the velocity and acceleration of point A of the train as observed by the engineer B, who is fixed to the locomotive. Use the axes given in the figure.

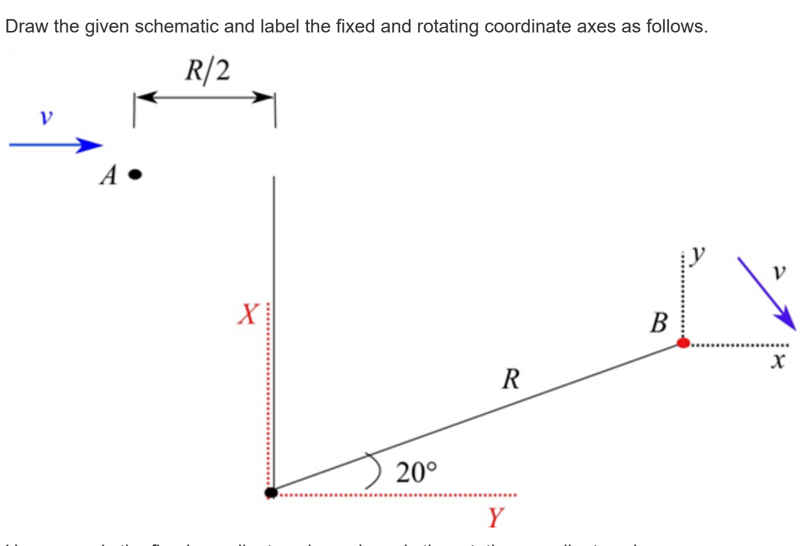


Problem 5/166

Step-by-step solution

Step 1 of 7

Draw the given schematic and label the fixed and rotating coordinate axes as follows.



Here, XY is the fixed coordinate axis, and xy is the rotating coordinate axis.

Comment

Step 2 of 7

Determine the velocity of the point A observed by the engineer at point B using the following formula:

$\mathbf{v}_A = \mathbf{v}_B + \boldsymbol{\omega} \times \mathbf{r} + \mathbf{v}_{rel}$ ...... (1)

Here, $\mathbf{v}_A$ is the absolute velocity of the point A with respect to origin O, $\mathbf{v}_B$ is the absolute velocity of the point B with respect to origin O, $\boldsymbol{\omega}$ is the angular velocity of the locomotive, $\mathbf{r}$ is the position vector of point A with respect to point B, and $\mathbf{v}_{rel}$ is the velocity of the point A observed by the person at point B.

Convert the uniform velocity of the train from $\mathrm{mi/hr}$ to $\mathrm{ft/s}$.

$$\begin{aligned} v &= 25\,\mathrm{mi/hr} \\ &= 25\,\frac{\mathrm{mi}}{\mathrm{hr}} \times \left| \frac{1609.344\ \mathrm{m}}{1\ \mathrm{mi}} \times \frac{3.280839\ \mathrm{ft}}{1\ \mathrm{m}} \times \frac{1\ \mathrm{hr}}{3600\ \mathrm{s}} \right| \\ &= 36.66\ \mathrm{ft/s} \end{aligned}$$

Write the vector notation of the velocity of point A.

$$\begin{aligned} \mathbf{v}_A &= v\ \mathbf{i} + 0\ \mathbf{j} \\ &= 36.66\ \mathbf{i}\ \mathrm{ft/s} \end{aligned}$$

Write the vector notation of the velocity of point B.

$$\begin{aligned} \mathbf{v}_B &= v \sin 20°\ \mathbf{i} - v \cos 20°\ \mathbf{j} \\ &= 36.66 \sin 20°\ \mathbf{i} - 36.66 \cos 20°\ \mathbf{j} \\ &= 12.5384\ \mathbf{i} - 34.449\ \mathbf{j} \end{aligned}$$

Comment

Step 3 of 7

Write the vector notation of the angular velocity of the locomotive.

$$\boldsymbol{\omega} \begin{aligned} &= -\omega\ \mathbf{k} \\ &= -\frac{v}{R}\ \mathbf{k} \end{aligned}$$

Write the position vector of point A with respect to point O.

$$\mathbf{r}_A = -\frac{R}{2}\mathbf{i} + R\,\mathbf{j}$$

Write the position vector of point B with respect to point O.

$$\mathbf{r}_B = R\cos 20°\,\mathbf{i} + R\sin 20°\,\mathbf{j}$$

Calculate the position vector of point A with respect to point B.

