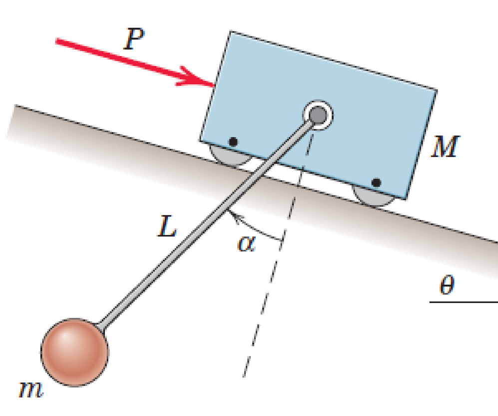
Engineering Mechanics | (8th Edition)

Problem

Determine the steady-state angle α if the constant force P is applied to the cart of mass M. The pendulum bob has mass m and the rigid bar of length L has negligible mass. Ignore all friction. Evaluate your expression for $P = 0$.



Problem 3/10

Walkthrough for problem
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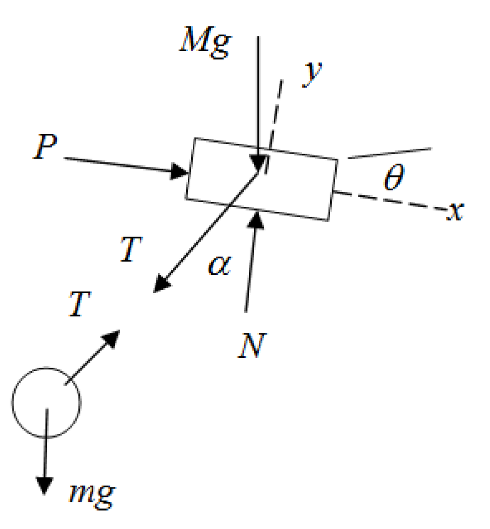
Step-by-step solution

Step 1 of 5

Draw the free body diagram of the system.



$mg \quad Mg \quad P \quad y \quad x \quad \theta \quad \alpha \quad T \quad N$

Comment

Step 2 of 5

Consider the equilibrium of forces acting on the bob in *x*-direction using free body diagram.

$$\sum F_x = 0$$
$$T\sin\alpha + mg\sin\theta = ma_x \quad \ldots\ldots (1)$$

Here, T is the tension in the rod, m is the mass of the bob, *g* is the acceleration due to gravity, a_x is the acceleration of the bob, θ is the angle made with the horizontal axis, α is the angle made with the vertical axis.

Consider the equilibrium of forces acting on the bob in *y*-direction using free body diagram.

$$\sum F_y = 0$$
$$T\cos\alpha - mg\cos\theta = 0 \quad \ldots\ldots (2)$$

Comment

Step 3 of 5

Consider the equilibrium of forces acting on the entire system in *x*-direction using free body diagram.

$$\sum F_x = ma_x$$
$$P + (M+m)g\sin\theta = (m+M)a_x$$
$$a_x = \frac{P + (M+m)g\sin\theta}{(m+M)}$$

Here, *M* is the mass of the cart, P is the force acting on the cart.

Substitute $\left[\frac{P+(M+m)g\sin\theta}{(m+M)}\right]$ for a_x in equation (1).

$$T\sin\alpha + mg\sin\theta = ma_x$$
$$T\sin\alpha + mg\sin\theta = m\left[\frac{P+(M+m)g\sin\theta}{(m+M)}\right]$$
$$T = m\left[\frac{P+(M+m)g\sin\theta}{(m+M)\sin\alpha}\right] - \frac{mg\sin\theta}{\sin\alpha}$$

Comment

Step 4 of 5

Calculate the angle α made by the bar with the vertical using the equation (2).

$$T\cos\alpha - mg\cos\theta = 0$$

Substitute $\left(m\left[\frac{P+(M+m)g\sin\theta}{(m+M)\sin\alpha}\right] - \frac{mg\sin\theta}{\sin\alpha}\right)$ for *T*.

$$\left(\left[\frac{P+(M+m)g\sin\theta}{(m+M)\sin\alpha}\right] - \frac{g\sin\theta}{\sin\alpha}\right)\cos\alpha - g\cos\theta = 0$$
$$\left(\left[\frac{P\sin\alpha + (M+m)g\sin\alpha\sin\theta - (M+m)g\sin\alpha\sin\theta}{(m+M)\sin^2\alpha}\right]\right)\cos\alpha - g\cos\theta = 0$$
$$\left(\left[\frac{P\sin\alpha}{(m+M)\sin^2\alpha}\right]\right)\cos\alpha - g\cos\theta = 0$$
$$\left[\frac{P\sin\alpha\cos\alpha}{(m+M)\sin^2\alpha}\right] - g\cos\theta = 0$$
$$P\cos\alpha = g\cos\theta(m+M)\sin\alpha$$
$$\tan\alpha = \frac{P}{g\cos\theta(m+M)}$$

Therefore, the angle made by the bar with the vertical is $\boxed{\tan^{-1}\left(\frac{P}{g\cos\theta(m+M)}\right)}$.

Comment

Step 5 of 5

Calculate the angle α when $P = 0$.

$$\tan\alpha = \frac{P}{g\cos\theta(m+M)}$$

Substitute 0 for *P*.

$$\alpha = \tan^{-1}\left(\frac{0}{g\cos\theta(m+M)}\right)$$
$$\alpha = 0°$$

Therefore, the angle made by the bar with the vertical when $P = 0$ is $\boxed{0°}$.

Comment

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