$$\begin{aligned} \mathbf{r} &= \mathbf{r}_{A/B} \\ &= \mathbf{r}_A - \mathbf{r}_B \\ &= -\frac{R}{2}\mathbf{i} + R\mathbf{j} - (R\cos 20°\mathbf{i} + R\sin 20°\mathbf{j}) \\ &= -\frac{R}{2}\mathbf{i} + R\mathbf{j} - R\cos 20°\mathbf{i} - R\sin 20°\mathbf{j} \end{aligned}$$

Comment

Step 4 of 7

Calculate the value of the $\boldsymbol{\omega} \times \mathbf{r}$.

$$\begin{aligned} \boldsymbol{\omega} \times \mathbf{r} &= \left(-\frac{v}{R}\ \mathbf{k}\right) \times \left(-\frac{R}{2}\mathbf{i} + R\,\mathbf{j} - R\cos 20°\mathbf{i} - R\sin 20°\mathbf{j}\right) \\ &= 0.5v\,\mathbf{j} + v\,\mathbf{i} + v\cos 20°\mathbf{j} - v\sin 20°\mathbf{i} \\ &= (1 - \sin 20°)v\,\mathbf{i} + (0.5 + \cos 20°)v\,\mathbf{j} \end{aligned}$$

Substitute $36.66\ \mathrm{ft/s}$ for v.

$$\begin{aligned} \boldsymbol{\omega} \times \mathbf{r} &= (1 - \sin 20°)v\,\mathbf{i} + (0.5 + \cos 20°)v\,\mathbf{j} \\ &= (1 - \sin 20°) \times 36.66\mathbf{i} + (0.5 + \cos 20°) \times 36.66\mathbf{j} \\ &= 24.12\ \mathbf{i} + 52.78\mathbf{j} \end{aligned}$$

Substitute $36.66\ \mathbf{i}\ \mathrm{ft/s}$ for $\mathbf{v}_A$, $(12.5384\ \mathbf{i} - 34.449\ \mathbf{j})$ for $\mathbf{v}_B$, and $(24.12\ \mathbf{i} + 52.78\mathbf{j})$ for $\boldsymbol{\omega} \times \mathbf{r}$ in the equation (1).

$$\begin{aligned} &\mathbf{v}_A = \mathbf{v}_B + \boldsymbol{\omega} \times \mathbf{r} + \mathbf{v}_{rel} \\ &36.66\ \mathbf{i} = 12.5384\ \mathbf{i} - 34.449\ \mathbf{j} + (24.12\ \mathbf{i} + 52.78\mathbf{j}) + \mathbf{v}_{rel} \\ &\mathbf{v}_{rel} = -18.33\ \mathbf{j}\ \ \mathrm{ft/s} \end{aligned}$$

Therefore, the velocity of the point A observed by the engineer at point B is $\boxed{-18.33\ \mathbf{j}\ \mathrm{ft/s}}$.

Comment

Step 5 of 7

Determine the acceleration of the point A observed by the engineer at point B using the following formula:

$\mathbf{a}_A = \mathbf{a}_B + \dot{\boldsymbol{\omega}} \times \mathbf{r} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}) + 2\boldsymbol{\omega} \times \mathbf{v}_{rel} + \mathbf{a}_{rel}$ ...... (2)

Here, $\mathbf{a}_A$ is the absolute acceleration of the point A with respect to origin O, $\mathbf{a}_B$ is the absolute acceleration of the point B with respect to origin O, $\dot{\boldsymbol{\omega}}$ is the angular acceleration of the locomotive (rotating axis), $\mathbf{r}$ is the position vector of point A with respect to point B, and $\mathbf{a}_{rel}$ is the acceleration of the point A observed by the engineer at point B.

Write the vector notation of the acceleration of the point A.

$$\begin{aligned} \mathbf{a}_A &= 0\ \mathbf{i} + 0\ \mathbf{j} \\ &= 0\ \mathrm{ft^2/s} \end{aligned}$$

Write the vector notation of the acceleration of the point B.

$$\begin{aligned} \mathbf{a}_B &= \mathbf{a}_r + \mathbf{a}_t \\ &= -\frac{v^2}{R}\cos 20°\ \mathbf{i} - \frac{v^2}{R}\sin 20°\ \mathbf{j} + \frac{dv}{dt}\sin 20°\ \mathbf{i} - \frac{dv}{dt}\cos 20°\mathbf{j} \end{aligned}$$

Substitute $36.66\ \mathrm{ft/s}$ for v, and $200\ \mathrm{ft}$ for R.

$$\begin{aligned} \mathbf{a}_B &= -\frac{v^2}{R}\cos 20°\ \mathbf{i} - \frac{v^2}{R}\sin 20°\ \mathbf{j} + \frac{dv}{dt}\sin 20°\ \mathbf{i} - \frac{dv}{dt}\cos 20°\mathbf{j} \\ &= -\frac{(36.66)^2}{200}\cos 20°\ \mathbf{i} - \frac{(36.66)^2}{200}\sin 20°\ \mathbf{j} + \frac{d\,36.66}{dt}\sin 20°\ \mathbf{i} - \frac{d\,36.66}{dt}\cos 20°\mathbf{j} \\ &= -\ 6.314\ \mathbf{i} - 2.298\ \mathbf{j}\ \mathrm{ft^2/s} \end{aligned}$$

Comment

Step 6 of 7

Calculate the value of the $\boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r})$.

$$\begin{aligned} \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}) &= -\frac{v}{R}\ \mathbf{k} \times (24.12\ \mathbf{i} + 52.78\mathbf{j}) \\ &= -\frac{36.66}{200}\ \mathbf{k} \times (24.12\ \mathbf{i} + 52.78\mathbf{j}) \\ &= -\ 4.42\ \mathbf{j} + 9.67\ \mathbf{i} \\ &= 9.67\ \mathbf{i} -\ 4.42\ \mathbf{j} \end{aligned}$$

Calculate the value of $2\boldsymbol{\omega} \times \mathbf{v}_{rel}$.

$$\begin{aligned} 2\boldsymbol{\omega} \times \mathbf{v}_{rel} &= 2\left(-\frac{v}{R}\ \mathbf{k}\right) \times -18.33\,\mathbf{j} \\ &= 2\left(-\frac{36.66}{200}\ \mathbf{k}\right) \times -18.33\,\mathbf{j} \\ &= -6.72\ \mathbf{i}\ \mathrm{ft^2/s} \end{aligned}$$

Comment

Step 7 of 7

Calculate the value of the angular velocity of the locomotive, $\dot{\boldsymbol{\omega}}$.

$$\dot{\boldsymbol{\omega}} = \frac{d\boldsymbol{\omega}}{dt}$$

Substitute $\left(-\frac{v}{R}\ \mathbf{k}\right)$ $36.66\ \mathrm{ft/s}$ for v.

$$\begin{aligned} \dot{\boldsymbol{\omega}} &= \frac{d\left(-\frac{v}{R}\ \mathbf{k}\right)}{dt} \\ &= \frac{d\left(-\frac{36.66}{200}\ \mathbf{k}\right)}{dt} \\ &= 0\ \mathrm{rad/s^2} \end{aligned}$$

Substitute $0\ \mathrm{ft^2/s}$ for $\mathbf{a}_A$, $(-6.314\ \mathbf{i} - 2.298\ \mathbf{j})$ for $\mathbf{a}_B$, $0\ \mathrm{rad/s^2}$ for $\dot{\boldsymbol{\omega}}$, $(9.67\ \mathbf{i} - 4.42\ \mathbf{j})$ for $\boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r})$, and $-6.72\ \mathbf{i}\ \mathrm{ft^2/s}$ for $2\boldsymbol{\omega} \times \mathbf{v}_{rel}$ in the equation (2).

$$\begin{aligned} &\mathbf{a}_A = \mathbf{a}_B + \dot{\boldsymbol{\omega}} \times \mathbf{r} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}) + 2\boldsymbol{\omega} \times \mathbf{v}_{rel} + \mathbf{a}_{rel} \\ &0 = (-6.314\ \mathbf{i} - 2.298\ \mathbf{j}) + (9.67\ \mathbf{i} - 4.42\ \mathbf{j}) + (-6.72\ \mathbf{i}) + \mathbf{a}_{rel} \\ &\mathbf{a}_{rel} = 3.36\ \mathbf{i} + 6.72\ \mathbf{j}\ \mathrm{ft^2/s} \end{aligned}$$

Therefore, the acceleration of the point A observed by the engineer at point B is $\boxed{3.36\ \mathbf{i} + 6.72\ \mathbf{j}\ \mathrm{ft^2/s}}$.

Comment